George Hall Bath 19th November 1748

Philosophia Britannica:

Henry Somerville

OR

A NEW and COMPREHENSIVE

SYSTEM

OF THE

Newtonian PHILOSOPHY,

ASTRONOMY and GEOGRAPHY.

IN A

COURSE of Twelve LECTURES,

With NOTES,

CONTAINING

The PHYSICAL, MECHANICAL, GEOMETRICAL, and EXPERIMENTAL PROOFS and ILLUSTRATIONS of all the Principal Propositions in every Branch of

NATURAL SCIENCE.

ALSO

A particular Account of the INVENTION, STRUCTURE, IMPROVEMENT and USES of all the considerable INSTRUMENTS, ENGINES, and MACHINES,

With *new Calculations* relating to their

NATURE, POWER, and OPERATION.

The Whole collected and methodized from all the principal Authors, and public Memoirs to the present Year;

And embellish'd with Seventy-five COPPER-PLATES.

By *B. MARTIN.*

Quæ toties Animos veterum torsêre Sophorum
Quæque Scholas frustra rauco Certamine vexant,
Obvia conspicimus, Nubem pellente Mathesi. HAL. in NEWT. Prin.

VOL. II.

READING,

Printed by C. MICKLEWRIGHT and C°. for the AUTHOR; and for M. COOPER in *Pater-noster-row, London*; R. RAIKES at *Gloucester*; B. COLLINS at *Salisbury*; and J. LEAKE, and W. FREDERICK at *Bath*. MDCCXLVII.

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TO THE

RIGHT HONOURABLE

JOHN,

Earl of *ORRERY.*

My LORD,

I BEG Leave to make YOUR LORDSHIP the humble Offering of thoſe LECTURES you have heretofore been pleaſed to approve and honour with your Preſence. YOUR LORDSHIP will not diſdain to caſt an

Eye on this Endeavour to diſplay ſome of that *wondrous Worth* and *matchleſs Sagacity* which has ſo much ennobled Human Nature, and dignified it with almoſt Divinity itſelf. I doubt not but ſome Sparks of that *Celeſtial Fire*, which inform'd the Soul of *NEWTON*, will ſtrike upon and mingle with the like congeneal Flame that glows in YOUR LORDSHIP's Breaſt.

THE Doctrine of SOUNDS preſents us with the *Philoſophic Grounds* of MUSIC and HARMONY: The Colours of Light, in regard to Quantity, obſerve the *Harmonic Law*; not a ſingle Ray can be reflected, but by the ſame *Divine Rule*: And there is ſomething extremely like *Muſic* in the Motions and Order of the *Spheres*. Conſonant, therefore, it is with the higheſt Reaſon, that theſe

Har-

Harmonious Subjects ſhould be recommended to the World under the AUSPICE of One ſo well known by *tuneful Accents*, and Skill *to ſtrike the Lyre*.

AGAIN, MY LORD, Does *Philoſophy* breathe *Religion* and *Devotion*, and furniſh us with the beſt Weapons againſt Vice and Immorality? Then let it be ſanction'd by the diſtinguiſh'd Name of that BOYLE, who in the early Years of Life ſet ſuch an illuſtrious Example of *Magnanimity* and *Chriſtian Heroiſm*, in the following *Reſolution*, as truly *pious*, as the Numbers are *poetical* in which it flows.

Mature in Years, if e'er I chance to tread
Where VICE *triumphant rears aloſt her Head*,
Ev'n there the Paths of VIRTUE I'll purſue.

And the Glory of making it good is every Day YOUR LORDSHIP'S Due.

But I dare not longer insist on Themes of Praise, (though ever so pleasing) to a Mind possess'd and enrich'd with every Virtue, as well as *native Innocence*.

EVERY thing, MY LORD, may be over-valued, and become the Subject of Flattery, except *Goodness* and *Wisdom*. The *Encomiums* of the Great, the Wise, and the Good, admit of no *Hyperbole*; these *shining Topics* ought to plead Excuse for prolix Admiration wherever we behold them, especially in an Age so little productive of such *Phænomena*. So COMETS, when they appear, set all Mankind a gazing; and such unusual Splendor detains our Eyes all the Time of their Appearance.

THE general Design of this Treatise being to facilitate the Way to real

real Science, will, I hope, render it ſo much the more acceptable to YOUR LORDSHIP, in regard of *My Young Lord* BOYLE, who bids ſo fair to deſerve and continue the Paternal Honours and Title of a Family that will always be had in Renown, while the Records of *Engliſh* Nobility, and the Annals of Heaven ſhall laſt. I am,

MY LORD,

With Reſpect and Duty,

YOUR LORDSHIP'S

Moſt Obedient Servant,

B. MARTIN.

ERRATA *in* Vol. I.

Page.	*Line.*	
101	25,	*for* $\frac{2}{5}$, *read* $\frac{3}{5}$.
104	18,	*for* 40 = 1, *read* 40 x 1.
206	36,	*read* Pl. XV. Fig. 15. *in the Margin.*
240	30,	*after* through, *read* the abovemention'd Space,
304	20,	*for* Weight, *read* Height,

ERRATA *in* Vol. II.

Page.	*Line.*	
23	30,	*after* Plate XXIX. *add* Fig. 5.
27	32,	*write* Plate XXIX. Fig. 7. *in the Margin.*
35	20,	*for* 1 to 12, *read* 1 to *n*.
61	22,	*for* 26, *read* 16.
82	14,	*write* Plate XXXIII. Fig. 2. *in the Margin.*
302	3,	*for* Liberklum, *read* Liberkhun.
448	11,	*for* Simson, *read* Simpson.

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LECTURE VI.

Of PNEUMATICS; *or Doctrine of the* AIR, *or* ATMOSPHERE *in general. Of* Artificial *or* Factitious AIR; *the* great Quantity *thereof in* Natural Bodies; *various Experiments relating thereto. Of the* Weight *of the* AIR; *of the Nature of the* BAROMETER *for estimating the same; an Account of the several Kinds*, viz. *the* Perpendicular, Diagonal, Horizontal, Pendent, Wheel, *and* Water BAROMETERS. *The best Way of* making *the* Common Barometer. *The Nature and Use of the* NONIUS, *applied thereto, explain'd. The Use of the Barometer* in measuring *the* Heights of Mountains, *&c. The* SPRING *or* ELASTICITY *of the Air accounted for, and explain'd. The Nature of the* SEA-GAGE *explain'd. The* ALTITUDE *of the* ATMOSPHERE *determined. The Art of* SAILING *in the* AIR *proved impossible. The* ABSOLUTE WEIGHT *of the Air determin'd by Experiment. Its* variable Pressure *on* HUMAN BODIES; *the* Quantity *thereof computed. An Account of fifty Experiments of the* AIR-PUMP *relating to the Weight, Spring, and other Properties of the* Air. *A particular Description of the* Air-Pump; *an* AIR-PUMP *of a* new Invention. *The* DIVING-BELL *explain'd. The* Nature *and* Use *of* THERMOMETERS *explain'd. The* New-

tonian STANDARD THERMOMETER. Farenheit's *new* Mercurial Thermometer *explain'd*. HYGROMETERS *of several Sorts explain'd. The Common* Air-Gun *explain'd. The* MAGAZINE AIR-GUN *particularly described.*

THAT Part of *Natural Philosophy* which treats of the *Nature, Properties, and Effects of the* ATMOSPHERE, or *Body of Air* encompassing the *Earth*, is call'd PNEUMATICS, from the *Greek* Word for *Wind* or *Breath*.

THE *Air* is generally esteemed a *Fluid*, but yet differs from the general Nature of Fluids in three Particulars, *viz.* (1.) In that it is *compressible*, which Property no other Fluid has. (2.) It cannot be *congeal'd*, or any how *fixed*, as all other Fluids may. (3.) It is of a *different Density* in every Part, decreasing from the *Earth*'s Surface upwards; whereas other Fluids are of an uniform Density throughout. The Air is therefore a Fluid *sui generis*, if it be properly any Fluid at all (LXXXV).

(LXXXV.) What is here said of the incongealable Quality of the Air, relates to the Impossibility of changing it from a *Fluid to a Fixed State* by Cold, as Water is congeal'd or converted into Ice; and melted Metals are brought to their fixed State: And in this particular limited Sense, the Air is *incongealable*, or uncapable of Fixation. But yet it is not absolutely so; for we find by various Experiments, that Air has a fixed State in the Composition of natural Bodies, from which when set at Liberty, it becomes a fluid elastic Air, like the common Air; and this again, from a *Fluid*, may be reduced to a *fix'd State* in Composition with other Matter, tho' not *per se*; for we know, as yet, of no fix'd Body consisting entirely of Air.

THAT

THAT the Air was created at first with the Earth itself, is not to be doubted; and that ever since there has been a *constant Generation* of Particles of Air by the mutual Action of Bodies upon each other, as in *Fermentations*, and all Kinds of *natural* and *artificial Chemistry*, Sir *Isaac Newton* thinks very reasonable to suppose; and Mr. *Boyle* has given numerous Experiments relating to the Production of *artificial* or *factitious Air*. (LXXXVI).

(LXXXVI.) Since Air is absolutely necessary for the Life of Man, and most Animals, yea, and Vegetables too, it was necessary at the first Formation of the Earth to render it a Habitation for Animals, and a proper Matrix for the Production of Plants. Now since there is a constant Generation of Air from all terrestrial Substances (as we shall shew by and by) it follows, that the original Atmosphere must be always increasing in Quantity and Bulk, unless we suppose all that is generated is again absorb'd or refix'd in the Substance of Bodies. And this alternate Transmutation of the State of Air is extremely manifest from numberless Experiments which have been made by Mr. *Boyle*, and Dr. *Hales*, of which I shall here give an Account of some of the principal of both Kinds, as follows.

2. The Production of artificial or factitious Air is caused either (1.) by slow Degrees from Putrefactions and Fermentations of all Kinds; or (2.) more expeditiously by some Sorts of chymical Dissolutions of Bodies; or (3.) and lastly, almost instantaneously by the Explosion of Gunpowder, and the Mixture of some Kinds of Bodies. Thus, if Paste or Dough with Leaven be placed in an exhausted Receiver, it will, after some Time, by Fermentation, produce a considerable Quantity of Air, which will appear very plainly by the Sinking the Quicksilver in the Gage. Thus also any Animal or Vegetable Substance, putrifying *in Vacuo*, will produce the same Effect.

3. Gunpowder, fired *in Vacuo*, instantly generates a large Quantity of Air in the Receiver, which causes the Quicksilver to subside. And in the remarkable Experiment of Dr. *Slare*, half a Drachm of Oil of Carraway-Seed, pour'd upon a

THAT the Air is a *heavy or ponderous Body*, must follow from the Nature of the Matter of

Drachm of the Compound Spirit of Nitre, produced such a prodigious Quantity of Air, as instantly blew up the Receiver, which was six Inches in Diameter, and eight Inches deep. The Pressure, therefore, of the Atmosphere on the exhausted Receiver, which it overcame, was above 400 *lb.* reckoning 15 *lb.* to a square Inch.

Plate XXVIII. Fig. 1.

4. But Dr. *Hales*, in his *Vegetable Statics*, has greatly excell'd in his Experiments of this Kind, and in the Methods of making them: One of which was by *Distillation*, the other by *Fermentation*. That by *Distillation* is as follows: The Matter to be distill'd is put into the Retort *r*, and then at *a* is cemented very fast the Glass Vessel *a b*, which was very capacious at *b*, and had an Aperture *c d*, or Hole at the Bottom. The Bolt-head *a b* being thus immersed in Water, with one Leg of an inverted Syphon put up as far as *z*, the Water would rise in the Bolt-head, and drive out the Air through the Syphon, which being taken out, the Water will remain in the Vessel to the Part *z*; at the same Time, while the Bolt-head is under Water, it is placed in the Vessel *x x*, which with the Bolt-head and Retort is carried to the Chymical Furnace, where the Retort has the Heat and Fire gradually communicated to it, and the Bolt-head *a b* and Vessel *x x* well screen'd from the Heat of the Fire.

5. As the Matter distill'd, all except the Air, would go down into the Water of the Bolt-head and Vessel; the Air that was generated or destroy'd by the Process would be shewn by causing the Surface of the Water in the Bolt-head to stand below or above the Point *z*, as at *y*, when all was set aside till it became quite cold. Thus if the Body distilling generates Air of an elastic Quality, that added to the former will not permit the Water *y* to rise so high as *z*, and the Space between *z* and *y* below will shew how much Air was produced from its fix'd State.

6. But if, when all is cold, the Surface of the Water *y* be seen above the Point *z*, it then shews that the distill'd Body did destroy, that is, imbibe or absorb, a Part of the natural Air above *z*; and the Space between *z* and *y*, fill'd with Water, will shew what Quantity was changed from a repellent elastic to a fix'd State, by the strong Attraction of the absorbing Particles of the distill'd Body. This Quantity of generated or absorbed Air it is easy to measure in Cubic Inches, by stopping the End of the Bolt-head with a Cork, and then from a Quantity of

which

which it doth consist; and since those Particles arise from Bodies of every Kind in or upon the

Water of a known Weight to fill it first to z, and afterwards to y; and the Difference of Weight in the two Bulks of Water gives the Number of Cubic Inches from a Table of specific Gravities, in the Manner we have formerly shewn.

7. The other Method which the Doctor made use of for estimating the surprizing Effects of Fermentation arising from various Mixtures of solid and fluid Substances, in generating and absorbing Air, was as follows: He put the Ingredients into the Bolt-head b, and then run the long Neck thereof into a tall cylindric Glass *a y*, and inclining both almost horizontally in a large Vessel of Water, the Water ran into the Vessel *a y*, and driving out Part of the Air, would possess its Place upon turning them up and placing both in a Vessel of Water *x x*, as you see in the Figure, where the Surface of the Water stands in the inverted Glass *a y* at the Point *z*. Plate XXVIII. Fig. 2.

8. If the Ingredients generated Air, then the Water would fall from *z* to *y*, and the empty Space *z y* was equal to the Quantity of generated Air; but if on Fermentation they absorbed or fix'd the active Particles of Air, then the Surface of the Water would ascend from *z* to *n*; and the Cylinder *z n* would be the Bulk of Air absorb'd, which is easily known in Cubic Inches.

9. When the Subjects for trying these Experiments were a burning Candle, burning Brimstone, Nitre, Gunpowder fired, living Animals, &c. the Doctor used to make use of a Pedestal, on the Top of which was a Plate whereon he laid the Matter to be fired; then inverting the tall cylindric Glass over it, and drawing the Water up to *z z* with an inverted Syphon, he set fire to the Matters lying on the Plate by means of a Burning-Glass, concentring the Sun's Rays in its Focus upon the same. See the Figure. Fig. 3.

10. But the Way that I make use of, and which is the most easy and expeditious possible, is instead of having the cylindric Glass close upon the Top at b b, to have it open by a small Neck, on which a brass Cap is cemented with a Female-Screw to receive a Stop-Cock, to take off the Communication of the external Air when Occasion requires. Thus the Use and Trouble of the Syphon is superseded; and in case of noxious Fumes, Vapours, &c. from *Aqua fortis*, burning Brimstone, &c. a Syringe screw'd on to the Stop-Cock will draw off the Air, and raise the Water to what Height you please, without the cumbersome Use of a large

Earth, 'tis evident the constituent Parts of Air are of a most *heterogeneous Nature*, and infinitely

Pair of Bellows, as the Doctor made use of.

11. I shall here subjoin the Quantity of Air which various Substances produce by Distillation, which I have collected from the Doctor's Experiments, and reduced to Cubic Inches.

A Cubic Inch of Hog's Blood	33
Tallow	25
Deer's Horn	234
Oyster-Shell	324
Heart-Oak	216
Pease	396
Amber	270
Oil of Anniseed	22
Oil of Cloves	88
Honey	144
Yellow Bees Wax	54
Coarsest Sugar	126
Newcastle Coal	360
Fresh Earth	43
Antimony	28
Pyrites	83
Sea-Salt mixed with Bone-Calx	64
Nitre, with *ditto*	180
Sal Tartar ditto	224
Rhenish Tartar	504
Calculus humanus	645
Stone in the Gall-Bladder	648

12. These are the principal Experiments by Distillation. Others were made by Fermentation in various Mixtures, some of which generated Air, others absorb'd it, and some did neither generate nor absorb Air. The principal Subjects, which of themselves absorb Air, are the Fumes of burning Brimstone or Matches, the Flame and Fumes of a burning Candle, the Breath of living and expiring Animals, as Rats, Mice, *&c.*

13. Thus the Doctor found that Linen Matches dipped in melted Brimstone, and fired under a Glass in a Quantity of 594 Cubic Inches, absorb'd 150, which was full one Fourth of the Whole. A Candle burning till it went out, its Fumes afterwards consumed a $\frac{1}{11}$ Part of the whole Quantity of Air, which was 594 Cubic Inches. A half-grown Rat expired in the confined Air in ten Hours, and absorb'd 45 Cubic Inches of Air, which was a $\frac{1}{13}$ Part of the whole 594.

various

various in their specific Gravities: Whence also it will follow, that as the Matter which composes

14. From what has been said we see with how much Reason Sir *Isaac Newton* philosophized on this Subject in the following Words: "True permanent Air arises by Fermentation or Heat from those Bodies the Chymists call *fixed*, whose Particles adhere by a strong Attraction, and are not therefore separated and rarified without Fermentation; those Particles receding from one another with the greatest repulsive Force, and being most difficultly brought together, which upon Contact were most strongly united." And again: "Dense Bodies, by Fermentation, rarify into several Sorts of Air; and this Air by Fermentation, and sometimes without it, returns into dense Bodies." See his *Optics*, Query 30, 31.

15. Now since Air is a heavy Body, a Cubic Inch whereof weighs very near $\frac{1}{3}$ of a Grain, it follows, that Air in its fix'd State in Bodies makes a Part of their Substance, and in some of them a very great Part too, as is known from the Quantity and Weight of the Air discharged upon the Analysis of the Bodies. This has escaped the Observation of Chymists, who have hitherto taught that all Bodies were ultimately resolvable into what they call *Four Elements* only, viz. *Water*, *Oil*, *Salt*, and *Earth*. But by the following Table it will appear, that Air is an Element of Natural Bodies in as proper a Sense as any of the other.

16. In the first Column of this Table you have the Bulk of the Body in Cubic Inches and Parts; in the second, the Number of Cubic Inches of generated Air; in the third is the Weight of the Body in Grains, in the fourth is the Weight of the generated Air; and the fifth shews what Part of the Whole the Air makes.

	C. Inch.	*C. Inch.*	*Grs.*	*Grs.*	*Prop.*
Deer's Horn,	$\frac{1}{2}$	117	241	33	$\frac{1}{7}$
Oyster-Shell,	$\frac{1}{2}$	162	266	46	$\frac{1}{6}$
Heart of Oak,	$\frac{1}{2}$	108	135	30	$\frac{1}{4}$
Indian Wheat,		270	388	77	$\frac{1}{5}$
Pease,	1	396	318	113	$\frac{1}{3}$
Mustard-Seed,		270	437	77	$\frac{1}{6}$
Amber,	$\frac{1}{2}$	135	135	38	$\frac{10}{35}$
Dry Tobacco,		153	142	44	$\frac{1}{3}$
Honey with *Calx* of Bones,	1	144	359	41	$\frac{1}{9}$
Yellow Wax,	1	54	243	15	$\frac{1}{16}$

the Body of Air, or Atmosphere, is *always variable*, so will its Weight or Gravity be likewise;

	C. Inch.	C. Inch.	Grs.	Grs.	Prop.
Coarse Sugar,	1 —	126 —	373 —	36 —	$\frac{1}{10}$
Newcastle Coal,	$\frac{1}{2}$ —	180 —	158 —	51 —	$\frac{1}{3}$
Nitre with *Calx* of Bones,	$\frac{1}{4}$ —	90 —	211 —	26 —	$\frac{1}{8}$
Rhenish Tartar,	1 —	504 —	443 —	144 —	$\frac{1}{3}$
Calculus humanus,	$\frac{1}{4}$ —	516 —	230 —	147 —	$\frac{1}{2}$

17. Thus we see that different Bodies contain different Quantities of fix'd Air, from a Sixteenth to one Half of the whole Substance. From hence we may be fully satisfied of the Truth of Sir *Isaac Newton*'s Reasoning in the 31st *Query* of his *Optics*, in these Words: "The Particles, when they "are shaken off from Bodies by Heat or Fermentation, so "soon as they are beyond the Reach of the Attraction of "the Body, recede from it, and also from one another, with "great Strength, and keep at a Distance, so as sometimes to "take up above a Million of Times more Space than they "did before in the Form of a dense Body: Which vast Con"traction and Expansion seems unintelligible by feigning the "Particles of Air to be springy and ramous, or roll'd up "like Hoops, or by any other Means than by a repulsive "Power.

18. That the Particles of Air cannot be thus coil'd up and detain'd in their elastic State in the Substance of Bodies, is easy to be shewn from Calculation. Thus, for Instance, one Cubic Inch of Oak yields 216 Cubic Inches of Air: Now suppose the Pressure of the Atmosphere be on every Square Inch about 15 *lb.* (as we shall shew) then in order to compress 216 Cubic Inches into one Cubic Inch, the Weight of 216 Times 15 *lb.* or 3240 *lb.* which would be the Force to confine it on each Side the Cube, which, as it has six Sides, will require $6 \times 3240 = 19440$ *lb.* or near twenty thousand Weight, to confine this Air in its elastic State in one Cubic Inch, supposing it to be all Air; but, as it is not, the Force will be greater still. This Force therefore of 19440 *lb.* must be exerted in every Cubic Inch of the Oaken Tree, which would rend it in pieces with a vast Explosion. It is therefore not to be doubted but Air in Bodies does exist in a fix'd and unelastic State; and that it is roused, and put into an active repellent State by means of Fire and Fermentation.

19. They who would see the numberless Uses that may be made of this important Doctrine of artificial Air, and the fur-

as

as we constantly experience by the BAROMETER, of various Kinds and Structure. (LXXXVII),

prizing Scenes of Knowledge which it lays open in the most abstruse and difficult Parts of *Physics*, may consult all the latter Part of the invaluable Book above-mentioned, *viz.* Dr. *Hales*'s *Vegetable and Chymico-Statical Experiments*, some of which we shall take notice of also in the Sequel of these Notes.

(LXXXVII) 1. The Weight of the Air is manifest from Reason and various Experiments. The Particles are affected by the Power of the Earth's Attraction, and must therefore all gravitate or tend towards its Centre, which is what constitutes Weight in them, and all other Bodies. The Experiments to shew the Weight of the Air are numerous which we shew on the Air-Pump, among which one is absolute and very exact, by weighing it in a Balance, in the same manner as all other heavy Bodies are weigh'd.

2. The Method I take for this is, I believe, the most exact and nice that can possibly be thought of. For since (as we have shewn) the Friction of the Balance is in Proportion to the Weight with which it is charged, the less the Weight is, the less will be the Friction, and consequently the more nice and exquisite will be the Experiment. In order to this I take a very thin large *Florence* Flask, whose Capacity is exactly known in Cubic Inches: This I exhaust of all the Air as near as can be, and then hang it to the End of a very fine and exact Hydrostatic Balance, which I counter-balance by Grain-Weights in a Scale hanging from the other End. When the Equilibrium is nicely obtain'd, I lift up the Valve, and let the Air rush into the Flask, which is sensibly heard, and seen to gravitate in the Glass, by causing it gradually to descend till it be fill'd with Air, and will then preponderate greatly. Then to restore the Equilibrium, I find by Experience 'tis necessary to add about 8 Grains for every Pint the Flask contains; which shews that a Gallon of Air weighs about a Dram, and a Bushel an Ounce *Troy*; and because one Pint $=$ 28 Cubic Inches nearly, therefore one Cubic Inch of Air weighs $\frac{8}{28} = \frac{2}{7}$ of a Grain, at a Mean.

3. As the Air is a heterogeneous Fluid, it will vary in its Weight according to its different component Parts, and also according to its different Altitudes, which it must have as an elastic and fluctuating Fluid. Since few Bodies are lighter than Water, and that Water is most easily rarified into Vapour, it follows, that the Atmosphere fill'd with aqueous Par-

SINCE

"SINCE the Particles of Air are such as being separated from Bodies beyond the Sphere of cor-

ticles will be lightest, as we generally find it is in moist rainy Weather; and also that it must oftener be in this light than in a heavier State: And that Instrument which shews the Variation of the Air's Gravity, or its different Weights at different Times, is call'd a BAROMETER, of which there are various Kinds, which are here described; but I shall first give an Account of the most simple Structure or Form of those Instruments, which is as follows. A Glass Tube, hermetically seal'd at one End, is to be fill'd with Quicksilver, well desiccated and purged of its Air. The Finger being then placed on the open End in immediate Contact with the Mercury, so as not the least Particle of Air is admitted, the Tube is inverted, and carefully immersed with the Finger on the open End in a Bason of the same prepared Mercury; then upon removing the Finger, the Mercury in the Bason will join that in the Tube; and the said Column of Mercury in the Tube will be seen immediately to subside, or sink down to a certain Pitch or Altitude, if the Tube be above 31 Inches long, as it ought to be.

Plate XXVIII. Fig. 4. 4. Let AB be such a Tube of 34 Inches Length, and $\frac{1}{4}$ of an Inch in Diameter, (as it ought to be for this Purpose) hermetically seal'd at A, and open at B; let CD be the Bason of Mercury, in which the Tube is immersed inverted, the Surface of the Mercury in the Bason EF, and in the Tube GH. Now, 'tis easy to understand, that if all this could be perform'd *in Vacuo*, as soon as the Tube was inverted, all the Mercury would descend into the Bason, because as a heavy Body it must tend towards the Centre of the Earth, till it meets with some Obstacle, as the Bason, to obstruct its Motion, and support it. I say, all this would happen *in Vacuo*, unless we can suppose any Power in the Tube sufficient to sustain the Column of Mercury; now there can be no such Power but that of *Cohesion*, which indeed, in Tubes of a small Bore, has been found able to sustain it; but in so large a Bore, as we suppose this Tube to have, that Power is by far too small to support so heavy a Column, which must therefore of Course sink into the Bason, and so stand upon the same Level in the Bason, and in the Tube.

5. But since the Mercury does not totally subside when this Experiment is made in the Air, the Column which remains in the Tube must owe its Suspension to the Air as its Cause, since nothing within nor without the Tube can be supposed

puscular

puscular Attraction, are strongly repell'd from those Bodies; this Repellency being mutual

(with any Shew of Reason) to produce such an Effect, besides itself). Now allowing the Air to be a gravitating Fluid, it must necessarily cause such an Effect, as the Suspension of Mercury in the Tube; for by its Gravity a Force of Pressure must be produced on all the Surface of Mercury in the Bason, which is contiguous to it, and on every Part equally. Now since the Mercury in the Bason is *in Equilibrio* with that in the Tube, it is plain the Weight of the Mercury in the Tube, and the Weight of the Air upon every circular Area of the Surface of the Mercury in the Bason, equal to the Orifice of the Tube, must be equal, for else they could not balance each other, as we find they do; the Column of Mercury therefore is sustain'd by the Counter-Pressure of a Column of Air of the same Base, and whose Altitude is equal to that of the Atmosphere.

6. That the Weight of the Columns of Air and Mercury, we have now been speaking of, are precisely equal to each other, will be farther evident, if we consider, that upon Supposition the Quicksilver were throroughly purg'd from Air, when it subsides in the Tube, it must leave a *Vacuum* in all that Part of the Tube above it, and so there is nothing to act upon its upper Surface to depress it; it will therefore always sink or rise to such an Altitude, as the various Gravity of the Air requires, and of which it is therefore an adequate Expression or Measure, as its Nature imports. This Invention was owing to that happy *Italian* Genius *Torricelli*, a Disciple of the famous *Galileo*. And hence it is very often call'd the *Torricellian Tube*, and the *Torricellian Experiment*, &c.

7. Since, as we have shewn, this suspended Column of Mercury exactly indicates the Gravity of the Air at all Times, it has employ'd the Attention of all Mankind, who very sensibly find themselves affected with the different State of the Air; but more especially has it merited the Consideration of Philosophers, who have taken all Opportunities to explore, by this Means, the two Extremes of the Air's Gravity, *viz.* when it is least and greatest of all, by observing the least and greatest Altitude of this mercurial Column, which by long Experience we find to be very nearly between 28 and 31 Inches, it being very rarely less or more than those Heights; whence $29\frac{1}{2}$ Inches is fix'd upon as the Mean-Altitude, expressive of the Mean-Gravity of the Air, which therefore let be represented by B H, and let the greatest Altitude be B I, between

by any impress'd Force to approach nearer to each other, the *repulsive Power* will re-act or resist the

G very small, the Motion of the Quicksilver, and consequently of the Ball G, will at Bottom be very considerable; but as the Weight G moves up and down, it turns the Pulley C D, and that a Hand or Index K L, by the Divisions of a large graduated Circle M N O P; by which means the minutest Variations of the Air are plainly shewn, if the Instrument be so very accurately made that the Friction of the several Parts be inconsiderable. This is one of the many curious Inventions of Dr. *Hooke*.

13. These are the principal Contrivances hitherto invented for enlarging the Scale of Variation in simple Mercurial Barometers. There are other Inventions of compound Barometers, *viz.* such as are made of Mercury and Water, or other Liquors for that Purpose; but they are so difficult to make, so faulty when made, and so troublesome to use, that it is not worth while to describe them. However, as the Reader may have an Idea of one of the best Sort, I shall here give him that which owes its Invention to the Reverend Mr. *Rowning*, together with his Demonstration of its Theory.

Plate XXVIII. Fig. 9.

14. A B C is a compound Tube seal'd at A, and open at C, empty from A to D, fill'd with Mercury from thence to B, and from thence to E with Water; let G, B, H, be in an horizontal Line, then it is plain from the Nature of the Syphon, that all the compound Fluid contain'd in the Part between H and G, must ever be *in Equilibrio* with itself be the Weight of the Air what it will, because the Pressure at H and G must always be equal. Whence 'tis evident, that the Column of Mercury D H is *in Equilibrio* with the Column of Water G E, and a Column of Air of the same Base conjointly, and will therefore vary with the Sum of the Variations of each of these; all which must now be computed.

15. The Variation of the Weight of the Air, which we will call V, is measured by the Space which the Mercury moves in the common Barometer in a given Time: Let x be the Space which the Water at E moves thro' in the same Time, and let the Diameter of the Tube A F be to that of the Tube F C as D to 1, then will the Space moved through at B be as $\frac{x}{DD}$, and therefore G E the Difference of the Legs E K and K B, will vary in its Weight by $x + \frac{x}{D}$. Also since the

said

said Force with an equal *Momentum*; and thus cause what we call the RENITENCY, ELASTICITY,

Space mov'd through by the Mercury at B and D is as $\frac{x}{D^2}$, the Difference D H will vary its Weight by $\frac{2x}{D^2}$. But this *Variation* of Weight is equal to both the former, and since $x + \frac{x}{D^2}$ is an Altitude of Water, if we put m to 1 as the Specific Gravity of Mercury to Water, we shall have $m : 1 :: x + \frac{x}{D^2} : \frac{x + \frac{x}{D^2}}{m} =$ Altitude of Mercury of the same Weight, consequently, $\frac{2x}{D^2} = V + \frac{x + \frac{x}{D^2}}{m}$; which Equation, when reduced, gives $x = \frac{VD^2 m}{2m - D^2 - 1}$, which gives this Analogy; as $x : V :: mD^2 : 2m - D^2 - 1$, so is the Scale of Variation in this, to that in the common Barometer.

16. Hence if $m = 14$, and $D = 1$; we have $x : V :: 14 : 26 :: 7 : 13$; which shews that when the Tubes A F and F C are of an equal Bore, the Variation in this is less than that of the common Barometer in the Ratio of 7 to 13. If $2m - D^2 - 1 = 0$, or $2m - 1 = D^2$, then $D = \sqrt{2m - 1} = 5,2$; whence it appears that when the Diameter of A F is to that of F C as 5,2 to 1, the Variation x will be infinite in respect of that in the common Barometer. If $D = 5$, then $x : V :: 175 : 1$; which shews how very large the Scale of Variation in this Barometer may be made in comparison of the common one. But I believe such a Structure as this will afford more Pleasure in Speculation than in Practice; and when all is done the Barometer of the common Form, as it is most simple, so it will be found the most easy and accurate of all others.

17. Before I conclude this Article, I shall just mention the Barometer invented by the Rev. Mr. *Caswell* of *Oxford*. Suppose ABCD be a Bucket of Water, in which is placed the Baroscope *xrexyosm*, which consists of a hollow Body *xrsm*, and Tube *exyo*, made of Brass, Tin, Glass, &c. The Bottom of the Tube *xy* has a Lead-Weight to sink it, so that the Top of the Body may just swim even with the Surface of Plate XXIX. Fig. 1.

or SPRING of the Air; which is so sensible by the

Water, by the Addition of some Grain-Weights. As the Instrument is put into the Water, with the Mouth downwards, the Water ascends into the Tube to the Height of *y u*; there is added on the Top a small concave Cylinder, or Pipe, to sustain the Instrument from sinking to the Bottom when the Air becomes heavier; *m d* is a Wire, and *m s*, *d e*, are two Threads, oblique to the Surface of the Water; of these Threads there may be several; and as the Water just touches the Top or Crown of the Instrument, when the Altitude of the Mercury is least in the Common Barometer, so as the Air increases in Weight, the Instrument sinks in the Water, and a small Bubble is form'd on the Thread, which continually ascends and descends thro' all the Length of the Thread. From a Calculation on the Theory, it appears, that this Barometer is above 1200 Times more exact than the Common Barometer. See the whole Calculation in the Professor's own Words in the *Phil. Transactions*.

18. Though I have made and tried the Barometer above described, and find it to answer the Theory very well, yet is it not fit for common Use, because it can only shew the extreme minute Variations of the Air's Gravity for the *present Time*, by reason it is affected by the Heat as well as Weight of the Air. While the Degree of Heat remains the same, nothing can exceed this Instrument as a Barometer; but as the Heat of the Air varies, so will the Elasticity of the included Air, which therefore will cause the Instrument to vary its Gravity, while that of the Air remains the same; and so cannot be of constant Use.

19. I have already hinted that the Common Barometer, after all, is the best Instrument to measure the Air's Gravity; which that it may do to the greatest Perfection, the following Things are necessary. (1.) That the Tube be at least of $\frac{1}{4}$ of an Inch Bore; $\frac{1}{3}$ of an Inch is a good Size. (2.) The Tube ought to be new, clean, and dry within when fill'd; in order to this, the Tube should be hermetically sealed at both Ends at the Glass-House when made; one End of which may be cut off with a File when you intend to use it. (3.) The Diameter of the Cistern that holds the Mercury, in which the Tube is immersed, should be as large as conveniently may be, that the Mercury therein may have nearly at all times the same Altitude; otherwise the Index will not be true. (4.) The Mercury must be very pure, and free from any Mixture of Tin, Lead, or other Metal. (5.) It ought to be

many

common Experiment of a *blown Bladder*, and purged from Air entirely, as it may be by boiling it, and filling the Tube with it while boiling-hot nearly. (6.) The Tube must be heated hot when fill'd, to avoid breaking by the boiling Mercury. (7.) It should be rubb'd very hard, to excite the Electric Virtue, which will expel the Particles of Air from the Surface within. (8.) There ought to be a *Nonius* (as it is call'd), applied to the Index of the graduated Plate, to measure more accurately the Rise and Fall of the Mercury.

20. This Artifice is of singular Use in this and many other Cases. It bears the Inventor *Nonius*'s Name, and its Nature and Manner of applying it is as follows. A B is the upper Part of the Barometer, in which the Surface of the Mercury is at C. F G is the usual Plate of 3 Inches Extent, from 28 to 31; and D E is the small Plate call'd the *Nonius*, so contrived as to slide by the other in such manner that its Index D may be always set on one Part to the Surface of the Mercury, and on the other End pointing to the Division in the Scale of Inches corresponding thereto. Again, the *Nonius* is divided into 10 equal Parts, which together are equal to 11 of the Divisions of the Scale; that is, DE = 11 Tenths of an Inch; and consequently each small Division of the *Nonius* is equal to 1,1; two of them to 2,2; three of them to 3,3; and so on. Whence 'tis easy to observe, that if the Index D points between any two Divisions of the Scale, as here between 29,7 and 29,8, we need only look back to see what Division of the *Nonius* coincides with a Division of the Scale, and that will shew how many Tenths of a Tenth, that is, how many Tenths beyond 29,7 in the present Case: But you observe the *Nonius* coincides with a Division of the Scale at the fifth Division; consequently, the Mercury stands at 29,75 Inches in the Scale; and so you proceed with the greatest Ease to the hundredth Part of an Inch, which is a great Degree of Exactness. Plate XXVIII. Fig. 10.

21. From what has been said we may easily see the excellent Use of the Barometer in measuring the Heights of Places, as Mountains, Towers, &c. For since (as we shall shew) the specific Gravity of Air (such as is near the Earth's Surface) is to that of Mercury as 1 to 12040, 'tis plain 12040 Inches of Air in Height will balance one Inch Height of Mercury; consequently, 1204 Inches, or 100 Feet, answers to $\frac{1}{10}$ of an Inch of Mercury. Therefore if a good Barometer be carried to the Top of a Mountain, or other high Place, the Mercury

many others on the *Air Pump*. (LXXXVIII.)

will subside near one Tenth of an Inch for every 100 Feet of perpendicular Ascent, and so will be a proper Index of the whole Height ascended.

22. But since Mercury is not quite 14 times heavier than Water, the Number 12040 is somewhat too large, and therefore a less Height than 100 Feet will answer to $\frac{1}{10}$ of an Inch Descent of Mercury in the Barometer; and what that is will be shewn from the Experiments made by Dr. *Nettleton* very exactly, as in the Table below.

		Altitude of ☿.			
	Height.	*Bottom.*	*Top.*	*Difference.*	*for* $\frac{1}{10}$
Tower of *Halifax*	102	29,78	29,66	0,12	85
Coal Mine	140	29,48	29,32	0,16	87
Another, *ditto*	236	29,50	29,23	0,27	87
A small Hill	312	29,81	29,45	0,36	86
Halifax Hill	507	30,00	29,45	0,55	91

23. Having the Height, to which the Mercury will stand at any one Elevation, it is easy to find at what Height it will stand at any other proposed. For since the Density of the Air decreases in a Geometrical Ratio, as the Altitudes increase in an Arithmetical one, the latter will be as the Logarithms of the former reciprocally: But the Weight of the Air is as the Density, and the Height of the Mercury in the Barometer is as the Weight, therefore the Elevations are as the Logarithms of the Height of the Mercury reciprocally; and consequently, if we take 30 Inches for the Standard Altitude, and 85 Feet for the Altitude requisite to make it fall $\frac{1}{10}$ of an Inch; then by saying, As the Logarithm of $\frac{1}{29,9}$ is to 85, so is the Logarithm of $\frac{1}{29,5}$ to the Elevation which will make it fall $\frac{1}{2}$ an Inch; and so for any other.

24. After this Manner, the Doctor has computed the following Tables.

A TA-

A TABLE shewing the Number of Feet ascending, required to make the Mercury fall to any given Height in the Tube, from 30 to 26 Inches. As also the Number of Feet descending, requir'd to make the Mercury rise, from 30 to 31 Inches.

Inch.	Dec.	Feet.	Dec.	Inch.	Dec.	Feet.	Dec.
31	0	834	79	27	9	1847	55
30	9	752	53	27	8	1938	97
30	8	670	01	27	7	2030	72
30	7	587	21	27	6	2122	80
30	6	504	15	27	5	2215	21
30	5	420	82	27	4	2307	95
30	4	337	21	27	3	2401	02
30	3	253	32	27	2	2494	44
30	2	169	10	27	1	2588	29
30	1	84	72	27	0	2682	33
30	0	00	00	26	9	2776	80
29	9	85	00	26	8	2871	62
29	8	170	29	26	7	2966	79
29	7	255	87	26	6	3062	32
29	6	341	73	26	5	3158	21
29	5	427	89	26	4	3254	46
29	4	514	34	26	3	3351	07
29	3	601	08	26	2	3448	05
29	2	688	11	26	1	3545	41
29	1	775	44	26	0	3643	14
29	0	863	08				
28	9	951	01				
28	8	1039	25				
28	7	1127	80				
28	6	1216	66				
28	5	1305	83				
28	4	1395	32				
28	3	1485	13				
28	2	1575	26				
28	1	1665	70				
28	0	1756	47				

A TABLE shewing the Number of Feet requir'd to make the Mercury fall one Tenth of an Inch from any given Height in the Tube, from 31 to 26 Inches.

Inch.	Dec.	Feet.	Dec.	Inch.	Dec.	Feet.	Dec.
31	0	82	26	27	9	91	42
30	9	82	53	27	8	91	75
30	8	82	79	27	7	92	08
30	7	83	06	27	6	92	41
30	6	83	33	27	5	92	74
30	5	83	61	27	4	93	07
30	4	83	89	27	3	93	41
30	3	84	16	27	2	93	76
30	2	84	44	27	1	94	12
30	1	84	72	27	0	94	47
30	0	85	00	26	9	94	82
29	9	85	29	26	8	95	17
29	8	85	58	26	7	95	53
29	7	85	86	26	6	95	89
29	6	86	16	26	5	96	25
29	5	86	45	26	4	96	61
29	4	86	74	26	3	96	98
29	3	87	03	26	2	97	36
29	2	87	33	26	1	97	73
29	1	87	63	26	0	98	10
29	0	87	93				
28	9	88	24				
28	8	88	55				
28	7	88	86				
28	6	89	17				
28	5	89	49				
28	4	89	81				
28	3	90	13				
28	2	90	45				
28	1	90	76				
28	0	91	08				

By reason of the *Spring of the Air*, its DENSITY must be always different in different Altitudes from the Earth's Surface; for the lower Parts of the Air, being pressed by the Weight of the superior Parts, will be made to accede nearer to each other, and the more so as the Weight of the

(LXXXVIII.) 1. That we may here exhibit a plain and clear Idea of the Force with which the Particles of Air repel one another, 'twill be necessary to proceed in the following Manner. If in any Distance A B, there are placed any Number of Particles at equal Intervals from one another; and in any other equal Distance C D, there are placed twice as many Particles at equal Intervals also; 'tis plain the Intervals between the Particles in C D will be but half so great as those between the Particles in the Line A B. Hence the Number of Particles in any equal Parts of A B, C D, will be inversely as their Distances from each other. Or, if we put N = Number of Particles, and I = to the Interval between each; then will N be always as $\frac{1}{I}$, for Lines.

Pl. XXIX. Fig. 2.

2. But for Superficies, since they are as the Square of their like Sides, we shall have N^2 as $\frac{1}{I^2}$; and in like Manner, since Solids are as the Cubes of their like Sides, we shall have N^3 as $\frac{1}{I^3}$. But N^2 is as the Density of the Superficies; and N^3 as the Density of the Solid; consequently the Density D, of a Superficies of this Sort, is as $\frac{1}{I^2}$; and of a Solid as $\frac{1}{I^3}$. And to facilitate the Idea, let A B C be a Superficies of such Particles, equal to a square Inch; and D F a Solid of a cubic Inch.

Fig. 3.

3. Next, let it be supposed that each of these Particles repels those, and those only, which are next to it; and let this repulsive Force (F) be inversely as the *n* Power of the Interval I, between the Centres of two adjacent Particles; that is, let F be as $\frac{1}{I^n}$. Hence 'tis manifest such an Assemblage of Particles must constitute an *elastic Fluid*, or such an one as, when compress'd, or acted upon by any external Agent, will, by Virtue of its innate repellent Power, re-act or

incumbent

incumbent Air is greater; and hence we see the *Density* of the Air is greatest at the Earth's Surface, and decreases upwards in *geometrical Proportion* to the Altitudes taken in *arithmetical Progression*. Now it is found that the Air is four Times more rare at the Height of seven Miles

make Resistance with an equal Degree of Force.

4. Now the Force of the superficial Parts is as the Density D, and the repellent Force F between two Particles, conjointly, or as $D \times F$; but D is as $\frac{1}{I^2}$, and F is as $\frac{1}{I^n}$; wherefore $D \times F$ is as $\frac{1}{I^2} \times \frac{1}{I^n} = \frac{1}{I^{n+2}}$, which therefore will express the elastic Force of the Fluid. Now the Density D of the Fluid in the cubic Inch is as $\frac{1}{I^3}$, whence $I^3 D$ is as 1, I^3 as $\frac{1}{D}$, and I as $\frac{1}{\sqrt[3]{D}}$; which substituted for I in the Expression of the elastic Force $\frac{1}{I^{n+2}}$, gives $D^{\frac{n+2}{3}}$; that is, the elastic or compressive Force is as the Cube Root of that Power of the Density, whose Index is $n+2$.

5. Hence if E the elastic Force be as the Density D, in any Fluid; then the general Expression $D^{\frac{n+2}{3}}$ becomes $E^{\frac{n+2}{3}}$, whence in that Case $\frac{n+2}{3} = 1$, and so $n+2=3$ and $n=1$. Consequently in such a Fluid F is as $\frac{1}{I}$, or the Particles repel each other with Forces that are reciprocally proportional to the Distance of their Centres. Such then is the Property of the Air, whose Density is always proportional to the Force which compresses it, as is proved by the following Experiment.

6. Let Mercury be pour'd into an inflected Tube ABCD, open at both Ends, to a small Height as BC. Then stopping the Orifice D very close with a Cork or otherwise, measure the Length of confined Air DC very nicely, and pour Mercury into the other Leg AB, till its Height above the Surface of that in CD be equal to the Height at which it Pl. XXIII. Fig. 4.

B 3 than

than at the Earth's Surface; and therefore at the Altitudes of 7. 14. 21. 28. 35. 42. 49. &c. the Rarity of the Air will be 4. 16. 64. 256. 1204. 4096. 16384. &c.

If the Air were of an equal Density throughout, the Height of the Atmosphere might be deter-

stands in the Barometer. Then it is plain the Air in the shorter Leg will be compress'd with a Force twice as great as at first when it possess'd the whole Space C D; for then it was compress'd only with the Weight of the Atmosphere; but now it is compress'd by that Weight, and the additional equal Weight of a Column of Quicksilver. Let E be now the Surface of the Mercury in the Leg C D, and upon measuring D E, the Space into which the Air is now compress'd, it will be found to be just half the former Space C D, that is, $DE = \frac{1}{2} DC$.

7. Hence it appears that the Spaces $S = DC$, and $s = DE$, which a given Quantity of Air possesses, under different Pressures p and P, are as those Pressures reciprocally; that is, $S : s :: P : p$. And because the Densities d, D, where the Quantity of Matter is given (*Annotat.* LVI. 9.) are reciprocally as the Magnitudes of Bodies, *viz.* $d : D :: s : S$; therefore the Densities of the Air are as the Compressing Forces directly, *viz.* $d : D :: p : P$. This Property of the Air is the Principle to which we owe the Invention and Contrivance of several very useful Instruments and Machines, some of which I will exhibit here, and others in the Sequel of this Work.

8. We have shewn in the last *Annotation* that the Pressure of the Air, in its State of Mean Gravity, will support a Column of Quicksilver to the Altitude of $29\frac{1}{2}$ Inches; and in (*Annot.* LXIII.) it was shewn that the specific Gravity of Mercury was to that of Water, as 14 to 1 nearly; therefore the said Mean Pressure of Air will sustain a Column of Water to the Height of $14 \times 29,5 = 413$ Inches $= 34$ Feet 5 Inches. But since Mercury is not quite 14 Times as heavy as Water, we may take 400 Inches for the Measure of the Mean Gravity of the Air on Water, and 29,5 for Mercury; and then we shall have $DC : DE :: P : 29,5$ in Mercury; or $DC : DE :: P : 400$, in Water; consequently $400\, DC = DE \times P$.

9. Again, let the Standard Altitude of Mercury or Water be $H = 29,5$ or 400, and let the Altitude $FG = b$; then

mined;

mined; for by Experiment we find the Length of a Column of Air 72 Feet high is equal in Weight to *one Inch* of Water of the same Base: Hence the Density of Air is to that of Water as 1 to 864. It is also found by Experiment, that the Weight of a Column of Air the Height of

will $P = H + h$, and then the above Equation will give this Analogy; As $S : s :: H + h : H$, whence $S : S - s :: H : h$, or $DE : EC :: H : h$; consequently, by having DE or CE given, you know the Altitude $h = FG$. Thus for Example: Let $DC = 10$ Inches, *it is required to find what Altitude of Water* FG *will by its Pressure raise the Surface at* C *one Inch?* Here $CE = 1$, $DE = 9$, and $H = 400$: Then $DE : CE :: H : h$, that is, $9 : 1 :: 400 : 44,4$; or $FG = 44\frac{1}{2}$ Inches nearly, or 3 Feet $8\frac{1}{2}$ Inches. Thus again, Query the Altitude FG that shall raise the Surface C 9 Inches, or $\frac{9}{10}$ of the Whole? Say, As $1 : 9 :: 400 : 3600 = FG$, or 300 Feet. Thus the Altitudes are found for every tenth Part of the whole Space DC, as in the following Table.

	Feet.	*Inches.*		*Feet.*	*Inches.*
1 ——	3	8	6 ——	50	0
2 ——	8	4	7 ——	77	9
3 ——	14	2	8 ——	133	4
4 ——	22	1	9 ——	300	0
5 ——	33	4	$9\frac{1}{2}$ ——	633	4

10. Hence is deduced the Nature and Structure of the SEA-GAGE, invented by Dr. *Hales*, and Dr. *Desaguliers*; whose Description thereof I shall here give. AB is the Gage-Bottle, in which is cemented the Gage-Tube Ff in the Brass-Cap at G. The upper End of the Tube F is hermetically seal'd or closed; the open lower End f is immersed in Mercury C, on which swims a small Thickness or Surface of Treacle. On the Top of the Bottle is screw'd on a Tube of Brass HG, pierced with several Holes to admit the Water into the Bottle AB. The Body K is a Weight hanging by its Shank L, in a Socket N, with a Notch on one Side at *m*, in which is forced the Catch *l* of the Spring S, and passing thro' the Hole L in the Shank of the Weight K, prevents its falling out when once hung on. On the Top, in the upper Part of the Brass Tube at H, is fix'd a large empty Ball, or full-blown Bladder F, which must not be so large, Pl. XXIX.

 the

the Atmosphere will be equal to the Weight of a Column of Water of the same Base, and 32 Feet, or 384 Inches high: Wherefore 864 multiplied by 384 will produce 331776 Inches, or a little above 5 Miles, for the Height of the Atmosphere, were the Density every where the same as at the Earth.

but that the Weight K may be able to sink the Whole under Water.

11. The Instrument, thus constructed, is used in the following Manner. The Weight K being hung on, the Gage is let fall into deep Water, and sinks to the Bottom; the Socket N is somewhat longer than the Shank L, and therefore, after the Weight K comes to the Bottom, the Gage will continue to descend, till the lower Part of the Socket strikes against the Weight; this gives Liberty to the C tch to fly out of the Ho'e L, and let go the Weight K; when this is done, the Ball or Bladder I instantly buoys up the Gage to the Top of the Water.

12. While the Gage is under Water, the Water having free Acces to the Treacle and Mercury in the Bottle, will by its Pressure force it up into the Tube F f, and the Height to which it has been forced by the greatest Pressure, *viz.* that at the Bottom, will be shewn by the Mark in the Tube which the Treacle leaves behind it, and which is the only Use of the Treacle. This shews into what Space the whole Air in the Tube F f is compress'd; and consequently, by the Rule (in *Article* 9.) the Height or Depth of the Water, which by its Weight produced that Compression, which is the Thing required.

13. If the Gage-Tube F f be of Glass, a Scale might be drawn on it with the Point of a Diamond, shewing, by Inspection, what Height the Water stands above the Bottom; which Scale is made from the Numbers in the foregoing Table, where the Division may be made for Hundredth Parts, as well as Tenths. But the Length of 10 Inches is not sufficient for fathoming Depths at Sea, since it appears by the Table that when all the Air in such a Length of Tube is compress'd into half an Inch, the Depth of Water is not more than 634 Feet, which is not half a Quarter of a Mile.

14. If to remedy this we make use of a Tube 50 Inches long, which for Strength may be a Musket-Barrel, and sup-

But

But since the Density of the Air decreases with the Pressure, it will be more rarefied and expanded the higher we go; and by this means the Altitude of the Atmosphere becomes indefinite, and terminates in pure *Æther*. But though we cannot assign the real Altitude of the Atmosphere,

pose the Air compress'd into an 100dth Part, or ½ an Inch; then by saying, As 1 : 99 :: 400 : 39600 Inches, or 3300 Feet; even this is but little more than half a Mile, or 2640 Feet. But since 'tis reasonable to suppose the Cavities of the Sea bear some Proportion to the mountainous Parts of Land, some of which are more than three Miles above the Earth's Surface; therefore to explore such great Depths, the Doctor contrived a new Form for his Sea-Gage, or rather for the Gage-Tube in it, as follows. BCDE is a hollow metalline Globe, communicating on the Top with a long Tube AB, whose Capacity is $\frac{1}{9}$ Part of that of the Globe. On the lower Part, at D, it has also a short Tube DE, to stand in the Mercury and Treacle. The Air contain'd in this compound Gage-Tube is compress'd by the Water, as before; but the Degree of Compression, or Height to which the Treacle has been forced, cannot here be seen through the Tube: Therefore to answer that End, a slender Rod of Metal or Wood, with a Knob on the Top, must be thrust up to the Top of the Tube AB, which will receive the Mark of the Treacle, and shew it when taken out. Pl. XXIX. Fig. 6.

15. If the Tube AB be 50 Inches long, and of such a Bore as that every Inch in Length should be a Cubic Inch of Air, and the Contents of the Globe and Tube together 500 Cubic Inches; then, when the Air is compress'd within a 100dth Part of the Whole, it is evident the Treacle will not approach nearer than 5 Inches of the Top of the Tube, which will agree to the Depth of 3300 Feet of Water, as above. Twice this Depth will compress the Air into half that Space nearly, *viz.* 2½ Inches, which corresponds to 6600 Feet, which is a Mile and a Quarter. Again, half that Space, or 1¼ Inch, will shew double the former Depth, *viz.* 13200 Feet, or two Miles and a half; which is probably very nearly the greatest Depth of the Sea.

16. A Gage of this Kind may be of very great Use in many other Cases. Thus the prodigious Force of Compression arising from Freezing may be accurately tried: Let a Bomb

it

it is certain from Observation and Experiment, that 45 or 50 Miles is the utmost Height where the *Density is sufficient to refract a Ray of Light*; and therefore that may be esteem'd the Altitude

of cast Iron six or eight Inches Diameter, and about one Inch thick, be fill'd with Water; then if a small Gage of this Sort be made and fix'd to a Stick, which is to be set upright in the Middle of the Bomb, so that the Gage-Bottle may be in the central Part; and if then the Hole of the Bomb be fast screw'd up, and the Bomb cover'd over with a *freezing Mixture* (which is made of equal Quantities of Salt and Snow, or pounded Ice) in a little Time the Water will begin to freeze all round the Inside of the Bomb, and by its Expansion will produce a greater Force upon the Water, and a greater Degree of Compression of the Air of course, than by any other Means yet known: And this may be continued till it shall burst the Bomb, when the Gage taken out of the globular Shell of Ice (for the Water will be frozen only on the Outside) will shew the exact Quantity of this Force of Compression.

17. Dr. *Hales* (the Author of this Contrivance) actually made the Experiment, but not having well secured the Gage, it was broken to pieces; but from computing the Force necessary to burst an Iron Bomb an Inch thick, it appeared that this Force was about equal to 1340 Atmospheres, or the Pressure of 1340 times the Weight of 33 Feet of Water. But this Computation was made upon Supposition that the Cohesion of cast Iron is the same with that of Iron-Wire; but as it must be considerably less, so the Number 1340 may be diminished to 1000; and then the Air must be compress'd into 1000 times less Space than it had in its natural State, and must in that Case have been more dense than Water: For its Density then to that of common Air was as 1000 to 1; whereas the Density of Water and Air are but as 860 to 1.

18. After the same manner may be tried the Force with which dried Pease, Beans, &c. expand with Moisture, when confined in a Bomb; for it must be a very strong Vessel indeed, since it has been found by Experiment they will burst a Gun-Barrel in swelling. In like manner also the elastic Force of factitious Air generated from Bodies by Fermentation may be estimated in a very nice and entertaining Manner: With many other Things of this Sort, which the ingenious Reader will readily excogitate of himself.

of

of the Air to the least sensible Degree of Density. (LXXXIX).

SINCE the Gravity of the Air is so various, that at one time it will sustain a Pillar of Mercury

(LXXXIX) 1. The Density of the Air on one hand, and the Rarity on the other, are both limited: No Condensation can reach so far as to cause a Penetration of Parts; the utmost Limit, therefore, of Density, must be a perfect *Plenum*, or a given Quantity of Air reduced into a Space absolutely full, or without any Pore; which is a Degree of Density that has not been, and probably never will be, in the Power of Art to effect.

2. On the other hand, the Rarity of the Air cannot proceed *ad infinitum*, but has its Limit from its Gravity: For though the Rarefaction of the Air be still greater as the Distance from the Surface of the Earth increaseth, its Spring at length will be so weaken'd, that the Force by which the Particles tend upwards from those next below them, will be less than the Force of Gravity by which they tend downwards. The Rarefaction of the Air must therefore be bounded, where these two opposite Forces come to balance each other.

3. Now though we cannot possibly define the Limits of the Atmosphere, yet we may still investigate how much the Air is rarefied at any proposed Altitude above the Earth's Surface: For doing which, several Methods have been proposed; some of which are very tedious, and difficult to be understood. I shall here illustrate Sir *Isaac*'s Theorem for that Purpose, which is very concise and plain. It requires only two different Densities of the Air, at two given Altitudes above the Earth's Surface, to be known, and which we easily obtain by Experiment as follows.

4. Take a Vial AEFB, fill'd two Thirds full of Water to CD; in which let a long Tube IG (open at both Ends) be immersed, and closely cemented to the Vial at AB, so that none of the included Air may escape. This done, blow a little Air through the Tube into the Vial, which increasing the Spring of the contained Air, will cause it to raise and support a Column of Water in the Tube, to such a Height H, that its Weight, together with that of a Column of Air pressing on its Surface H, is equivalent to the increased Spring of the confined Air.

5. The Vial and Tube thus prepared are to be carried up to the Top of a Tower, Mountain, or some high Place; and

31 Inches high, when at another it will raise it but to the Height of 28 Inches, in the *Barometer*; it follows, that we may take 29½ Inches of

in the Ascent, since the Column of Air pressing on the Water at H is constantly shorten'd, so its Force of Pressure will be diminished. The Spring of the Air in the Vial will therefore keep the Column of Water constantly rising in the Tube; so that when you have ascended the Height of about 72 Feet, the Water in the Tube will have risen from H to L, through the Space of one Inch; and so in Proportion for any other Altitude, as I have several times found by trying the Experiment.

6. From hence it appears, that the Altitude of one Inch of Water is equivalent to the Altitude of 72 Feet, or 864 Inches of Air; and therefore the specific Gravity of Air is to that of Water as 1 to 864, or, as Sir *Isaac* has stated it, 860. Now since the specific Gravity of Water is to that of Mercury as 1 to 14, therefore the specific Gravity of Air to that of Mercury will be as 1 to $860 \times 14 = 12040$; and since the Height of Mercury supported by the Air in the Barometer is 2,5 Feet; if we say, As $1 : 12040 :: 2,5 : 2,5 \times 12040 = 30100$ Feet, which would be the Height of the Air were it every where as dense as at the Earth, or about $5\frac{3}{4}$ Miles.

7. But since the Air is not uniformly dense, we must seek its Height by another Method to be taught by and by. In the mean time, as the Air's Density constantly decreases, we shall shew how to find the Ratio of its Density at any Altitude to that at the Earth's Surface. Thus, since the Densities are as the compressing Force, which is as the Altitude of the incumbent Column of Air, and since the Weight of Mercury is to Water as 1 to 14, it is plain that the Air which supports a Column of Mercury 2,33 Feet, will sustain a Column of Water to the Height of 33 Feet. The Density on the Earth's Surface then is as 33.

8. Again; it is evident, since 860 Feet Altitude of Air is equal in Weight to 1 of Water, therefore at the Height of 860 Feet above the Earth, the Air (continuing in the same State) would sustain only 32 Feet of Water. At the Height therefore of 860 Feet, the Density of the Air is as 32.

9. Hence the Density at any other Altitude is easily found by the Hyperbola *f a i b*, and its Asymptotes SF and S*t*, S being the Center of the Earth, and A its Surface: Then the

Mercury

Mercury for the *mean Altitude*, and consequently its Weight for the *mean Weight* of a Pillar of Air of the same Base. But a Column of Mercury

Earth's Semidiameter SA = 4000 Miles nearly, or 21120000 Feet. Take AB = 860 Feet, and let the Density of the Air be required for any other Height, as AC = 7 Miles, or 36960 Feet. In the Points A, B, C, erect the Perpendiculars AH, BI, CK, which let be made proportional to the Densities of the Air in the Points A, B, C; that is, let AH : BI :: 33 : 32, and AH : CK :: 33 : x; and from the Points H, I, let fall the Perpendiculars Ht, Iu.

10. Then putting SA = Aa = r, SB = a, SC = b, and SB : SA :: Aa : Bb = $\frac{rr}{a}$; thus Cc = $\frac{rr}{b}$; then Aa — Bb = $\frac{ar-rr}{a}$, and Aa — Cc = $\frac{br-rr}{b}$. And put $m : n :: 33 : 32 ::$ AH : BI. Then, by the Nature of the Hyperbola, we have the Area $thiu$ as the Logarithm of $\frac{St}{Su}$, and the Area $tbkw$ as the Logarithm of $\frac{St}{Sw}$; or the Area $thiu : tbkw ::$ L. $\frac{m}{n}$: L. $\frac{m}{x}$. But (by *Coroll.* to *Prop.* XXII. *Lib.* 2. of the *Principia*) it is, $thiu : tbkw ::$ Aa — Bb : Aa — Cc :: $\frac{ar-rr}{a} : \frac{br-rr}{b} :: \frac{a-r}{a} : \frac{b-r}{b}$.

11. Now, because $a-r$ = AB = 860, and $b-r$ = AC = 36960, we have $\frac{860}{21120000} : \frac{36960}{21156960} ::$ L. $\frac{m}{n}$: L. $\frac{m}{x}$; whence we have L. $\frac{m}{x} = 0{,}573190$; therefore L. m — L. x = 0,573190; whence L. m — 0,573190 = L. x = 0,945324, the Number answering to which is 8,817 = x = CK, the Density required. Or, the Density at A is to the Density at C as AH to CK, or as 33 to 8,817, which is nearly as 4 to 1.

12. Since the Densities m, n, x, are defined Logarithms, it is evident they must be in a *geometric Progression*. But to shew these Things more generally: Let SC = x be a variable Distance, and its Fluxion CE = $\dot{x}$; let the Density CK = y, the compressing Force in the Altitude C as v, and the Power of Gravity as g. Then will the specific Gravity of the Air

whose

whose Base is *one Square Inch*, and Altitude 29½, weighs about 15 *lb.* which is equal to the Pressure of Air on every *Square Inch*; and therefore upon

be there as gy; for it will be as the Density y when the Gravity g is given, and as the Gravity when the Density is given; and when neither is given, it will be conjointly as both.

13. Since the whole Weight or Pressure of a Column of any homogeneous Fluid, of a uniform Density, is as its specific Gravity multiplied by its Magnitude, (by *Annot.* LVI. 10.) and if the Base be the same as the said Gravity multiplied by the Altitude, and therefore its Fluxion as the specific Gravity multiplied by the Fluxion of the Altitude; therefore we have $gy\dot{x} = -\dot{v}$, because the Density of the Air through the very small Space CE may be look'd upon as uniform; and since the Pressure decreases as the Altitude x increases, therefore it is that we make the Fluxion of it negative, *viz.* $-\dot{v}$.

14. If the Gravity g be as $\frac{1}{x^m}$, and the Density y as any Power n of the compressing Force v, *viz.* if $y : v^n$, and therefore $v : y^{\frac{1}{n}}$, by taking the Fluxions we have $\frac{1}{n} y^{\frac{1-n}{n}} \dot{y} = \dot{v}$. In the Place of g and $\dot{v}$ in the Equation $gy\dot{x} = -\dot{v}$, let their Values be substituted, and we have $\frac{1}{n} y^{\frac{1-n}{n}} \dot{x} = \frac{-\dot{x}}{x^m}$.

15. If we put $n = 1$, that is, if the Density be as the compressing Force, we have $\frac{\dot{y}}{y} = \frac{-\dot{x}}{x^m}$. Now, since any Quantities x, $x + a$, $x + 2a$, $x + 3a$, in arithmetical Progression, have all their Fluxions equal and the same, viz. $\dot{x}$; therefore if any Quantities $\frac{1}{x^{m-1}}$ be taken in such a Progression, their Fluxions $\frac{-\overline{m-1} \times \dot{x}}{x^m}$ or $\frac{-\dot{x}}{x^m}$ will be all the same or constant; therefore $\frac{\dot{y}}{y} = -\frac{\dot{x}}{x^m} = 1$; consequently $\dot{y} : y$, that is, the Fluxions of the Densities are as the Densities themselves; which therefore are in Geometrical Progression, as is manifest from the Doctrine of Fluxions.

every

every *Square Foot* it will be 2160 *lb.*; and allowing 14½ *Square Feet* for the Surface of the Body of a middle-sized Man, it must sustain a Pressure of

16. If in the same Hypothesis you put $m = 1$, or suppose the Gravity to be every where uniform, or given; then $\frac{\dot{y}}{y} = \frac{-\dot{x}}{x}$. If now we take $\frac{-\dot{x}}{x}$ constant, or make $\dot{x} : x$, then will the Distances x be in Geometrical Progression; and in that Case also we have $\frac{\dot{y}}{y} = 1$, or $\dot{y} : y$; whence also the Densities y are in Geometrical Progression.

17. The Fluent of the above Equation $\frac{1}{n} y^{\frac{1-2n}{n}} \dot{y} = \frac{-\dot{x}}{x^m}$ is $\frac{1}{1-n} y^{\frac{1-n}{n}} = \frac{-1}{m-1} x^{1-m} + Q$, a constant Quantity. Here, 'tis plain, it cannot be $n = 0$, for then $y^{\frac{1}{0}}$ = Infinite; nor can $m = 1$, because then it would be $x^{1-m} = x^0 = 1$, and so the Density y would be every where the same, or constant; neither can $n = 1$, for then $y^{\frac{1-n}{n}} = y^{\frac{0}{n}} = 0$. To determine the Value of Q, we must first define the Altitude SF, where the Density vanishes, or $y = 0$, and call it $a = SF$; then we have $Q = \frac{-1}{m-1} a^{1-m}$; and hence $\frac{1}{1-n} y^{\frac{1-n}{n}} = \frac{a^{1-m} - x^{1-m}}{m-1}$; where 'tis plain $\frac{1-n}{n}$ ought to be a positive Number, and less than Unity, that while the Distances x increase, the Densities y may decrease.

18. If the Altitude at which the Density y vanishes be supposed infinite, then $Q = 0$, and the Equation is $\frac{1}{1-n} y^{\frac{1-n}{n}} = \frac{1}{m-1} x^{1-m}$. For if in the Equation (*Art.* 17.) $y = 0$, and x = Infinite, then $\frac{a^{1-m}}{m-1} - \frac{x^{1-m}}{m-1} = 0$; therefore $\frac{a^{1-m}}{m-1} = \frac{x^{1-m}}{m-1}$, and so $a = x$ = Infinite, contrary to Supposition.

31320

31320 Pounds, or 14 *Tons*, when the Air is of a mean Gravity. This prodigious Force would crush us into a very small Compass, were it not

19. If now the Gravity be reciprocally as the Squares of the Distance, or $m = 2$, we have the Equation $\frac{1}{1-n} y^{\frac{1-n}{n}} = \frac{1}{m-1} x^{1-m}$ become $\frac{1}{1-n} y^{\frac{1-n}{n}} = \frac{1}{x}$; whence y will be reciprocally as $x^{\frac{n}{n-1}}$, which is a general *Formula* for any Hypothesis of the Ratio of the compressing Power and Density. Thus, if you suppose the compressing Force in the duplicate Ratio of the Density, that is, $y^2 : v$; then $y = v^{\frac{1}{2}}$, and $n = \frac{1}{2}$, and therefore $\frac{n}{1-n} = 1$, whence y will be reciprocally as x. Hence all those Cases of *Scholium* to *Prop.* XXII. *Lib.* I. of the *Principia* are derived, and any others at Pleasure.

20. The Density of the Air decreasing indefinitely, it is evident there is no certain Limit or Boundary of the Atmosphere, which gradually rarefies into pure *Æther*, or *Aura*, as it is often call'd. But since one principal Effect of the Air is the Refraction of Light, and since the Particles of Light are the smallest Bodies we know of in Nature, 'tis reasonable there to fix the Boundary of what we may properly call *Air*, in the Altitude where it begins to have the Power of producing this least Effect in Nature, *viz.* the refracting a Ray of Light.

Pl. XXX. Fig. 2. 21. To discover this Altitude of the Air we have the following Method. Let ADF be the Surface of the Earth, S the Sun below the Horizon, SB a Ray of Light touching the Earth, which is reflected by a Particle of Air, in the highest Part at B, in the horizontal Line BA to a Spectator at A. The Angle SBN is the Depression of the Sun below the Horizon in this Case, which, because it is at the Moment Twilight ends, is known from Observation to be about 18 Degrees. But because BA is also a Tangent, the Angle ACD = SBN = 18 Degrees; and the Angle ACB = $\frac{1}{2}$ ACD = 9 Degrees; which would be true, did the Ray SB pass through the Atmosphere without Refraction; but because it does not, but is refracted or bent towards H, the Angle ACB must be

that

that it is equal on every Part, and counter-balanced by the equal Re-action of the Spring of the Air within us. (XC.)

diminished by the horizontal Refraction, which is about half a Degree; whence the Angle ACB = 8° 30′.

22. Therefore in the right-angled Triangle ACB we have all the Angles given; and one Side, (*viz.* AC = 4000 Miles; or the Semidiameter of the Earth,) to find the Side BC; thus;

As the Sine ABC	= 81° 30′	=	9.995203
Is to the Side AC	= 4000	=	3.602060
So is Radius	90°	=	10.000000
To the Side BC	= $4044\frac{1}{2}$	=	3.606857

wherefore BC − HC = HB = $44\frac{1}{2}$ Miles, the Height of the Atmosphere required.

(XC.) 1. Since a cubic Inch of Mercury weighs very nicely 8,1 *oz. Averdupois*; a Pillar of Mercury, whose Base is one square Inch, and

				lb.	*oz.*	
Altitude	28	Inches, will weigh		14	3	*ferè.*
	$29\frac{1}{2}$			14	15	
	31			15	11	

2. So that the Air, at a Mean Gravity, is equivalent to the Pressure of 15 *lb.* upon every *square Inch*; and therefore upon every *square Foot* it will be equal to 2160 *lb.* and 2160 × 14,5 = 31320 *lb.* or 14 Tons nearly, the Weight or Pressure sustain'd by a middle-sized Man. When the Air is lightest, this Pressure is $13\frac{1}{3}$ Tons; and when, heaviest, it is $14\frac{3}{10}$ Tons; the Difference is 1,1 Tons, = 2464 *lb.* the Weight with which we are compress'd more at one Time than another.

3. This great Difference of Pressure must greatly affect us in regard to the animal Functions, and consequently in respect to our Health. If a Person, for instance, be asthmatical, he will find his Disorder increase with the Levity of the Air; for since a pure, dense, elastic Air, which is very heavy, is only capable to distend his Lungs in Respiration, when the Air is less compress'd by its diminish'd Weight, it will have less Elasticity, and so be less capable of expanding the Lungs; the Valetudinarian will therefore find his Difficulty of Breathing increase in Proportion.

THE Weight of the Air is proved by a great Variety of curious Experiments, the principal of which here follow.

4. Again, the Reason why we think the Air lightest in fine Weather, when it is really heaviest, is because the greater Pressure constringes and braces the Fibres and Nerves, and brings them to a due Tone, by which Means all the Blood-Vessels act with their full Power and natural Vigour; hence a proper Velocity is given to the Fluids, and a greater *Momentum* to overcome Obstructions in the Capillaries; thus by a brisk Circulation of the Fluids, and a due Compression of the Solids, we find ourselves firm and well, alert and light, and therefore fancy the Air is so.

5. Whereas, on the contrary, when this Pressure is lessen'd by near 2500*lb.* the Fibres are relax'd, the contractile Force of the Vessels diminish'd, a languid Circulation ensues, Obstructions, Viscidities, &c. happen, and produce Agues, Fevers, Aches, &c. in some; and in all, a Sort of Indolence or gloomy Inactivity, and Heaviness; and therefore we imagine that it results from the Heaviness of the Air, when it is just the contrary.

6. If it be required to find the Weight of the whole Atmosphere on the Earth's Surface, we may proceed thus: Suppose the Earth's Diameter in round Numbers 8000 Miles, the Area of a great Circle will be $8000 \times 8000 \times 0{,}7854 =$ 50266400 square Miles, which multiplied by 4, gives 201065600 square Miles for the Surface of the Earth; but because we took the Diameter a little too large, we may take 200,000,000 for the Number of square Miles in the Earth's Surface; in one square Mile are $(5280 \times 5280 =)$ 27878400 square Feet, therefore on the Earth's Surface we have 5575680000000000 square Feet, which multiplied by 2160 (the Pressure on each square Foot, *Article* 2.) gives 12043468800000000000*lbs.* for the whole Pressure. *N. B.* Since 2240*lb.* make a Ton, the Pressure 2160*lb.* upon a square Foot, is very near a Ton Weight.

7. I shall now present the Reader with a Solution of a very
Pl. XXX. Fig. 3. curious Problem, viz. *To find the Thickness* FH *of an hollow Ball or Globe* FDME, *made of any given Metal,* &c. *whose specific Gravity is known, such that it shall swim immersed in part or wholly in any homogeneous Fluid, whose specific Gravity is also known.* Let AB be the Surface of the Fluid, and let the Globe FDE swim therein, immersed to the Depth LM;

(1.) By

(1.) By actually *weighing it in a nice Balance*; where we ſhall ſee that *one Gallon of Air* will weigh a *Dram* very nearly. (2.) By filling a Glaſs Tube with Mercury, and inverting it in a Baſon of the ſame Fluid, where it will appear that a Column will be ſupported in the Tube by the ſole Weight or Preſſure of the Air, to upwards the Height of 28 Inches. (3.) By taking the Air off from the Surface of the Quickſilver in the Gage of the Air-Pump, which then immediately riſes by the Preſſure of the external Air. (4.) By exhauſting a Receiver placed over the Hole of the Braſs Plate on the Pump, which will then be kept faſt on by the Preſſure of the incumbent Air. Or, (5.) More demonſtratively by exhauſting a ſmall Receiver under one larger, and letting in the Air at once upon it; which will then be faſten'd to the Plate, as before, though not

and let the ſpecific Gravity of the Metal be to that of the Liquor as 1 to 12.

8. Then putting the Diameter $FM = D$, $HN = d$, $LM = x$; we have the ſpherical Shell equal to the Sphere FDE — Sphere HIK, that is, $\frac{pD^3}{6} - \frac{pd^3}{6}$; alſo the Segment of the Liquor DME, is $\frac{pDx^2}{2D} - \frac{px^3}{3D}$; and, in Caſe of an Equilibrium between theſe Quantities, we have $\frac{pD^3 - pd^3}{6} : \frac{pDx^2}{2D} - \frac{px^3}{3D} :: n : 1$. Whence we ſhall get $D^3 - d^2D = 3Dx^2n - 2nx^3$; and thence $D^3 - 3Dx^2n + 2nx^3 = d^2D$; or $D^2 - 3x^2n + \frac{2nx^3}{D} = d^2$; whence at laſt $d = \sqrt{D^2 - 3x^2n - \frac{2nx^3}{D}}$. Therefore $\frac{D-d}{2} =$ the Thickneſs of the Shell required.

placed over the Hole. (6.) By placing the Hand on the open Receiver, and exhausting, the Weight of the Air on the Hand will be extremely sensible. (7.) By placing a plain Piece of Glass on the said open Receiver, which, when the Air is a little exhausted, will be broke into Pieces by the Weight of the Air. (8.) A Bladder tied over the same Glass will be broke in the same manner. (9.) The Air exhausted from a thin Bottle under a Receiver, and then suddenly let in, will, by its Weight, instantly reduce it to very small Pieces. (10.) A Bottle broke by the same means another way. (11.) By putting a Piece of Wood under Quicksilver in the Receiver, and then exhausting the Air, and letting it in again, it will by its Weight force the Quicksilver into the Pores of the Wood, and very sensibly increase its Weight. (12.) The exhausted Brass Hemispheres prove not

9. If we suppose the Body to swim in the Fluid wholly immersed, then $x = D$, and $d = \sqrt{D^2 \times \overline{1-n}} = D\sqrt{1-n}$. Now admit FDE be a Sphere of Copper 10 Feet in Diameter, and that the fluid Medium be Air, whose specific Weight to that of Copper is as 1 to $860 \times 9 = 7640$; hence $n = \frac{1}{7640}$, and therefore $1-n = 1 - \frac{1}{7640} = \frac{7639}{7640}$; and because D = 10 Feet, or 120 Inches, therefore $120\sqrt{\frac{7639}{7640}} = d = 119{,}992$; and so $\frac{D-d}{2} = 0{,}004 =$ FH, the Thickness of the Metal requisite for the Globe to swim in Air.

10. But in order to this, one Thing more is necessary, *viz.* that *the Concavity of the Globe be a pure Void or* Vacuum; for if it be fill'd with Air only, the Globe will sink in the Air, be it ever so thin; because in that Case it must be heavier than an equal Bulk or Globe of Air. Hence we see how

only

only the prodigious Weight of the Air, but also the Quantity thereof very exactly. (13.) By exhausting Glass Bubbles swimming in Water, and letting the Air in again, it will force the Water into the Bubbles, and make them sink. (14.) The Syringe with its Weight descending *in Vacuo*, and ascending again upon the Admission of Air, does very prettily prove the *Pressure of the Air*, and the *Rationale* of Syringes in general. (XCI).

impossible a Thing is that *Aerial Navigation*, which *Franciscus de Lanis* and other Miracle-Mongers have amused us with, before true Philosophy appear'd to deliver us from those vain Speculations, and fruitless Attempts that may be gronnded thereon.

(XCI) 1. I shall here give the *Rationale* of the several *Phænomena* of the Experiments on the Air-Pump, as they are shewn in the Order of my Lectures on this Subject. The *First* of which is, *to shew the Absolute Weight of the Air by weighing it in a Balance*; of which we have already given an Account in *Annot*. LXXXVII. 2.

2. The *Second* is, *fixing a small Receiver on the Plate of the Air-Pump, by exhausting the Air out of it*. The Reason of which is, that the Pressure of the Air acts now alone on the Outside of the Glass, and perpendicularly on its Top, and presses it down with a Force equal to so many times 15 *lb.* as there are Square Inches in the Top of the Glass, or in the largest horizontal Section of it. The Spring of the Air, (which is always equipollent to the Pressure) being now taken away from within the said Receiver, will leave it to sustain the entire Force of Pressure, which will therefore fix it fast down to the Plate.

3. The *Third* Experiment *fixes the Glass firmly on the Plate, not, as before, over the Hole, but on one Side of it*. This is to undeceive People in regard to the common erroneous Notion of a *Suction*, which they suppose is something within-side of the Glass that draws it down as the Air passes out through the Hole. But when they see the Glass placed on one side the Hole under a Receiver, and that as the Air is drawn out of

THAT Water rises in *Pumps*, *Syphons*, and all Kinds of *Water-Engines*, by the *Pressure of the Air* only, is made evident by taking off the said Pressure (in the exhausted Receiver) from a Bason of Mercury, which then will not rise in the Pipe of the Syringe on drawing up the Piston, as it will in the open Air.

the Receiver it will by its Spring all escape from under the Glass at the same time, and then when the Air is let into the Receiver all at once, it falls on the little Glass, and fixes it down in such manner that it is plainly seen to sink into the Leather upon the Plate; I say, when all this is seen and consider'd, it entirely eradicates that vulgar Error, and sets the Truth in a clear Light.

4. The *Fourth* Experiment *fixes a Person's Hand on the Top of an open Receiver*. This is done by the Pressure of the Air on the Top or Back of the Hand, when the Spring of the Air is wanting within the Receiver to counter-act it. This great Pressure is very sensible to the Hand, though not hurtful; and the Skin and Flesh is visibly press'd down between the Metacarpal Bones. The Spring of the Air in the Hand at the same time exerts itself, by extending the Skin and Flesh of the Part of the Hand on the Glass as far down as possible, by which means the Blood flows thither in great Quantity, as in Cupping, and makes the Part look very red. If the Area of the Top of the Receiver be 4 Square Inches, the Hand will be press'd or kept on by a Weight equal to 60 *lb*.

5. The *Fifth* Experiment is *fixing the Brass Hemispheres together by the Pressure of the external Air, in such manner as to require two strong Men to pull them asunder*. This is done by exhausting the Air from their Cavity, and thereby taking away the Spring, leaving the Pressure to act alone. If the Diameter of the Hemispheres be 4 Inches, the Area will be 12,556 Square Inches, which multiplied by 15 gives 188,3 *lb.* by which they are compress'd together.

6. The *Sixth* Experiment *shews the Spring of the Air throwing the Air out of a Glass-Bubble through the Water in which it is placed, in Form of large round Bubbles of Air*. This is done by taking the Pressure of the Air off the Surface of the Water in the Jar under the Receiver; and by that means the Spring of the Air, having nothing to counter-act or confine it, will exert itself, and cause the Air to escape out of the

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THE SPRING *of the Air* is demonstrable by various Experiments: As, (1.) By the great Expansion of a small Quantity of Air in an emptied Bladder, when the Air is taken off from the external Parts in the Receiver. (2.) By the Extrusion of a Fluid out of a Glass Bubble, by the Expansion of the Bubble of Air contain'd therein.

Bubble, and from all Parts of the Water, in very small Globules rising up to the Top; whence, by the way, it will appear, that Water is a very porous Body, and all its Interstices possess'd by Air, which is now expanded into visible Volumes or Globules, and seen to make its Escape.

7. The *Seventh* Experiment is but a Part of the former, and shews, *that upon letting the Air again into the Receiver, it falls on the Surface of the Water, and by that means compresses the whole Body of Water, and drives Part of it into the evacuated Glass-Bubble, which then becomes heavier than Water, and sinks to the Bottom.* As there is but very little Air left in the Bubble, its Spring will be very weak, and so will yield to the Force of the external Air compressing the Water, and therefore will give the Water Admittance till it becomes so far compress'd as to have a Spring equal to that of the outward Air, or to that wich was in the Bubble at first. Its Density will then be the same also; and its Bulk, compared with the whole Bulk of the Bubble, will shew what Part of the whole Quantity of Air remained after Exhaustion.

8. The *Eighth* Experiment *shews that the same Bubble, placed with its Neck upon a hollow Glass over a small Bason, under the Receiver, upon exhausting the Air the small Quantity of Air in the Bubble will again expand itself, and drive out all the Water.* The Pressure of the Air, which before kept the Water in the Bubble, being now taken away, the Spring of the Air in the Crown of the Bubble gradually exerts itself, and at last expels all the Water. From this Experiment it plainly appears, that the Spring of the Air is equal to the Pressure, because the Spring drives out all the Water which the Pressure forced into the Bubble.

9. The *Ninth* Experiment *is the Expulsion of the Contents of an Egg through a small Hole in the little End by the Spring of the Air contain'd in the great End of the Egg.* While the Egg is new and good, there is always a small Quantity of Air contain'd in the great End between the Shell and the Skin or *Pu-*

(3.) By the Expulsion of the White and Yolk of an Egg through a small Hole in the little End, by the Expansion of the Air contained in the great End; and also, (4.) By raising up the Skin of the Egg (after the Yolk is taken away, and one half of the Shell) by the Expansion of the said included Bubble of Air, so as almost to fill the Half-Shell,

tamen, which, upon taking off the Pressure of the Air from the Hole, will expand itself, and drive out the *White and Yolk* through the said Hole in the little End.

10. The *Tenth* Experiment *shews, that when half the Shell of the Egg thus emptied is taken off, the said Bubble of Air will, upon Exhaustion, so expand itself by its Spring as to raise up the Skin of the Egg, and throw it so far out as to make the Resemblance of the entire Egg.* This will happen only when the Egg is quite new; for as the Egg grows stale, the Air will lose its Spring by degrees, and the Egg will become putrid or addle. It is observed by Naturalists, that this included Bubble of Air is absolutely necessary for the Production and Maturation of the Chick, which is effected by the Warmth and Fermentation occasion'd by the constant *Incubation of the Hen.*

11. The *Eleventh* Experiment *is to shew the great Quantity of Air contain'd in all solid Bodies.* For when a Piece of Brass, Iron, Stone, &c. is put into the Water of a Jar under the Receiver, and the Air drawn out, the Spring of the Air contain'd in the Pores of those solid Bodies, will, by expanding the Particles, cause them to appear on the Surface in numberless Globules, and exhibit a curious Spectacle to the Eye, like the pearly Drops of Dew on the Piles of Grass; all which suddenly disappear by letting the Air in again.

12. The *Twelfth* Experiment *shews, that a Piece of Cork with a Weight added to it, to make it just sink in the Water, will be raised to the Top, or made to swim, by exhausting the Air.* For the Bubbles of Air which are expanded from its Pores, and adhering to its Surface, render it lighter than Water, in which Case it must necessarily rise to the Top, or swim.

13. The *Thirteenth* Experiment *shews, that Glass-Images and Bubbles, which sink in Water, will, on exhausting the Receiver, rise to the Top and swim.* For the Bodies of these Images, &c. being hollow, are fill'd so far with Water as to make them just sink; and the rest of the Cavity being pos-

Shell. (5.) Glaſs Bubbles and Images fill'd with Water, ſo as to make them juſt ſink in Water, will, upon exhauſting the Air from the Surface, riſe to the Top of the Veſſel. (6.) Alſo a Bladder fill'd with Air, and juſt made to ſink with a Weight, will, upon Exhauſtion, ſoon riſe by the Expanſion of the contain'd Air. (7.) The Spring

ſeſſed of Air, this Air will, upon taking off the Preſſure of the external Air, exert its Spring, and drive out the Water from the Images and Bubbles; they then become lighter than the Water, and riſe to the Top. When the Air is let in again, the Water re-enters their Bodies, and they ſink down again.

14. The *Fourteenth* Experiment *ſhews, that a Bladder nearly emptied of Air, and ſunk with a Weight to the Bottom of a Jar of Water, will, upon Exhauſtion, riſe to the Top and ſwim.* The Reaſon of which is, that when the Preſſure of the external Air is taken off, the Spring of the little incloſed Air will dilate and expand the Bladder to its full Bulk; and then the Quantity of Water equal to its Bulk will be heavier than the Weight and Bladder, and ſo will buoy them up to the Top, according to the Laws of *Hydroſtatics*, which ſee.

15. The *Fifteenth* Experiment *raiſes Beer or Ale into a large white Head or Froth to the Top of the Jar.* This happens on account of the great Tenacity of the Fluid; for when the Preſſure of the Air is taken off, the Air in the Beer expands itſelf into large Globules, to which the Particles of Beer adhering on every Side render them too heavy to riſe from the Surface, and fly away in the Air. The Bubbles of Air being thus raiſed are, as it were, conglutinated or ſtuck together by the adheſive Quality of the Liquor; and thus riſe in great Quantities, the upper Part being raiſed and ſuſtain'd by the Expanſion of that below. When the Air is let in, the Air-Bubbles contract, ſubſide, and retire within the Pores of the Fluid. In the ſame Manner Soap-Water, Yeaſt, &c. will riſe in a Head.

16. The *Sixteenth* Experiment *is exhibiting the Phænomena of Boiling Water in the exhauſted Receiver.* To this End the Water muſt be as hot as the Finger can well bear when put under the Receiver. Upon exhauſting, the Air-Bubbles will be ſeen to riſe very ſoon, and at firſt very ſmall; they ſoon appear bigger, and at laſt are ſo large, and riſe with ſuch Ra-

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of the Air will ſhew itſelf alſo by raiſing heavy Weights laid on a Bladder, half fill'd with Air, in a proper Veſſel under the Receiver. (8.) Beer, Cyder, Water, and porous Bodies, do emit great Quantities of Air under the exhauſted Receiver. (9.) Fiſhes are made ſo light or buoyant by increaſing the Spring of the Air in

pidity, as greatly to agitate the Water, and cauſe it to appear in all the Circumſtances of Boiling; which Agitation of the Water will continue till the Air be let in again, and then it will ceaſe, and all will be quiet and ſtill as at firſt. Some People imagine the Water grows hotter by boiling under the Receiver, as it does over the Fire; not conſidering that Water boils only by the great Expanſion and Rarefaction of the Air it contains, from whatever Cauſe it proceeds, as from the Heat of Fire, from taking off the Preſſure of the Atmoſphere by the Air-Pump, &c.

17. The *Seventeenth* Experiment *ſhews, that a ſhrivel'd Apple will be plump'd out, and made to look fair, under the exhauſted Receiver.* The Reaſon of which is the Expanſion of the Air in the Subſtance of the Apple, when the Preſſure is taken off from its Surface; for though ſome Parts of the Skin be pervious to the included Air, (as appears by the little Streams riſing from the Pores if the Apple be placed in Water) yet the greateſt Part of the Surface is not, and will not therefore ſuffer the Air to paſs out, but will yield and expand to its utmoſt Dimenſions, (and ſometimes burſt) on which all the Wrinkles diſappear, and the Apple puts on a youthful Face, till the Air be again let in, when it inſtantly returns to its former State of Decay and ſhrivel'd Countenance.

18. The *Eighteenth* Experiment *exhibits the beautiful Appearance of Air riſing from all Parts of a vegetable Subſtance very copiouſly through the Water in Vacuo.* For when the Preſſure of the incumbent Air is taken off, the Spring of the Air, contain'd in the Air-Veſſels of Plants, will, by expanding the Particles, cauſe them to riſe from the Orifices of all the Veſſels, and that for a long time together, by which is ſhewn what a great Quantity of Air is contain'd in all vegetable Subſtances; and ſince it is ſeen to come out of the Sides or all over the Surface of a Piece of Stick, as well as from its tranſverſe Sections, it is a convincing Proof, that the Texture of the Stems of Plants and Trees conſiſts of Veſſels in a

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their Bladders, upon Exhaustion, that they rise to the Top of the Water, and cannot again descend to the Bottom. (10.) Shrivel'd Apples are made to look fair and smooth by the Spring of the contain'd Air filling out the Wrinkles. (11.) The Spring of the Air in a square Bottle, cemented close, will immediately burst it in pieces, upon

longitudinal and also in an horizontal Position.

19. The *Nineteenth* Experiment *shews the Method of injecting a vegetable Substance with Quicksilver.* Thus if a Piece of Stick be cut even at each End with a Penknife, and immersed in Mercury, upon pumping out the Air from the Receiver it will at the same time come out of the Pores of the Wood through the Mercury, as will be visible at each End. When the Air is let in again, it falls on the Surface of the Mercury, and forces it into the Pores of the Wood to possess the Place of Air. When the Wood is taken out and weigh'd, it will be found several times heavier than before; it will have changed its Colour, being now of a blueish Hue all over; and, if split or cut transversely, the Quicksilver will be seen glittering in all its Pores, and through every Part.

20. The *Twentieth* Experiment *is the breaking of a Bladder by the Weight of the Air.* For if the Bladder be tied over one End of an open Receiver, as the Air is exhausting the Spring will be weaken'd, and give way to the Pressure of the Air on the Bladder, in which Case the Bladder will put on a concave Figure, which will be nicely spherical; and this will continue increasing, till the Strength of the Bladder be overcome by the Pressure, when it will break with a very great Report.

21. The *Twenty-first* Experiment *is the breaking a Glass-Bottle by the Pressure of the Air.* For this Purpose the Bottle ought to be of a square Form, and not cylindrical or globular; it should also be not very thick, if small. Then the Bottle is screw'd on to the Hole in the Plate of the Pump, and the Air drawn out; by this means the Bottle sustains the Pressure from without, so long as its Strength will permit; then the Parts yield, and the Bottle is instantly reduced into very small Pieces.

22. The *Twenty-second* Experiment *breaks a Bottle by the Spring of the Air.* For the Mouth of the Bottle being securely seal'd up, so that no Air from within can escape, it is put under the Receiver; and as the Air is drawn off from its

exhausting

exhausting the incumbent Air. (12.) But that curious Experiment which shews the Force of the Spring of the Air to be equal to its Weight or Pressure, is by raising the Mercury, by the Expansion of a small Quantity of confined Air, to the same Height in an exhausted Tube above the Pump, as that which it is raised to in the

Surface, the Spring of the Air included will take place, and act more and more forcibly against the Sides of the Glass, which having now nothing but its own Strength to defend it, as soon as that is overcome the Parts give way, and the Glass is burst in pieces.

23. The *Twenty-third* Experiment *is to shew, that a Bladder being emptied of its Air, all to a very little, and then suspended in the Receiver, the little Portion of Air will expand itself in such manner upon Exhaustion, that at last it will distend and fill out the Bladder to its utmost Bulk, and make it appear as one full-blown.* The Reason of which is apparent from what has been so often repeated above; as also, of its contracting again when the Air is let in.

24. The *Twenty-fourth* Experiment *shews, that the Syringe will descend from the suspended Piston* in Vacuo, *when the Hole at bottom is stopp'd, and a small Weight added to overcome the Friction.* If the Hole be stopp'd in the open Air, and the Piston drawn up, it will be resisted by the Pressure of the incumbent Column of Air; but *in Vacuo*, where this Air is taken away, the Piston may freely rise; or, which is all one, the Syringe may descend; as it will, if a small Weight be added to overcome the Friction of the Piston. When the Air is let in again, it will be seen to push up the Syringe upon the Piston again.

25. The *Twenty-fifth* Experiment *shews, that Water rises in Pumps, and Quicksilver in the Barometer, by the Pressure of the Air only.* For a Glass Tube being screw'd on to the above-mention'd Syringe, and immersed in the Mercury in the open Air, if the Piston be then lifted up, it will attenuate the Air contained in the Glass Tube, by giving it a greater Space to expand in, and by this means lessen its Spring. The Pressure then of the external Air will raise so much Mercury into the Tube, as its Weight added to that of the Spring of the included Air is an equipollent Force, and then an Equilibrium will ensue: But if the Mercury be placed under the exhaust-

Mercurial

Mercurial Gage by the Pressure of the Atmosphere below it.

THE great Action of animal Life, *viz. Breathing*, by *Inspiration* and *Expiration* of Air, is owing to the *Pressure* and *Spring* of the Air conjointly, as is evident by the *Contraction* and *Expansion* of a Bladder in a small Receiver, with a Blad-

ed Receiver, and the Piston lifted up, no Mercury will then be seen to rise; which plainly shews the Cause, *viz.* the Air's Pressure is in that Case taken away.

26. The *Twenty-sixth* Experiment *shews, that the Spring of the Air has a Force equal to the Pressure of the Air, by raising the Quicksilver to the same Height.* For if a Tube open at both Ends be cemented into a Glass Vial, nearly fill'd with Quicksilver, and placed under the exhausted Receiver, as the Air is gradually exhausted you will see the Mercury rise from the Vial into the Tube above the Pump, by the Spring of the included Air, to the same Height as it is in the Gage-Tube below by the Pressure, and that during the whole Time of the Exhaustion. And this will always happen, let the Quantity of Air in the Vial be ever so small, or what it will; the Phænomenon depending not upon the Quantity, but the Strength of the Spring.

27. The *Twenty-seventh* Experiment *shews the Method of making an artificial Fountain* in Vacuo, *by the Air's Pressure.* For this Purpose a very tall Glass Tube is hermetically closed on the Top, and at Bottom by means of a Brass Cap screw'd on to a Stop-Cock, and that to the Plate of the Pump; then, when all the Air is exhausted, the Cock is turn'd, and taken off the Plate, and immersed in Mercury or Water: Then, upon turning the Cock again, the Fluid by the Pressure of the Air will be seen very beautifully to play up in the Tube in the Form of a Fountain.

28. The *Twenty-eighth* Experiment *shews, that the Magnetic Virtue from the Stone, or a touch'd Piece of Iron, affects the Needle* in Vacuo, *in the same manner as in open Air.*

29. The *Twenty-ninth* Experiment *shews, that the Attraction of Cohesion is the same* in Vacuo *as in the open Air.* For this Purpose a large Glass Tube, drawn out into a very fine Capillary at Top, when fill'd with Water will sustain it to a certain Height in the Air: If the same be placed under the Receiver, and the Air drawn out, the Water will remain suf-

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der tied on at Bottom to repreſent the Diaphragm.

HENCE the Neceſſity of Air for *Reſpiration* and *animal Life* in moſt Sorts of Creatures, which die very ſoon in the *exhauſted Receiver:* Though ſome Animals will not be kill'd in this manner; as *Flies*, *Frogs*, *Toads*, ſome ſort of *Fiſhes*, &c.

pended as before; which ſhews it to be wholly owing to the Force of Attraction.

30. The *Thirtieth* Experiment *ſhews, that Bodies, which equilibrate each other in the Air, loſe their Equilibrium* in Vacuo. Thus if a Piece of Lead at one End of a fine Balance, and a Piece of Cork at the other, are *in Equilibrio* in the Air, and thus placed under the Receiver, as ſoon as the Air begins to be exhauſted, ſo ſoon the Equilibrium will begin to be deſtroy'd, till at laſt, when all the Air is taken away, the Cork will deſcend, and ſhew itſelf really heavier than the Lead. The Reaſon of which is evident from Hydroſtatic Laws; for both Bodies being weigh'd in Air, each would loſe the Weight of an equal Bulk of Air, conſequently the Cork will loſe a greater Weight than the Lead in the Air; and therefore when the Air is taken away, the Weight that is reſtored to it being greater than what the Lead has retriev'd, will cauſe it to preponderate, or weigh down the Lead *in Vacuo*. And hence we ſee, that *a Pound of Feathers is really heavier than a Pound of Lead,* if weigh'd in the Air.

31. The *Thirty-firſt* Experiment *ſhews the Air to be the Medium of Sounds*. For if a Bell be ſcrew'd on to the Air-Pump, it will ring in the Air, and be heard under a thin Receiver: But when the Air is exhauſted, the Sound is not heard, which plainly proves it to be propagated by means of the Air, and this is farther evinced by letting the Air gradually into the Receiver, becauſe if, in the mean time, you keep ſhaking the Bell, the Sound will increaſe in proportion as the Glaſs is fill'd with Air.

32. The *Thirty-ſecond* Experiment *ſhews, that the Air is neceſſary for the Exiſtence of Fire and Flame.* Thus if Charcoal thoroughly lighted, and a Candle burning, be placed under the Receiver, as the Air is exhauſted the Coals will begin to decline and die away, and the Candle will go out by degrees.

33. The *Thirty-third* Experiment *ſhews the Riſe of Vapours and Smoke to be owing to the Air*; becauſe when the Air is ta-

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THAT Air passing through the Fire, and heated Brass Tube, is unfit for animal Respiration, is shewn by the sudden Death of any Animal put into a Receiver fill'd therewith. Also Candles and living Coals, put into this adust Air, immediately go out. Hence the noxious and pestilential Qualities of *Damps* and *suffocating Exhalations*,

ken away, the Vapours, which at first rise very plentifully from the wet Leathers of the Plate so as to obscure the Receiver, begin to fall when the Air becomes greatly attenuated; and the Smoke, which at first rose from the Candle extinct, now begins to descend; and when the Air is all exhausted, the Receiver becomes quite clear, and free from all Appearance of Smoke or Vapour. Hence, by the way, we see the Reason why, when the Air grows lighter, it lets fall the Vapours, and the Weather becomes misty, hazy, and wet or rainy.

34. The *Thirty-fourth* Experiment *shews the Explosion of Gunpowder is owing to the Air*. For if it be kindled *in Vacuo*, the Air, that so suddenly expands itself from the Powder, and gives such a Shock to the common Air, now finds none to encounter, and so makes no sensible Appearance, otherwise than by the sinking of the Mercury a little in the Gage by its Spring.

35. The *Thirty-fifth* Experiment *shews how Halo's are produced by refracted Light*. Thus if a Candle be held on one Side of the Receiver, and the Eye placed at some Distance on the other, as soon as the Air begins to be exhausted, and becomes attenuated and repleted with Vapours to a proper Degree, the Light of the Candle will be refracted through that Medium in Circles of various Colours, very much resembling those seen about the Moon in a hazy Air at Night.

36. The *Thirty-sixth* Experiment *shews how the Lungs of an Animal are affected* in Vacuo; *in what Manner it dies, and is revived again*. For this Purpose a small Bladder is tied to a Pipe, and screw'd into a Bottle, which then represents the Lungs in the Thorax. This Pipe is perforated quite through to the Bladder, and is therefore analogous to the *Trachea* or Wind-pipe. The Air confin'd in the Bottle about the Bladder is in the same Circumstances with that in the Breast about the Lungs. When this *Apparatus* is placed under the Receiver, one or two Exsuctions will attenuate the Air in the Re-

halations, ſo frequent and fatally experienced in Mines, and other ſubterranean Places.

THAT Air in its natural State is neceſſary *for Fire* and *Flame*, is obvious from the *ſudden Extinction of a Candle, a live Coal*, &c. in the *exhauſted Receiver*. Alſo Gunpowder fired there-

ceiver and Bladder, upon which the Spring of included Air in the Bottle will compreſs the Bladder, as that in the Thorax does the Lungs; and a few more Turns will cauſe the Bladder to be compreſs'd together. The Lungs being thus compreſs'd, the Animal is ſenſible of a prodigious Weight, the Circulation of the Blood through the Lungs is ſtopp'd, the Creature is all over convulſed, and at laſt expires in the greateſt Agonies of a moſt cruel Death. When the Air is let in again, the Bladder gradually expands, as do the Lungs of the Animal; and if it has not lain too long, the Blood will again paſs through them, and the Animal will recover its ſuſpended Life.

37. The *Thirty-ſeventh* Experiment *ſhews Air to be abſolutely neceſſary to moſt Sorts of Animals*. This we do by exhauſting the Air from a Cat, a Rat, a Mouſe, a Bird, *&c.* which ſoon die in the Manner above deſcribed. It is not always, indeed, that Gentlemen can thus ſuffer their Curioſity to get the Aſcendant ſo far over their Humanity, as to deſire ſo ſhocking a Spectacle. The Ladies (greatly to their Honour) ſhew more Conſideration, in generally voting againſt it.

38. The *Thirty-eighth* Experiment *ſhews Air is not abſolutely neceſſary to the Life of ſome Animals*: For it is well known that pumping the Air from a Toad, an Eel, a Viper, and all Sorts of Inſects, ſeems not immediately to affect them. Indeed, the winged Inſects cannot fly, but they will crawl and run about very briſkly. Some ſay Fiſh will die for want of Air; I confeſs I never could kill any. They appear greatly diſturb'd, ſwoln, and ſickiſh at firſt; but Mr. *Hawkeſby* ſays he has kept them a Week *in Vacuo*, and they recover'd their firſt Illneſs, and were at the Week's End as lively and alert as thoſe which had been kept as long in the Air.

39. The *Thirty-ninth* Experiment *ſhews no Winged Animal can fly without Air*. For this Purpoſe a large Butter-fly is a proper Subject, for as ſoon as it is put under the Glaſs it will fly and flutter about, but when the Air is taken away, no-

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in will not take Flame, or be *explosive*, but *melt and die away*.

THAT the different Velocities with which heavy and light Bodies descend in the Air, is owing to the Air's Resistance only, is manifest from the equal Velocity or Swiftness with which all Bodies

thing more of that kind is seen. If a fine Silk be tied about one of the Horns of this Animal, and it be thus suspended in the Middle of the Receiver, it will at first fly towards every Side of the Glass, but when the Air is exhausted, it cannot get out of the perpendicular Position into which it is brought by its Gravity, though it will be constantly endeavouring to do it.

40. The *Fortieth* Experiment *is that of Adust or Burnt Air*; this Air is brought into the Receiver thro' the Fire; and if a Candle be put down into it, it instantly goes out, and will do so for many times together; but every time the Candle burns longer than before; which seems to shew, that this Air is somewhat of the same Nature with that in Mines, commonly call'd *Damps*, and is, like that, purified again by Fire.

41. The *Forty-first* Experiment *shews that Adust Air is instant Death to most Sorts of Animals*. Thus a Sparrow put into this Air tumbles down with a kind of Vertigo, is convulsed, and dies directly; much after the same Manner as Men fall down dead in the contaminated Air of Mines, deep Wells, &c.

42. The *Forty-second* Experiment *shews that all Bodies descend equally swift* in Vacuo. Thus a Guinea and Feather let fall from the Top of a tall exhausted Receiver, come down to the Bottom in the same time, or both together. But when let fall from thence in the Air, the Feather will descend much slower than the Guinea, and with an oblique or indirect Motion.

43. The *Forty-third* Experiment *shews all Fermentation and Putrefaction depend on Air*. Thus Apples, Pears, Plums, Cherries, &c. which in the Air soon grow mellow, putrid and rotten, will, if kept in an exhausted Receiver, placed under Water, be preserv'd a long time untainted, appear fresh and in their native Bloom. Thus Eggs also, which in the Air soon grow stale, putrid, and addle, will *in Vacuo* retain their Goodness, and be fit for use after a great while.

descend in the *exhausted Receiver*, as is shewn in the Experiment with a *Guinea* and a *Feather*.

AIR is likewise necessary for the Existence and Propagation of Sounds; for a Bell placed under the Receiver, and rung, will not be heard when the Air is drawn out; but in condensed Air, the Sound will be augmented in proportion to the Condensation.

THAT *Fermentation, Putrefaction*, &c. depend on the Air, and are promoted by it, is shewn

Which is the Reason why many People keep them in Pots of Butter, Lard, &c. to preserve them from the Air.

44. The *Forty-fourth* Experiment *shews how necessary Air is for the Germination and Growth of Plants and Vegetables.* For if the same Seed be planted in two different Pots of Earth at the same time, and one of them be kept in an exhausted Receiver, the Difference between the Appearance and Growth of each will be sufficiently sensible to any that shall try the Experiment.

45. The *Forty-fifth* Experiment *shews, that the Writing made with Phosphorus upon Paper, laid on the Plate of the Pump, will* in Vacuo *appear luminous*, and not be extinguish'd like common Fire. It will also send up lucid Fumes or Clouds to the Top of the Receiver.

46. *If the Paper be wetted by Patches, on which the Lines have been drawn with Phosphorus, instead of a Cloud it will give Flashes* in Vacuo. For these Experiments with Phosphorus the Room should be made very dark.

47. The *Forty-seventh* Experiment *shews, that upon some Chemical Mixtures a strong Effervescency, Ebullition, and Accension will happen* in Vacuo. Thus if to an equal (but small) Quantity of Oil of Vitriol, Oil of Tartar *per Deliquium*, and Oil of Cloves, you put two or three small Pieces of Phosphorus, the Mixture will take Fire in the open Air, and is put out by the Addition of a little Water. It will not only shine, but boil up into a Flame *in Vacuo*.

48. *Melted Lead, and other Metals, set to cool* in Vacuo, *have their Surfaces concave*; whereas they are convex in the open Air. The Reason of which is the same as of the Expansion of Water when it congeals into Ice. Thus Ice be-

by

by preserving Fruit in their natural Bloom and Perfection through the Winter in an *exhausted Glass*.

THE Use of the *Diving-Bell* depends on the *Pressure* and *Spring* of the Air: For since the Space which Air takes up is reciprocally as the Power compressing it, 'tis evident that at the Depth of 33 Feet of Water, where the Pressure of the Atmosphere is doubled, the Bell will be half fill'd with Water, at the Depth of 66 Feet

comes specifically lighter than Water, and swims in it; as any solid Metal is specifically lighter than when melted: Thus a leaden Bullet swims in melted Lead. What Agent Nature employs in the Affair of Congelation, is perhaps as yet unknown to Mortals; but whatever it be, 'tis certain that one Part of its Operation is to sever the Particles, and fix them at a greater Distance from each other in the fix'd, than they are in the fluid State.

49. *The Chemical Process of Crystallization will not succeed in Vacuo.* If Salts be mix'd with Water and evaporated to a Pellicle, and then placed under an exhausted Receiver, and set in a cool Place as usual, it will not shoot into Crystals, as in the open Air it readily will.

50. The *Fiftieth* Experiment shews, that if a Piece of Wood be cemented in the lower Part of the Neck of the open Receiver, and Mercury be pour'd upon it, after two or three Exhaustions the Pressure of the Air will be so great on the Mercury, as to cause it to descend through the Pores of the Wood in Form of a beautiful Shower; which will shine (if it be well cleansed and the Weather dry) in a dark Room. The Air also will follow the Quicksilver through the Pores of the Wood, and cause the Gage to sink.

51. To these Experiments of a *Vacuum*, I shall add the following Particulars relating to the CONDENSATION of Air: As, (1.) That the Vessel ought to be very strong to bear the Force of the Air's Spring thus increased; for which Reason they are generally made of Brass. (2.) If Glass be used for a Condenser, it will not indeed suffer so great a Degree of Condensation, but the Experiment will be pleasanter, by viewing the Subject placed in the condensed Air. (3.) The Spring

it will be two Thirds fill'd; at the Depth of 99 Feet it will be three Fourths fill'd; and so on. Whence appears the Necessity of having the Vessel in the Form of a *Bell*, that the perpendicular Height of the Water may be as little as possible. Hence also we see how necessary it is to have a very gentle Descent of the Bell, that the Divers may have Time to admit the Air, so greatly condensed, by proper Degrees, lest it should burst the fine Vessels of their Bodies, and

of the Air will be greater in proportion to its Condensation; and therefore (4.) The Sound of the Bell will be twice and thrice as loud as in the common Air, if the Air be made twice or thrice as dense by Injection. (5.) A round Vial will be broke by condensed Air, that could not be broke by the Pressure of the common Air. (6.) Though Animals soon die by not having the natural requisite Quantity of Air, yet they will not be easily kill'd by having that Quantity increased by Condensation. (7.) If Air be condensed upon Water in a Bottle, it will cause it to spout through the Tube of Communication to a very great Height, *viz.* to 30 Feet, if only one Atmosphere be injected; to 60 Feet, if two; and so on. (8.) A Bladder, that will sustain the Spring of common Air, will be broke by the Spring of condensed Air. In short, the Force of condensed Air may be so far increased, as to countervail or antagonise the greatest Power of Nature that we can apply. (9.) Water with Air condensed upon it will conceive a much greater Degree of Heat than in the common Air, where it will boil much sooner than in condensed Air. (10.) So great may the Degree of Heat acquired in Water this way be, as to melt soft Solder; and therefore Vessels should have their Parts put together with hard Solder, that are used about these Experiments.

52. From this vast Power of confined and elastic Air and Steam it is that we account for the prodigious Effects of *Pappius*'s DIGESTER in dissolving Bones and reducing them to a Jelly, so as to become a wholsome and savoury Diet; for which Purpose they are put into a metalline Vessel, with a Cover, which is fast and strongly screw'd down, and Air-tight. The Digester nearly fill'd with Water and Bones

kill

kill them: Together with several other Particulars relating to the Nature and Manner of using this Machine, which will be more fully explain'd in the Note below (XCII).

is set over a gentle Fire, which by degrees rarefies the Water into Steam, which with the included Air in a short Space of Time acts upon the Bones with so great an Energy, as to effect their utter Dissolution, and cause them to mix and incorporate so intimately with the Water, or Broth, as to make a perfect *Coagulum* or Jelly when all is cold, which may be then sliced out with a Knife. They who would see more of the wonderful Effects of this Instrument may consult the Author's own Book upon the Subject.

(XCII) 1. That the Reader may have a just Idea of the *Campana Urinatoria* or DIVING-BELL, according to the latest Improvements by Dr. *Halley* and Mr. *Triewald* of *Stockholm*, I have here exhibited two Figures of the same. The first is that of Dr. *Halley*'s Form, which was 3 Feet wide at Top, 5 Feet at Bottom, and 8 Feet high; and contain'd about 63 Cubic Feet, or near 8 Hogsheads, in its Concavity.

2. This was coated with Lead, so heavy that it would sink empty; and the Weight was distributed about the Bottom IK, that it would go down in a perpendicular Position and no other. In the Top was fix'd a strong but clear Glass D, to let in the Light from above; and likewise a Cock, as at B, to let out the hot Air that had been breath'd; and below, as LM, was fix'd a circular Seat for the Divers to sit on; and lastly, from the Bottom was hung, by three Ropes, a Stage for the Divers to stand upon to do their Business. This Machine was suspended from the Mast of a Ship by a *Sprit*, which was sufficiently secured by *Stays* to the Mast head, and was directed by *Braces* to carry it over-board clear of the Side of the Ship, and to bring it in again. Pl. XXX. Fig. 4.

3. To supply the Bell with Air under Water with two Barrels, such as C, of about 63 Gallons each, were made and cased with Lead, so that they might sink empty, each having a Hole in its lowest Part to let in the Water, as the Air in them is condensed in their Descent, and to let it out again when they were drawn up full from below. And to a Hole, in the Top of the Barrels, was fix'd a Hose or hollow Pipe, well prepar'd with Bees-Wax and Oil, which was long enough to fall below the Hole at the Bottom, being

THE *Spring* of the Air is most evidently concern'd in that Chirurgical Operation we call *Cupping*; for when a *Vacuum* is made by a Syringe in the Cupping-Glass applied to any Part, the *Spring* of the Air in the Flesh under the Glass does

sunk with a Weight appended, so that the Air in the upper Part of the Barrels could not escape, unless the lower Ends of these Pipes were first lifted up.

4. These Air-Barrels were fitted with Tackle, proper to make them rise and fall alternately, like two Buckets in a Well; in their Descent, they were directed by Lines fasten'd to the under Edge of the Bell to the Man standing on the Stage to receive them, who by taking up the Ends of the Pipes above the Surface of the Water in the Bell, gave Occasion for the Water in the Barrels to force all the Air in the upper Parts into the Bell, while it enter'd below, and fill'd the Barrels. And as soon as one was discharged, by a Signal given, it was drawn up, and the other descended, to be ready for Use.

5. As the cold Air rush'd into the Bell from the Barrel below, it expell'd the hot Air (which was lighter) thro' the Cock B, at the Top of the Bell, which was then open'd for that purpose. By this Method, Air is communicated so quick, and in such Plenty, that the Doctor tells us, he himself was one of five who were together at the Bottom, in nine or ten Fathoms Water for above an Hour and an half at a Time, without any Sort of ill Consequence; and he might have continued there, as long as he pleased, for any thing that appear'd to the contrary.

6. In going down, 'tis necessary it should be very gently at first, that the dense Air may be inspired to keep up, by its Spring, a Balance to the Pressure of the Air in the Bell. Upon each 12 Feet Descent, the Bell is stopp'd, and the Water that enters is driven out by letting in three or four Barrels of fresh Air. By this Means, the Doctor says, he could (by taking off the Stage) lay the Bottom of the Sea, just within the Compass of the Bell, so far dry, as not to be over Shoes thereon.

7. By the Glass above so much Light was transmitted when the Sun shone, and the Sea was clear and even, that he could see perfectly well to write and read, and much more to take up any Thing under the Bell; and by the Return of the Air-

strongly

ſtrongly act, and by that means cauſes the Fleſh to diſtend and ſwell into the Glaſs, while the Preſſure of the Air on the Parts without the Glaſs accelerates the Motion of the Blood and Fluids, towards the Part where it is diminiſh'd or taken

Barrels, he could ſend up Orders, written with an Iron Pen, on ſmall Pieces of Lead, directing they were to be moved from Place to Place.

8. But in dark Weather, when the Sea was rough and troubled, it would be as dark as Night in the Bell; but then the Doctor found he could keep a Candle burning in the Bell, as long as he pleaſed; it being found by Experiment, that one Candle conſumes much about the ſame Quantity of confined Air as one Man does, *viz.* about a Gallon *per* Minute.

9. The only Inconvenience the Doctor complain'd of was, that upon firſt going down they felt a ſmall Pain in their Ears, as if the End of a Quill were forcibly thruſt into the Hole of the Ear. This may proceed from its being ſome Time before the Air can get from the Mouth, thro' the ſmall Canal of the *Euſtachian Tube*, which leads to the inner Cavity of the Ear; where, when it comes, it makes an *Equilibrium* with the outward Air, preſſing on the *Tympanum*, and thus the Pain, for a ſhort Time, ceaſes; then deſcending lower, the Pain of the Ears returns, and is again abated; and ſo on till you come down to the Bottom, where the Air is of the ſame Denſity continually.

10. One of thoſe Divers (who thought to out-wit Dame Nature for once) put a Piece of chew'd Paper in his Ears, which, as the Bell deſcended, was ſo forcibly preſſed into his Ears, that it was with great Difficulty the Surgeon could extract it. Thus a Bottle with only common Air in it, and cork'd down tight, if it be let down to a conſiderable Depth of Water, will be found, upon drawing it up again, to have had the Cork forced in by the Preſſure of the Water at that Depth.

11. This Bell was ſo far improv'd by the Doctor, that he could detach one of his Divers to the Diſtance of 80 or 100 Yards from it, by a Contrivance of a Cap or Head-piece, ſomewhat like an inverted Hand-Baſket, as at F, with a Glaſs in the fore Part, for him to ſee his way thro'. This Cap was of Lead, and made to fit quite cloſe about his Shoulders; in the Top of it was fix'd a flexible Pipe communicating with

off by the Glass.

SINCE we know that *Heat* augments the repellent Power in the Particles of a Fluid, and by that means increases its Elasticity, and thereby causes it to expand itself into a large Space; and

the Bell, and by which he had Air when he wanted, by turning the Stop Cock near his Head-piece. There was also another Cock at the End in the Bell to prevent any Accident happening from the Person without.

12. This Person was always well clothed with thick Flannels, which were warm'd upon him before he left the Bell, and would not suffer the cold Water to penetrate to hurt him. His Cap contain'd Air enough to serve him a Minute or two; then by raising himself above the Bell, and turning the Cock F, he could replenish it with fresh Air. This Pipe he coil'd round his Arm, which served him as a Clue to find his Way to the Bell again.

13. This Diving-Bell receiv'd its last Improvement from Mr. *Martin Triewald*, F.R.S. Captain of *Mechanics* and *Military Architecture*, to his *Swedish* Majesty; the Manner and Form whereof is shewn in a Figure of his own drawing. A, B, is the Bell, which, as appears by the Scale of Feet under it, is much less than Dr. *Halley*'s, and therefore will come cheaper. It is sunk with leaden Weights D, D, appended at the Bottom; the Substance of the Bell is Copper, and tinn'd within all over; and as in the Rivers and Coasts of the *Baltic* Sea, the Water is very clear, so he has illuminated the Bell with three strong convex Lenses G, G, G, with Copper-Lids H, H, H, to defend them.

Pl. XXX. Fig. 5.

14. The Iron Ring, or Plate E, serves the Diver to stand upon when he is at Work; and it is suspended at such a Distance from the Bottom of the Bell, that when the Diver stands upright, his Head is just above the Water in the Bell, and it is much better there than higher up in the Bell, because the Air is colder, and consequently more fresh and fit for Respiration near the Surface of the Water, than towards the Top of the Bell.

15. But when there is Occasion for the Diver to be wholly in the Bell, and his Head of Course in the upper Part, Mr. *Triewald* has contrived, that even there, when he has breath'd the hot Air as long as he well can, by means of a spiral Copper Tube *b*, *c*, placed close to the Inside of the Bell, he

that

that *Cold* has a quite contrary Effect; we learn the Use of the THERMOMETER in indicating the *various Degrees of Heat and Cold in the Air*, by the different Altitudes of the Spirit of Wine in that Instrument (XCIII).

may then draw the cooler and fresher Air from the lowermost Parts; to which End, a flexible Leather Tube, about two Foot long, is fix'd to the upper End of the Tube at *b*, to the other End of which is a turn'd Ivory Mouth-piece, for the Diver to hold in his Mouth, to respire the Air from below by; and this he may do in any Posture of standing, sitting, bowing his Body, &c.

(XCIII.) 1. A THERMOMETER being design'd to indicate the *various Degrees of Heat and Cold* by the elastic or expansive Power of Bodies of the Fluid sort, so many Ways, Methods, and Forms of constructing such an useful Instrument have been thought of, and invented at several Times for this Purpose; at first *Air*, then *Oil*, then *Spirits of Wine*, and lastly *Quicksilver* have been every Way attempted and tortur'd in this Experiment.

2. The Spring of Air being sooner affected by *Heat* and *Cold* than that of any other Fluid, was first thought upon as the best Expedient to answer this End; and so it really would be, were it not that the Weight or Pressure of the Atmosphere affects it also at the same time; and by acting sometimes with, sometimes against it, renders the Effect by Heat or Cold very uncertain, and therefore the Instrument useless. For Example: The Air in the Bottle AF will, by its Expansion, when the Air grows warmer, raise the Water higher in the Tube than the Point H, and if the Air be lighter at this time it will press less on the Surface of the Water at H, and so will suffer it to rise still higher. But if the Air be heavier it will act against the Spring, and not permit it to raise the Water so high. The same may be observed with respect to its Contraction by Cold; wherefore such an Instrument, for common or constant Use, will not do at all, tho' perhaps none is better calculated for some extemporaneous Uses, as measuring the Degree of Coldness in different Cellars, or of Warmth in divers Rooms upon the same Floor. Pl. XXX. Fig. 1.

3. It was upon this Account found necessary to have recourse to some other Fluid, which, secured from the Pres-

sure

ſure of the Air in a Tube hermetically ſeal'd, might expand and contract ſolely by the Heat and Coldneſs of the Air about it. And becauſe moſt Fluids are ſubject to freeze or thicken in great Degrees of Cold, it was ſoon conſider'd that Spirits of Wine, a little tinged with Cochineal, would beſt anſwer the Purpoſe, and accordingly Thermometers were generally made therewith, and became of common Uſe.

4. Tho' the Spirit of Wine Thermometers would do very well to ſhew the comparative Heat of the Air, yet this was far ſhort of the Virtuoſo's Views, who wanted to explore the various and vaſtly different Degrees of Heat in other Bodies, as *boiling Water*, *boiling Oils*, *melted Metals*, and even *Fire* itſelf, and Degrees of Cold too, beyond what the Spirit Thermometer can ſhew. For Spirit in a moderate Degree of Heat will burſt the Tube; and in an intenſe Degree of Cold will freeze, as the *French* Philoſophers found, who went to meaſure a Degree upon the Surface of the Earth under the North Polar Circle.

5. It having been found by Experiment, that Linſeed Oil required four times the Degree of Heat to make it boil as Water did, it was quickly ſubſtituted inſtead of Spirits for Philoſophic Uſes. This Sir *Iſaac Newton* always uſed, and by it diſcover'd the comparative Degree of Heat which makes Water boil, which melts Wax, which makes Spirit of Wine boil, and melts Tin and Lead; beyond which we do not find the Oil-Thermometer has been applied; for which reaſon (as alſo for its ſullying the Tube) it has been leſs uſed of late, and given way to

6. The MERCURIAL THERMOMETER which will ſuſtain any Degree of Heat or Cold, as far as any Inſtrument of this Kind can be expected to do. Mr. *Farenheit*, of *Amſterdam*, was the Contriver of this Thermometer, and tho' ſeveral Artificers made them as well as he, yet they ſtill go by his Name. Dr. *Boerhaave* uſed only this Thermometer. As the Mercury very freely and uniformly expands itſelf from hard Froſt to the Heat of Summer, ſo one Sort of thoſe Thermometers are contrived with a Scale, to include thoſe Extremes only, and the Beginning of the Diviſions, or 0, is fix'd to that Altitude of the Quickſilver, as is obſerv'd when Water juſt begins to freeze, or Snow to thaw; for which reaſon that is call'd the *Freezing Point* in the Scale. This Thermometer is ſmall, ſhort, put in a neat Frame, and carried in the Pocket any where.

7. But the *Grand Thermometer* of FARENHEIT is graduated after a different Manner, as deſtin'd to a more critical and extenſive Uſe. In this the Bulb, or large Part at the Bottom,

tom, is not *spherical* (as in common ones) but *cylindrical*, to the End, that the Heat may penetrate and reach the inmost Parts as soon as possible, so that the whole may expand uniformly together. Hence it is, that in the cylindric Bulb, the Fluid will expand and rise immediately, whereas in the spherical Bulb, it is seen first to fall (by the sudden Expansion of the Ball, before the Fluid is heated) and then to rise, by the Expansion of the Fluid when heated. I have here given a Figure, both of *Farenheit*'s *Mercurial Thermometer*, and also of Sir *Isaac Newton*'s made with Linseed Oil.

8. I take this of Sir *Isaac*'s to be the best fitted of any for a *Standard Weather Thermometer*; and even for any Degree of Heat which the various States of the human Body exhibit; and also for those different Degrees which Vegetation requires in the Green-House, Hot-Bed, *&c.* In all which Cases 'tis necessary there should be one common, unerring, and universal Measure, or Standard, which at all times, and in every Place, will shew the same Degree of Heat, by the same Expansion of the Fluid, according to which the Scale should be made in every Standard Thermometer.

9. In order to this, the Tube proposed should be very nicely weigh'd when empty, and then the Bulb, and about a tenth Part of the Length of the Tube above it, is to be fill'd with Quicksilver; then it is to be weigh'd again, and the Excess of this, above the former Weight, will give the Weight of the Quicksilver pour'd in; this will give the Weight of 100dth Part. Let a Mark be made with a File upon the Tube at the Surface of the inclosed Quicksilver.

10. Then weigh out 9 or 10 Parcels of Quicksilver, each equal to 100dth Part of that first put in the Tube, and having pour'd the several Parcels in one after another upon the inclosed Quicksilver, and marked the Tube successively at the Surface of each Parcel, you'll have the Tube divided into proper Intervals, which, if the Bore of the Tube be every where the same, will be equal to each other; if not, they will be unequal; and each of these Intervals is to be divided into 10 others, increasing or decreasing as the Intervals do.

11. When this is done, the Capacity of the Tube is divided into *Thousandth Parts* of that of the Ball, and the contiguous Part of the Tube reaching up to the first Mark. The Tube is now to be put into a Frame, and by the Side of it is to be placed a Scale, divided into *Thousandth Parts*, exactly corresponding to those on the Tube; and writing 1000 over-against the first Mark, you write 1010 over-against the second, 1020 against the third, and so on, as you see in the Figure.

12. The

12. The Standard Thermometer-Tube, and its Scale, being thus constructed, is then to be fill'd with some proper Fluid, as *Lindseed Oil*, where great Degrees of Heat are not proposed; and Mercury is to be used, when they are. When the Fluid is pour'd in, it is to be adjusted in such a Quantity, that it may stand just at the principal Point, mark'd 1000, in *Water just freezing*. And here great Precaution is to be used; for many Trials must determine this Point to which the Fluid must always rise by slow Degrees, and with an uniform Motion.

13. When this Point is well secured, all the Trouble is over the Ball, being then immersed in *boiling Water, Spirits, Oils, melted Metals, &c.* in *Snow, Freezing Mixtures*, &c. the Expansions, by all the various Degrees of Heat and Cold, will be shewn by the Numbers against the Heights to which the Fluid rises in the Tube in each Case, these are to be wrote on the Side of the Scale; and since the same Degree of Heat will cause the same Expansion of the same Fluid at all Times, 'tis evident, if Thermometers were every where constructed in this Manner, the Observations made by them in any Part of the World, may be compared together, which cannot otherwise be done; whence this Part of Philosophy would receive its final Perfection.

14. By one of those Standard Thermometers well made, many more might soon be constructed with any expanding Fluid, without the Trouble of graduating their Tubes by equal Quantities of Quicksilver. For having fill'd the Balls, and a convenient Part of the Tube, with the proposed Fluid, place them all together in a Vessel of cold Water; and while it is warming as gently as possible, when the Oil in the Standard Thermometer shall arrive successively at the several Divisions of its Scale, at the same Instant of Time mark the new Tubes at the several Heights of their Fluids, and form a Scale for every Tube, that shall correspond to those Marks. Then, while the Liquors subside by cooling gently, examine whether they nicely agree at the several Marks. To determine the *Freezing Point* in all, they are to stand together in the Water till it just begins to freeze: Or, having all the other Points duly, that may be deduced very exactly by the Rule of *Proportion*.

15. A Thermometer that shall vary very sensibly by every small Variation of Heat and Cold, as those of the Atmosphere, must have a large Ball in Proportion to the Bore of the Tube; and that the Heat or Cold may sooner penetrate the innermost Parts of the Liquor, the Ball should not be spherical, but oblong and flatted like a *French Flask*; and the

Lengths

Lengths of the Tubes should be proportion'd to the Degrees of Heat they are intended to discover.

16. Sir *Isaac Newton* graduated his Standard Thermometer on both Sides, as shewn in the Figure. Those on the Right Hand measured the *Heat of the Oil*; as those on the Left measur'd the Bulk thereof: But since the latter, as well as the former, begins from a Cypher at the Freezing Point, and is regularly continued upwards by the common Divisions 10, 20, 30, 40, *&c.* it will equally serve both Purposes; since the Degree of Heat will always be proportion'd to the Expansion of the Bulk of the Fluid above or below the Freezing Point.

17. By this Division therefore on the Left Hand, I shall express some of the principal Articles of Sir *Isaac Newton*'s Scale of the various Degrees of Heat, as in the Tablet below.

D. of Heat.	
0	Water just freezing, and Snow just thawing.
1 to 4	The Heats of the Air in Winter.
4 to 8	The Heats of the Air in Spring and Autumn.
8 to 12	The Heats of the Air in Summer.
13	The greatest Summer-Heat.
26	The greatest Heat of the external Parts of the Human Body.
31	Water just tolerable to the Hand at Rest.
$36\frac{1}{2}$	Water hardly tolerable to the Hand in Motion.
43	Melted Wax just growing stiff and opake.
$51\frac{1}{2}$	Melted Wax just before it bubbles or boils.
54	Spirit of Wine just begins to boil.
72	Water begins to boil.
75	Water boils vehemently.
86	A Mixture of $\frac{1}{12}$ of Lead, $\frac{2}{5}$ of Tin, and $\frac{1}{2}$ Bismuth, melts.
103	A Mixture of equal Parts of Tin and Bismuth melts.
122	A Mixture of $\frac{1}{3}$ of Tin and $\frac{2}{3}$ of Lead melts.
154	The Heat which melts Tin.
174	The Heat which melts Bismuth.
206	The least Heat which melts Lead.
290	The Heat with which burning Bodies shine in a dark Night.
410	The Heat of a small Coal-Fire.
450	The Heat of a small Wood Fire.

THE

THE *Moisture* and *Dryness* of the Air are shewn by the HYGROMETER, which is made several ways, but that with a *Cord* is most common and useful; for that by shrinking with Moisture will turn an Index one way, and extending with Dryness will turn it the contrary way, over the graduated Limb of a Circle (XCIV) (XCV).

18. Dr. *Hales* considers the Freezing Point as one Boundary to Vegetation, *viz.* on the Side of Cold; and the other Boundary he fixes to that Degree of Heat with which Wax will begin to melt, because a greater Degree of Heat will, instead, of collecting and assimilating the nutritive Particles, dissipate them, even those which are most viscid and glutinous; and therefore the Plant will rather fade than vegetate in such Degrees of Heat.

19. This Space the Doctor divided into 100 equal Parts in his Thermometers: But his Numbers, express'd in those of the Standard Thermometer, are for several Particulars mention'd by the Doctor as follows. For *Myrtle*, $4\frac{4}{4}$; *Oranges*, $6\frac{1}{2}$; *Ficoides*, $7\frac{1}{2}$; Indian *Fig*, $8\frac{1}{4}$; *Aloe*, 10; *Cereus*, 11; *Euphorbium*, 12; *Piamento*, 13; *Ananas*, $14\frac{1}{2}$; *Melon-Thistle*, $15\frac{1}{4}$; *Air under the Glass of a Hot-Bed*, 17; *the Hot-Bed itself*, 28. If the Hot-Bed exceed the Heat of 40 or thereabouts, it will scorch the Plants and kill them. The Heat of Milk from the Cow is 28, that of Urine 29, and of Blood in a Fever nearly 40.

20. As *Farenheit*'s Thermometer is come into such general Use, I have here placed it by the Standard Thermometer, that the Divisions on each may be reduced to the other's respectively by bare Inspection, and the Use of both be still preserved. If the Reader would see all the different Sorts of Thermometers, or rather all the different Methods of graduating them, he may be fully satisfied by consulting Dr. *George Martine*'s Treatise on this Subject.

(XCIV) 1. An HYGROMETER, sometimes call'd a NOTIOMETER, is any Instrument or Contrivance, by which we can estimate the Quantity of Moisture or Vapours in the Air; or by which we can compare the various Degrees of its Humidity and Siccity at different Times. For this Purpose different Subjects have been at times essay'd, but none as yet have been found satisfactory or lasting.

2. Thus Cotton, Spunge, &c. hung at the End of a nice

I SHALL

I SHALL finish this Lecture with giving you an Account of the Structure and Use of the common AIR-PUMP, and of one of a *new Invention* of my own. The common or large Air-Pump is represented where *aa*, *aa*, are the two Brass Barrels, in which the Pistons *cc*, *cc*, move by Chains fasten'd to each of them, and to a Wheel moving on the Axle *f*; when the Engine is put into Motion by the Winch *bb*. *gg*, *gg*, are two Pl. XXIII. Fig. 4.

Balance, in an exact Equilibre, will by contracting Moisture from the Air become heavier, which will therefore be shewn by its descending; and when the Air becomes drier, it ought to part with the Moisture and become lighter; but this it will not readily do, and is therefore of little Use. Salts have been likewise used this way, but to no purpose.

3. It would be endless to take notice of all the Methods that have been attempted by Philosophers, and all without Success. However, as some are better than others, and will endure for a considerable Time very well, I shall here give an Account of one which is the best of any I have hitherto thought of. It is made of a String either of Hemp or Cat's-Gut, (as all the best Sort are) and shews the increasing *Moisture* of the Air by its *Twisting* and *Shortening*, and the *Dryness* by *Untwisting* and *Lengthening*.

4. Thus, Let ABC be the lower Part of a twisted Line or Cord, hanging from the Height of the Room against one Side thereof on the Wall or Wainscot; let there be described a large Circle, graduated into an 100 equal Parts, such as KLMN; in the Center of which is a Pin, with a small Pulley IB, carrying an Index OP. If now a Cord be put round the Pulley, and a small Weight or Ball D be suspended at the lower End to keep it strait, then as the Cord gathers Moisture from the Air, it will twist and become shorter; the Consequence of which will be, that in contracting it will turn the Pulley IB, and this by its Index will point to the Numbers on the graduated Circle, which will shew the Degree of Moisture or Dryness by the Contraction or Relaxation of the Cord. Pl. XXX. Fig. 6.

5. Again: If the Ball D hang over the Center E of another graduated Circle CFGH placed horizontally, carrying an Index EF upon its Divisions, it will shew the same Thing

Pillars

Pillars or Pieces of Wood supporting the Frame of the Pump-Wheel, which is screw'd upon them by Nuts under the little Pieces of Wood *e*, *e e*. The Tube or Pipe mark'd *b b* is call'd the *Swan-Neck*, made of Brass: By this the Air passes from under the Receiver *o o*, through a small Hole *k* in the Middle of the Brass Plate *i i i* on the Top of the Pump; to a Brass Piece in the Box *d d*,

by the twisting and untwisting of the Cord BC, as in the Circle above; so that this may be look'd upon as a *Double Hygrometer*, and so simple in its Structure, that any Person may make it; and that it will answer very well for a considerable Time, I am fully satisfied by Experience: And I believe a better than this was never made.

(XCV) 1. There remains yet one more Pneumatic Machine to be described, which has made a considerable Noise in the Philosophic World, but has never been of any Use in Civil Life; I mean, the famous Invention of the AIR-GUN, of which there are two Sorts; one the *Common Air Gun*, the other the *Magazine Air-Gun*: Of both which I shall give the following short Account.

Pl.XXXI. 2. The *Common* AIR-GUN is made of Brass, and has two Barrels; the Inside Barrel KA of a small Bore, from which the Bullets are shot; and a larger Barrel ECDR on the Outside of it. There is a Syringe SMNP fix'd in the Stock of the Gun, by which the Air is injected into the Cavity between the two Barrels through the Valve EP. The Ball K is put down into its Place in the small Barrel with the Rammer, as in another Gun. At SL is another Valve, which being drawn open by the Trigger O, permits the Air to come behind the Bullet, so as to drive it out with great Force.

3. If this Valve be open'd and shut suddenly, one Charge of condensed Air may make several Discharges of Bullets; but if the whole Air be discharged on one single Bullet, it will drive it out more forcibly. This Discharge is effected by means of a Lock *k l* placed here, as usual in other Guns; for the Trigger being pull'd, the Cock *k* will go down, and drive a Lever *o*, that will open the Valve, and let in the Air upon the Bullet K.

4. The *Magazine* AIR-GUN is the Invention of an ingenious Artist, whose Name is *L. Colbe*. By his Contrivance ten Bul-

which

which being perforated length-ways to the Middle Point under each Barrel, does there, through a small Hole, by a Bladder-Valve, transmit the Air from the Receiver into each Barrel to be pump'd out by passing through the Hole in the descending Piston. These Holes in the Pistons and Bottoms of the Barrels are cover'd with Valves, to prevent the Return of the Air into the Receiver. *lll* is the Mercurial Gage, or common Barometer, immersed in a Bason of Mercury *mm* fix'd in the Bottom of the Frame, and at top communicates with the Receiver, which therefore shews how much the Receiver is exhausted by the Rising of the Mercury in the Tube, by a graduated Scale affixed thereto. The

lets are so lodged in a Cavity near the Place of Discharge, that they may be drawn into the shooting Barrel, and successively shot so fast as to be nearly of the same Use as so many several Guns. In the Figure you have a Section of the Gun, as big in every Part as the Gun itself; and so much of the Length as is necessary to form a compleat Idea of the Whole.

5. AEE is Part of the Stock; G is the End of the injecting Syringe, with its Valve H opening into the Cavity between the Barrels, as before. KK is the small shooting Barrel; which receives the Bullets from the Magazine ED, which is of a serpentine Form, and closed on the End D when the Bullets *b*, *b*, *b*, *b*, are lodged in it. The circular Part *sIskMi* is the Key of a Cock, having a cylindric Hole through it IK, which is equal to the Bore of the small Barrel, and makes a Part of it in the present Situation. Pl.XXXI.

6. When the Lock is taken off, the several Parts Q, R, T, S, W, &c. come into View, by which means the Discharge is made by pushing up the Pin P*p*, which raises and opens a Valve V, to let in the Air against the Bullet I from the Cavity F, F, F; which Valve is immediately shut down again by means of a long Spring NN of Brass. This Valve V being a conical Piece of Brass, ground very true in the Part which

Stop-cock *nn*, alſo, communicates with the Receiver, and conſequently with the Swan-Neck and Mercurial Tube: Its Uſe is, by turning the Cock, to re-admit the Air, when there is Occaſion. The Receiver is ground true on the Bottom, and is fix'd on the Pump at firſt by means of wetted Leathers, to exclude the Air, inſtead of Cement formerly uſed for that purpoſe.

BUT with how much more Conveniency, and leſs Expence, Pneumatical Experiments of all Kinds may be perform'd, by a *New*, *Elegant*, and *Portable Air-Pump*, which I have lately contrived and made, will be eaſy to apprehend from a bare View of the Figure thereof: In which A B is the Head or Part containing the Wheel,

receives it, will of itſelf be ſufficient to confine the Air.

7. To make a Diſcharge you pull the Trigger *zz*, which throws up the Seer *yx*, and diſengages it from the Notch *x*, upon which the ſtrong Spring W W moves the Tumbler T, to which the Cock is fix'd. This by its End *u* bears down the End *v* of the tumbling Lever R, which by its other End *m* raiſes at the ſame time the flat End *l* of the horizontal Lever Q; and by this means, of courſe, the Pin P *p* is puſh'd up, which ſtands upon it, and thus opens the Valve V, and diſcharges the Bullet. This is all evident from a bare View of the Cut.

8. To bring in another Bullet to ſucceed I inſtantaneouſly, there is a Part call'd the Hammer H, which by a ſquare Hole goes on upon the ſquare End of the Key of the Cock, and turns it about ſo as to place the cylindric Bore of the Key I K in any Situation required. Thus when the Bullet is in the Gun, the Hammer ſtands as in the Figure, where the Bore of the Key coincides with that of the Barrel K K; but when the Ball is diſcharged, the Hammer H is inſtantly brought down to ſhut the Pan of the Gun, by which Motion the Bore of the Key is turn'd into the Situation *ik*, ſo as to coincide now with the Orifice of the Magazine; and upon lifting the Gun upright, the Ball next the Key tumbles into its Cavity, and falls behind two ſmall Ends *s*, *s*, of two tender Springs, which like

Plate XXXI.

which

which alternately raises and depresses the Pistons C D in the Barrels E F, which are strongly press'd down by the said Part A B, supported on the two Pillars G H, fix'd into the Bed or Bottom of the Machine I K L. On this Bottom stands the Receiver M N on a large smooth Brass Plate, in the Middle whereof is a Hole, by which the Air passes out of the Receiver into a small Tube on the under Part of the Frame, and goes to the Piece O, which communicates with the perforated Brass Piece on which the Barrels stand, and from which they receive the Air to be exhausted. On the middle Part of this Brass Piece is a Perforation, over which is placed a small Receiver P Q, and under it a Bason of

Fingers Ends detain it. The Key in this Position is seen in the Figure. Then opening the Hammer again the Bullet Plate I, is brought into its proper Place near the discharging Valve, XXXI, and the Bore of the Key makes again a Part of that of the shooting Barrel.

9. It evidently appears how expeditious a Method this is of charging and discharging a Gun; and were the Force of condensed Air as great as that of Gunpowder, such an Air-Gun would actually answer the End of many Guns, and prove the best Defence against Highwaymen or Robbers that People are aware of; because when they have Reason to suspect them, they might then make five or six Discharges before the Thief can come within Pistol-shot.

10. In the Air-Gun, and in all other Cases where the Air is required to be condensed to a very great Degree, it will be requisite to have the Syringe of a small Bore, *viz.* not exceeding $\frac{1}{2}$ an Inch in Diameter; because, as has been shewn, the Pressure against every *square Inch* is about 15 *lb.* and against every *circular Inch* it is therefore about 12 *lb.* If therefore the Syringe be one Inch in Diameter, when one Atmosphere is injected, there will be a Resistance of 12 *lb.* against the Piston; when 2, of 24 *lb.*; and when 10 are injected, there will be a Force of 120 *lb.* to overcome; whereas 10 Atmospheres act against the circular Half-Inch Piston (whose Area

Mercury R, in which a ſmall Tube RS (hermetically ſealed at one End, and fill'd with Quickſilver) is inverted; and therefore as the ſmall Receiver PQ is exhauſted, (at the ſame time with the large one MN) the Approach of the *Vacuum* will be ſhewn by the *Deſcent of the Quickſilver* in the Tube RS. By the Stop-cock T the Air is again let into the Receiver. I take this to be the *laſt Improvement* this Machine is capable of, as to its Form; for it conſiſts of only ſuch Parts as are Eſſential. And thus conſtructed, it may, together with its Receivers, be contain'd in a Box of a ſmall Size, and comes to but a ſmall Price in compariſon of the other Forms. (XCVI.)

is but $\frac{1}{4}$ Part ſo big) with a Force but a $\frac{1}{4}$ Part ſo great, *viz.* 30 *lb.*; or 40 Atmoſpheres may be injected with ſuch a Syringe, as well as 10 with the other. In a Word, the Facility of working will be *(cæteris paribus)* inverſely *as the Squares of the Diameters of the Syringe.*

(XCVI) 1. I ſhall conclude this Subject with a few Articles relating to the Rarefaction of the Air in the Recipient in working the Machine; for the Reader muſt not ſuppoſe that all the Air can be exhauſted, if the Pump be ever ſo good, or work'd ever ſo long. The Reaſon is evident when we conſider, that the Air which is exhauſted is only puſh'd out by the Spring of that which remains behind: If therefore every Particle were ſuppoſed to be exhauſted, the laſt would be expell'd without an Agent, or there would be an Effect without a Cauſe, which is abſurd.

2. Let the Capacity of any Receiver be to that of the Barrel as C to 1. Alſo let the Rarefaction of the Air which remains in the Receiver be to the common Air as R to 1, after any Number of Turns or Exſuctions N. Then, upon raiſing the Piſton, the Air will ruſh into the Barrel, and ſo will now be rarified in the Ratio of C to C + 1, or of 1 to $\frac{C+1}{C}$; and ſince this is the Ratio of the Rarefaction by every Exſuction,

Exsuction, 'tis evident it will be the common Ratio of a Geometrical Series of Rarefactions produced by the several Turns of the Winch, *viz.* The Series $1 : \left.\frac{\overline{C+1}}{C}\right|^{1} : \left.\frac{\overline{C+1}}{C}\right|^{2} : \left.\frac{\overline{C+1}}{C}\right|^{3} : \left.\frac{\overline{C+1}}{C}\right|^{4}$, &c. to $\left.\frac{\overline{C+1}}{C}\right|^{N} =$ last Term of the Series, which therefore as it expresses the last Rarefaction will be equal to R, that is $R = \left.\frac{\overline{C+1}}{C}\right|^{N}$

3. Hence from the known Property of Logarithms we have $L.R = N \times L.\frac{C+1}{C} = N \times \overline{L.\,\overline{C+1} - L.C}$; whence, $\frac{L.R}{L.\,\overline{C+1} - L.C} = N$. Wherefore if $C = 1$, that is, if the Capacity of the Receiver be equal to that of the Barrel, we shall have $N = \frac{L.R}{L.2}$. Consequently, if R express any Degree of Rarity proposed, as 1, 2, 3, 4, 5, 6, &c. we have N the Number of Turns or Exsuctions to effect it.

4. And from hence the following Table is constructed; in which the first Column expresses the Rarity of the Air in the Receiver, and the second the Number of Turns to produce it.

Rarity.	*Number of Turns.*	*Rarity.*	*Number of Turns.*	*Rarity.*	*Number of Turns.*
1	0.	60	5,907	900	9,814
2	1.	64	6.	1000	9,966
3	1,585	70	6,129	1024	10.
4	2.	80	6,322	2000	10,966
5	2,322	90	6,492	2048	11.
6	2,585	100	6,644	3000	11,551
7	2,807	128	7.	4000	11,966
8	3.	200	7,644	4096	12.
9	3,170	256	8.	5000	12,288
10	3,322	300	8,229	6000	12,551
16	4.	400	8,644	7000	12,773
20	4,322	500	8,966	8000	12,966
30	4,907	512	9.	8192	13.
32	5.	600	9,229	9000	13,136
40	5,322	700	9,451	10000	13,288
50	5,644	800	9,644	16384	14.

5. From this Table we may observe, that if any Numbers in the first Column be taken in Geometrical Progression, the corresponding Numbers of the second will be in Arithmetical Progression. Thus against 2, 4, 8, 16 in the first, you see 1, 2, 3, 4 in the second Column.

6. When the Capacity of the Receiver exceeds that of the Barrel, then the Number of Turns N to produce a given Rarefaction R will be greater than before. Therefore if the Number of Turns in this particular Case, which let us call n, be multiplied by some Number m, it will produce a Number of Turns N that shall affect the Rarefaction R in any Receiver proposed. Now since $n = \frac{L.\,R}{L.\,2}$, we shall have $n \times m = \frac{m \times L.\,R}{L.\,2} = N = \frac{L.\,R}{L.\,\overline{C+1} - L\,C}$; whence we have $m = \frac{L.\,2}{L.\,\overline{C+1} - L.\,C}$.

7 From hence a Table of Multipliers expressing the Value of m, when the Receiver is in any given Proportion larger than the Barrel, is easy to be constructed. Of which the following is a Specimen.

Capacity of Receiver.	*Multiplier*	*Capacity of Receiver.*	*Multiplier.*	*Capacity of Receiver.*	*Multiplier.*
1	1.	20	14,207	300	208,291
2	1,710	30	21,139	400	277,605
3	2,409	40	28,071	500	346,920
4	3,106	50	35,003	600	416,235
5	3,802	60	41,934	700	485,549
6	4,497	70	48,866	800	554,864
7	5,191	80	55,798	900	624,179
8	5,835	90	62,729	1000	693,494
9	6,579	100	69,661		
10	7,273	200	138,976		

8. By means of these two Tables, those who know nothing of *Algebra* may find how many Turns are necessary to rarify the Air in the Receiver to any given Degree, when the Ratio of the Receiver's Capacity to that of the Barrel is known. For Example: Let the Receiver be 10 times as big as the Barrel, and let it be required to find how many Turns of the Winch will rarify the Air 100 times. First seek the Number

Number that will do it when the Receiver is equal to the Barrel; which I find by the firſt Table is 6 Turns, and 644 Parts of 1000 of another. Then, againſt 10 in the ſecond Table, I find the Multiplier 7,273, by which if I multiply 6,644, I ſhall have a Product 48,322, which will expreſs the Number of Turns required.

9. The Aſcent of the Quickſilver in the Gage of the Common Pump is proportional to the Quantity of Air drawn out, either upon the Whole, or upon any ſingle Turn of the Winch: And the Deficiency from the Standard Altitude of $29\frac{1}{2}$ Inches is always proportional to the Quantity of Air remaining in the Receiver; as may be eaſily deduced from what has been ſaid of the *Denſity*, *Spring*, and *Preſſure* of the Air.

10. The Gage of a *Condenſer* will have the Spaces unpoſſeſs'd of Quickſilver at the End decreaſing in *Harmonical Proportion:* For ſince equal Quantities of Air are injected by the Syringe at each Stroke of the Piſton, the Quantity of Air in the Condenſer will increaſe in Arithmetical Progreſſion, and ſo will its Denſity, and of courſe the Denſity of that in the End of the Gage, becauſe the Quickſilver is preſs'd on each Side equally; but the Spaces diminiſh as the Denſities increaſe, as we have elſewhere ſhewn. Therefore *the Spaces are inverſely as a Series of Terms in Arithmetical Progreſſion*, and conſequently are in *Muſical Proportion*; for that this is a Property of *Muſical Terms* will be ſhewn in *Annot*. CIX.

LECTURE VII.

The Doctrine of WINDS *and* SOUNDS,

Of WIND *in general. The* THEORY *of* Winds *by Dr.* HALLEY. *Of the Constant, or General* TRADE WINDS; *of the* MONSOONS; *the Cause of* VARIABLE WINDS. *Of* AERIAL TIDES. *Of the* VELOCITY *of* Wind. *Of the* MOMENTUM *or* FORCE *of* Wind. *A* CALCULATION *thereof, and its* APPLICATION *to the* SAILS *of a* WIND-MILL. *The best* FORM *and* POSITION *of the* SAILS. *A* CALCULATION *of the* FORCE *of* BELLOWS *in impelling* Wind. *A* New Invention of WATER-BELLOWS. *The* Nature *of* SOUND *in general. The* SENSATION *of* SOUND. *The* ORGAN OF HEARING *described. Of the* WAVES *or* PULSES *of* AIR. *Their Various* PROPERTIES *explain'd. The* NEWTONIAN DOCTRINE *of* VIBRATIONS *and* TREMORS *of* SOUNDING BODIES *explain'd. The* WAVES *of* WATER *accounted for. Of the* VELOCITY *of* SOUNDS. *The* DISTANCE *to which they may be heard. Of* ECHO'S. *Of the* SPEAKING TRUMPET *of the best Form. Of* OTACOUSTIC INSTRUMENTS. *Of the* NOTE, TONE, *or* TUNE *of* SOUNDS. *Of* CONCORDS *and* DISCORDS; *the* RATIONALE *of the* DIATONIC SCALE

SCALE *of* MUSIC. *The* Mathematical THEORY *of* MUSICAL CHORDS, *and of* HARMONIC PROPORTIONS. *Of the* Sympathetic VIBRATIONS *of* MUSICAL STRINGS, *and other Bodies.*

IN this Lecture I shall consider the Nature of WIND and SOUND in general; and of the Vibrations of *Musical Strings* and *Sonorous Bodies*, with regard to the Science of MUSIC.

WIND is a *Stream* or *Current of Air*: As the Air is a Fluid, its natural State is that of *Rest*, which it endeavours always to keep or retrieve by an universal *Equilibrium* of all its Parts. When, therefore, this natural *Equilibrium* of the Atmosphere happens by any means to be destroy'd in any Part, there necessarily follows a Motion of all the circumjacent Air towards that Part, to restore it; and this Motion of the Air is what we call *Wind*. (XCVII.)

(XCVII) 1. I shall here give the principal Phænomena of the Wind, as they are deduced from Dr. *Halley*'s admirable History thereof in the *Philosophical Transactions*, and illustrate the same by his Map of the World drawn up for that Purpose.

2. The First is, That in the great *Pacific* or *Western Ocean*, the *Atlantic* and *Ethiopic* Seas, there is a general Easterly Wind all the Year long, without any considerable Variation; excepting that it is subject to be deflected therefrom some few Points of the Compass towards the North or South, according to the Situation of the Place. The Reason is, because the Parts under the Equator are more heated and rarified than any others, as above mention'd.

3. The Second is, That on each Side the Equator, to

HENCE,

HENCE, with respect to that Place where the *Equilibrium* of the Air is disturb'd, we see the Wind may blow from every Point of the Compass at the same time; and those who live *Northwards* of that Point have a *North Wind*; those who live *Southwards*, a *South Wind*; and so of the rest: But those who live on the Spot, where all these Winds meet and interfere, are oppress'd

about 27 or 30 Degrees, the Wind does more and more decline from the East to the North-East on one Side, and South-East on the other; occasion'd by the two contrary Motions of the Air, arising from Heat and Cold, as above explain'd. These Winds are indicated by the Position of the Arrows in the *Atlantic* and *Pacific* Ocean in the Map.

4. Towards the *Caribbee* Islands, on the *American* Side of the *Atlantic* Ocean, the aforesaid North-East Wind becomes still more and more Easterly, so as sometimes to be East, sometimes East by South, but mostly Northward of the East a Point or two, seldom more. It is likewise observed that the Strength of these Winds does gradually decrease as you sail to the Westward.

5. All along upon the Coast of *Africa* on the Western Side, the Wind sets in upon the Land from various Points of the Compass, North-West, West, South by West, South-West, and almost South, especially toward the *Cape of Good Hope*; all which is easily seen in the Map.

6. In the *Atlantic* Ocean, towards the North of the Line, between 4 and 10 Degrees of Latitude, and 20 and 30 of West Longitude, there is a Tract of Sea where the Winds are not properly said to be *constant* or *variable*; for it seems to be condemn'd to perpetual *Calms*, attended with terrible Thunder and Lightning, and Rains, so frequent that our Navigators from hence call this Part of the Sea the RAINS, as by others they are call'd the CALMS and TORNADOES, as in most of our common Maps. The Reason of this seems to be, that this being the Place where the Easterly and Westerly Winds commence, the Air is divided and held as it were *in Equilibrio* between both; by which means it is render'd more rare than the rest, and too light to sustain the Vapours raised into it, so that it lets them descend in *continual Rains*. See the Parting of the Air in the Map.

with

with turbulent and boisterous Weather, Whirlwinds and Hurricanes; with *Rain*, *Tempest*, *Lightning*, *Thunder*, &c. For *sulphureous Exhalations* from the *South*, Torrents of *Nitre* from the *North*, and *aqueous Vapours* from every Part, are there confusedly huddled and violently blended together; and rarely fail to produce the *Phænomena* abovemention'd.

7. In the *Indian* Ocean the Winds are partly *General*, as in the *Atlantic* and *Ethiopic* Oceans; and partly *Periodical*, that is, such as blow one Half of the Year one Way, and the other Half of the Year near upon the opposite Points: And these Points and Times of shifting are different in different Parts of this Ocean. These Winds are call'd by Seamen *Monsons* or *Monsoons*.

8. Between 10 and 30 Degrees, from *Madagascar* to *New Holland*, the general Trade-Winds about South-East by East are found to blow all the Year long in the same Manner, and for the same Reasons as in the other Oceans above-mention'd.

9. During the Months of *May*, *June*, *July*, *August*, *September*, *October*, the aforesaid South-East Winds extend to within two Degrees of the Equator; after this, for the other six Months, the contrary Winds set in, and blow from the North-West from the Latitude of 3 to 10 Degrees South.

10. From about three Degrees South Latitude, over all the *Arabian* and *Indian* Seas and Gulf of *Bengal*, from *Sumatra* to the Coast of *Africa*, there is another Monsoon, blowing from *October* to *April* on the North-East Points; but in the other Half-Year, from *April* to *October*, from the opposite Points of South-West and West-South-West, and that with rather more Force than the other, accompanied with dark rainy Weather; whereas the North-East blows clear.

11. The Sea between *Madagascar* and *Africa*, and Southwards to the Equator, is subject to the same Change of Wind, or Monsoons, whose Course from *April* to *October* is South-South-West; which, as you go more Northerly, becomes more and more Westerly, till at last they fall in with the West-South-West Winds mention'd in the last Articles. What Winds blow the other Half-Year in those Parts, the Doctor could not obtain any satisfactory Account of; only that they were Easterly, and as often to the North as to the Southward thereof.

MANY

MANY are the particular Causes which produce Wind by interrupting the Equipoise of the Atmosphere; but the most general Causes are two, *viz.* HEAT, which, by *rarifying* the Air, makes it *lighter* in some Places than it is in others; and COLD, which, by *condensing* it, makes it *heavier*. Hence it is, that in all Parts over the *Torrid Zone*, the Air being more rarified by a greater Quantity of the Solar Rays, is much lighter than in the other Parts of the Atmosphere, and most of all over the Equatorial Parts of the Earth. And

12. To the Eastward of *Sumatra* and *Malacca*, on the North Side of the Equator along the Coast of *Camboia* and *China*, the Monsoons blow, and change at the same Times as before; only their Directions are much more Northerly and Southerly than the others, as is easy to observe in the Map. These Winds reach to the *Philippine* Islands Eastward, and to *Japan* Northwards; and are not so constant to their Points as the others above-mention'd.

13. Between the same Meridians, on the South Side the Equator, from *Sumatra* to *New Guinea* Eastward, the same Northerly and Southerly Monsoons are observ'd; only the Northerly are here North-westerly, and the Southerly blow from the South-East. They are not more constant than the others; and besides, they keep not the same Times, but change a Month or six Weeks later.

14. The Shifting of these contrary Winds, or Monsoons, is not all at once; and in some Places the Time of the Change is attended with *Calms*, in others with *variable Winds*, and particularly those of *China*, at ceasing to be Westerly, are very subject to be tempestuous; and such is their Violence, that they seem to be of the Nature of the *West-India* Hurricanes, and render the Navigation of those Parts very unsafe at that Time of the Year. These Tempests the Seamen call the *Breaking up of the Monsoons*.

15. The Cause of the *Monsoons*, or Periodical Winds, is owing to the Course of the Sun Northward of the Equator one Half of the Year, and Southward the other. While he passes through the six Northern Signs of the Ecliptic, the va-

since the Parts at the Equator are most rarified which are near the Sun; and those Parts are, by the Earth's diurnal Rotation *Eastward*, continually shifting to the *West*; it follows, that the Parts of the Air which lie on the *West* Side of the *Point of greatest Rarefaction*, and, by flowing towards it, meet it, have less Motion than those Parts on the *East* of the said Point, which follow it; and therefore the Motion of the *Eastern Air* would prevail against that of the *Western Air*, and so generate a *continual East Wind*, if this were

rious Countries of *Arabia*, *Persia*, *India*, and *China* are heated, and reflect great Quantities of the Solar Rays into the Regions of the ambient Atmosphere, by which means it becomes greatly rarified, and has its Equilibrium of course destroy'd; to restore which, the Air, as well from the Equatorial Parts Southwards, where it is colder, as from the colder Northern Climes, must necessarily have Tendency or Motion towards those Parts, and so produces the Monsoons for the first six Months, during which Time the Heat of those Countries is greatest.

16. Then for the other six Months, the Sun traversing the Ocean and Countries towards the Southern Tropic, while in the six Southern Signs, causes the Air over those Parts to be now most heated and rarified; and consequently the Equatorial Air to alter its Course, or the Winds to veer quite about, and blow upon the opposite Points of the Compass.

17. These are the general Affections of constant and regular Winds; none of which are found not subject to some Variation and Exception, on account of the different Circumstances of Heat, Cold, Land, Water, Situation, &c. concerning all which I shall refer the Reader to the Doctor's own large historical Account of the Winds, publish'd in the *Transactions*, or *Miscellanea Curiosa*, Vol. I.

18. From what has been said, 'tis easy to understand, that since so large a Portion of the Atmosphere as is over the Torrid Zone, and Parts about it, is in such continual Agitation and alternate Motion, those Agitations in an elastic Fluid must extend every way to a great Distance, and produce Effects of the same Kind in a various Manner; by which means

all

all the Effect of that *Rarefaction.* But we are to consider, that as all the Parts of the Atmosphere are so greatly rarified over the Equator, and all about the Poles greatly condensed by extreme Cold, this heavier Air from either Pole is constantly

the Air in all other Latitudes and Climes will suffer a Perturbation more or less, and have a perpetual Tendency to Motion in various Directions, depending on the Situation of Country, the Degrees of Heat and Cold in the Climate, the Position of Hills, Vales, &c. besides what may be owing to the Accension and Explosion of Meteors, the Eruption of subterranean Air, and a hundred other Causes: I say, from all this it is easy to infer, *that our Climate, wherever we live, must necessarily be attended with variable Winds, almost perpetually.*

19. I shall only add farther, that since the Atmosphere is a *gravitating fluid Substance*, it must be subject to the attracting Power of the Sun and Moon, as well as of the Earth; and therefore when the Influence of those Luminaries, either singly or conjointly, is opposite to that of the Earth, the same Effects must follow in the Body of fluid Air, as we have shewn were produced in the ambient Fluid of Water, *viz.* that the Atmosphere shall be of an oblong Figure, or of different Altitudes in different Parts; and that these *Tides of Air* have nearly all the same Affections with those of the Ocean before explain'd, excepting only in this, that they must be as much greater as the Density of Water exceeds that of Air, *viz.* in the Ratio of 860 to 1.

20. Now because of an Equality of Pressure or Weight in the Atmosphere in unequal Altitudes of Air, we can never be sensible of an *Aerial Tide*, either of *Ebb* or *Flood*, by the Barometer; and can only know it by the Position of the Heavenly Bodies. However, as this prodigious Protuberance of the Atmosphere is constantly following the Moon, it must of course produce a Motion in all Parts, and so produce a Wind more or less to every Place, which as it conspires with, or is opposed to the Winds arising from other Causes, makes them greater or less. And I believe something of this may be deduced from Observations made of the State of the Air at the Times of the *New and Full Moons.* And that this was the Case in respect to the two last great Storms, Dr. *Mead* has observed in his Tract *De Imperio Solis & Lunæ.*

flowing

flowing towards the Equator, to restore the Balance destroy'd by the *Rarefaction* and *Levity* of the Air over those Regions: Hence, in this respect alone, a constant *North* and *South Wind* would be generated (XCVIII).

(XCVIII) 1. I find by Experience, that People have in general but an obscure Idea or confused Notion of the Cause of this perpetual Current of Air from East to West, or of a constant East Wind under the Equator; therefore in order to elucidate this Matter, I shall represent it in, and explain it by, a Figure. Let CBADE be Part of a Section of the Atmosphere over the Equator, C the East, E the West, A the Point to which the Sun S is vertical, and R the Point of greatest Rarefaction, or that where the Air is most of all heated, and consequently lightest. Plate XXXIII. Fig. 1.

2. That this Point R is on the Eastern Side of the Point A is not difficult to be conceived, when what is said concerning the Tide in *Annot.* LXXXIV. is well consider'd. And because the Air at R is by Supposition lighter than where it is colder at C and D, it is plain that, in order to maintain an Equilibrium, (which is necessary in a fluid Body) the Air by its greater Weight will have a Tendency from C and D towards R, and rise to a Height there greater than at C or D, in proportion as its Density is less.

3. Now this being the Case, it is evident, the Sun being always between the Points R and D, will be heating the Air on that Part; and those Regions between R and C, having been deserted by the Sun, will grow cold: Consequently, the Air between C and R, as it is colder, will likewise be heavier than that between R and D which is hotter, and so will have a greater *Momentum*, or Quantity of Motion, towards the Point R; and since this Point R is constantly moving after the Point A Westward, the Motion of the Western Air towards it will be in part diminish'd by that means; and being also inferior in Quantity to the Motion of the Eastern Air, the latter will prevail over it, and be constantly following the said Point R from East to West, and thus produce a *continual East Wind.*

4. It may perhaps be here said, that though the Motion of the Air be less from D to R, yet it is something, and so there ought to be a Western Wind, at least in some Degree, and to some Distance Westward of the Point R. To which I answer, That the Nature of a Fluid will not permit two

Now

Now it is eaſy to underſtand, that by a Compoſition of theſe two Directions of the Air from the *Eaſt* and *North*, a *conſtant North-Eaſt Wind* will be generated in the *Northern* Hemiſphere, and a *conſtant South-Eaſt Wind* in the *Southern* Hemiſphere, to a certain Diſtance on each Side the Equator, all round the Earth. And this Caſe we find to be verified in the *General Trade Winds*, which conſtantly blow from the *North-Eaſt* and *South-Eaſt*, to about 30 Degrees on each Side the Equator, where thoſe Parts are over the open Ocean, and not affected with the Reflection of the Sun-Beams from the heated Surface of the Land; for in this Caſe the Wind will always ſet in upon the Land, as on the Coaſt

contrary Motions to reſtore or ſuſtain an Equilibrium, (I mean in regard of the whole Body of it) for wherever one Part of the Fluid is determined to move, all the reſt muſt neceſſarily follow it; otherwiſe the Equilibre of the Air would be deſtroy'd in one Part, to make it good in another; a Defect which Nature cannot be guilty of. Thus we ſee the Tides of the Ocean always follow the Courſe of the Moon from Eaſt to Weſt, without any Motion of the Waters from the Weſt towards the Moon, in the open Oceans: And the Point R can only be conſider'd as the *Aerial Tide*, or *Flood of High Air*; and has nearly the ſame Phænomena with Aqueous Tides.

5. This being clearly underſtood, all the reſt is eaſy; for no one can find it difficult to conceive how the cold Air from each Pole muſt neceſſarily ſet in towards the Equator directly, where meeting, and interfering with the Eaſtern Current, it does with that compound a new Direction for the moving Air, which lies between both the former, *viz.* a *North-Eaſt* Current on the North Side, and a *South-Eaſt* one on the South Side: All which naturally reſults from the *Doctrine of the Compoſition of oblique Forces*. (See *Annot.* XXIV.)

of

of *Guinea*, and other Parts of the *Torrid Zone*, we know it does (XCIX).

As the Motion of the Air has a greater or lesser Velocity, the Wind is *stronger* or *weaker*; and it is found from Observation, that the Velocity of the Wind is various, from the rate of 1 to 50 or 60 Miles *per* Hour (C).

(XCIX) Mr. *Clare*, in his *Motion of Fluids*, has a very pertinent Experiment for illustrating this Matter. It is thus: Let there be a very wide Dish or Vessel of Water, in the Middle of which is to be placed a Water-Plate fill'd with warm Water; the first will represent the Ocean, the other an Island rarifying the Air above it. Then holding a Candle over the cold Water, blow it out, and the Smoke will be seen to move towards the warm Plate, and rising over it will point out the Course of the Air from Sea to Land. And if the ambient Water be warm'd, and the Plate fill'd with cold Water, and the smoaking Wick of a Candle held over the Plate, the contrary will happen.

(C) 1. The Experiment to prove this, is to chuse a free open Place, where the Current of Air, or Wind, is not at all interrupted, but flows uniformly, or as much so as the undulatory State of the Atmosphere will admit; in such a Place, a Feather, or some very light Body, is to be let go in the Wind, and then by a Half-Second Watch, or Pendulum, you observe nicely to what Distance it is carried in any Number of Half-Seconds; or in how many Half-Seconds it has pass'd over a given or measured Space; this will give the Rate of Velocity in the Wind *per* Second, and of course *per* Hour.

2. The late Rev. Dr. *Derham*, who was most accurate in making Experiments of this Sort, approves of this Method before that of the *Mola alata* or *pneumatica* invented by Dr. *Hook* (of which see an Account in the *Philosophical Transactions* N°. 24.) And he tells us (in N°. 313.) that he thus measured the Velocity of the Wind in that very great Storm of 1705, *August* 11. and by many Experiments he found, that it was at the Rate of 33 Feet *per* Half-Second, or of 45 Miles *per* Hour; whence he concludes, that the most vehement Wind (as that of 1703 in *November*) does not fly at the Rate of above 50 or 60 Miles *per* Hour; and that at a Me-

THUS much may suffice for a general Account of the *Nature and Origin of Winds*: We proceed

dium the Velocity of Wind is at the Rate of 12 or 15 Miles *per* Hour.

3. And sometimes the Wind is so slow as not to exceed the Velocity of a Person riding or walking in it; and in that Case, if the Person goes with the Wind, he finds no Wind at all, because there is no Difference of Velocity, or no relative Wind, which is that only that we are sensible of whilst in Motion; the Reason of which we see in *Annotat.* XX.

4. The best Method, that I know of, to bring the Force of the Wind to a Mathematical Calculation and Certainty, is by the following new contrived ANEMOSCOPE, of which I had the first Hint from my ingenious and generous Friend Dr. *Burton* of *Windsor*. ABCDEFGHI is an open Frame of Wood, firmly supported by the Shaft or Postern I. In the two Cross-Pieces HK, LM, is moved an horizontal Axis QM, by means of the four Sails *ab*, *cd*, *ef*, *gh*, in a proper Manner exposed to the Wind. Upon this Axis is fix'd a Cone of Wood, MNO; upon which, as the Sails move round, a Weight S is raised by a String on its Superficies, proceeding from the small to the largest End NO. Upon the great End or Base of the Cone is fixed a Ratchet-Wheel *ik*, in whose Teeth falls the Click X to prevent any retrograde Motion from the depending Weight.

5. From the Structure of this Machine, 'tis easy to understand, that it may be accommodated to estimate the variable Force of the Wind, because the Force of the Weight will continually increase, as the String advances on the conical Surface, by acting at a greater Distance from the Axis. And therefore, if such a Weight be put on, on the smallest Part at M, as will just keep the Machine in *Equilibrio* with the weakest Wind; then, as the Wind becomes stronger, the Weight will be raised in Proportion, and the Diameter of the Base of the Cone NO may be so large in Comparison of that of the smaller End or Axis at M, that the strongest Wind shall but just raise the Weight to the great End.

6. Thus for Example, let the Diameter of the Axis be to that of the Base of the Cone NO as 1 to 28, then if S be a Weight of *one Pound* at M on the Axis, it will be equivalent to 28 *lb.* or ¼ of an Hundred, when raised to the greatest End. If therefore, when the Wind is weakest, it supports 1 *lb.* on the Axle, it must be 28 times as strong to raise the Weight to the Base of the Cone. Thus may a

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now to the *Doctrine of Sounds*. We know by the Experiment of the Bell in the exhausted Re-

Line or Scale of 28 equal Parts be drawn on the Side of the Cone, and the Strength of the Wind will be indicated by that Number therein from which the String shall at any time hang.

7. Furthermore, the String may be of such a Size, and the Cone of such a Length, that there shall be 16 Revolutions of the String between each Division of the Scale on the Cone; so will the Strength of the Wind be express'd in Pounds and Ounces. And if greater Exactness be required, let the Periphery of the Cone's Base be divided into 16 equal Parts, then whenever the *Equilibrium* happens, the String will leave the Conic Surface against one of those Divisions, and thus shew the Force of the Wind to a Dram *Averdupois* Weight.

8. Having premised thus much relating to the Structure and Nature of the Instrument, I shall now proceed to a more particular Examination of the Theory of Wind-Mills, by reassuming what we have formerly said on that Head (See *Annotat.* XLV.) Therefore let lm (parallel to the Axis QM) $= a$, represent the whole Force of the Wind on the Sail; this Force is reduced to ln, and this again to no, which acts normally to the Axis, and turns the Sail. Also we have shewn, that, putting $mn = x$, this Force which turns the Sail is express'd by $\frac{aax - x^3}{a}$; and that when it was a *Maximum*, $x = \sqrt{\frac{aa}{3}} = a\sqrt{\frac{1}{3}}$; and the Angle $lmn =$ 54° 44′.

9. Hence we observe, that when the Mill is in its greatest Perfection, $ln = \sqrt{aa - xx} = \sqrt{a^2 - \frac{a^2}{3}} = a\sqrt{\frac{2}{3}}$, hence the whole Force in the Direction lm is to the same reduced, in the Direction ln, as lm^2 to ln^2, or as a^2 to $\frac{2}{3}a^2$, or as 1 to $\frac{2}{3}$, *viz.* as 3 to 2.

10. Again, the whole Force in the Direction lm is to the same a second Time reduced in the Direction no, as a^2 to $\frac{a^2x - x^3}{a}$, that is, as a^2 to $aa\sqrt{\frac{4}{27}}$, or as 1 to $\sqrt{\frac{4}{27}} = \frac{200}{519} = \frac{5}{13}$ nearly; or, the Force thus reduced is to the

84 ceiver, that Sound has a necessary Dependance on the Air; and if we reflect on the Nature of

whole Force as 5 to 13, when the Sails are posited in the best Manner.

11. Now in order to determine the absolute Force of the Wind, we must compare it with that of Water, as follows. Since Air and Water are both Fluids, if they move with equal Velocities, their Effects in a given Time will be as the Quantities of Matter, that is, (putting the *Roman* Letters for those Particulars in Water, and *Italics* for the same in Air) if $V = \mathit{V}$, then $E : \mathit{E} :: M : \mathit{M}$. But in equal Quantities of Matter, *viz.* $M = \mathit{M}$, their Effects will be as the Squares of the Velocities, *viz.* $E : \mathit{E} :: V^2 : \mathit{V}^2$ (See *Annot.* XLIX. 17.) Therefore, when neither the Velocity, nor Masses of Matter are known, the Effects will be in a given Time in a *Ratio* compounded of both; that is, $E : \mathit{E} :: MV^2 : \mathit{MV}^2$.

12. But we have shewn $M : \mathit{M} :: DB : \mathit{DB}$, (See *Annot.* LVI. 9.) therefore $E : \mathit{E} :: DBV^2 : \mathit{DBV}^2$ in a given Time. Let us now suppose $B = \mathit{B}$; then because $D : \mathit{D} :: 860 : 1$, (See *Annot.* LXXXIX. 6.) we have $E : \mathit{E} :: 860\,V^2 = : \mathit{V}^2$; and lastly, if we suppose the Effects to be equal, *viz.* $E = \mathit{E}$, then we have $860\,V^2 = \mathit{V}^2$. Therefore if we put $V = 1$, we have $860 = \mathit{V}^2$; and so $\mathit{V} = \sqrt{860} = 29{,}326$; that is, *The Velocity of Air ought to be somewhat more than 29 Times greater than that of Water to strike a given Surface with the same Force.*

13. Indeed Mr. *Belidor* makes $\mathit{V} = 25\tfrac{1}{2}$, because he has strangely mistaken the specific Gravity of Air to be $\frac{1}{640}$, instead of $\frac{1}{860}$, on which Account all his Calculations on this Head are very faulty. If V denotes any equable Velocity of Water, the Height H of a Fall necessary to produce that Velocity is thus found, As $32 : \sqrt{16} (= 4) :: V : \sqrt{H}$; or thus, As $1024 : 16 :: V^2 : H = \frac{16V^2}{1024} = \frac{V^2}{64}$; or putting $V = 1$, we have $H = \frac{1}{64}$. Now a cubic Foot of Water, whose Height is 1, strikes with a Force $= 62{,}5\,lb$; therefore the Force of a Column, whose Height is H, striking against a Surface of one Square Foot is $62{,}5\,H = \frac{62{,}5}{64}$, or $= \frac{62{,}5\,V^2}{64}$, when the Velocity is not given.

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the Particles of a sonorous Body and those of Air, we shall find that Sound is nothing but the

14. But (by *Art.* 12.) $V^2 = \frac{\mathcal{V}^2}{860}$, therefore $\frac{\mathcal{V}^2}{860} \times \frac{62,5}{64} = 62,5\,H = 0,976\,lb.$ the Force of a Stroke of a Column of Water whose Velocity is $V = 1$ and of Air, whose Velocity is $\mathcal{V} = 29,3$, and Height $H = \frac{1}{64}$ of a Foot; therefore $\frac{\mathcal{V}^2}{860} \times 0,976 = 0,00113\,\mathcal{V}^2$ will be a constant Multiplier to reduce the Force of Wind blowing with any Velocity $\mathcal{V}$, on any given Number of square Feet or Area A, to Pounds *Averdupois* Weight. For Example, suppose the Velocity of the Wind at the Rate of 20 Feet *per* Second; here $\mathcal{V} = 20$, and $\mathcal{V}^2 = 400$, and $0,00113\,\mathcal{V}^2 = 0,00113 \times 400 = 0,452$ of a Pound on a square Foot; and therefore on 10 square Feet it will be 4,52 *lb.*; on 100 square Feet it will be 45,2 *lb.*; on 1000, 452 *lb.*; and so on.

15. Hence, to compute the Force of Wind on the Sails of a Mill we proceed as follows: Admit the Length of a Sail be 30 Feet, and Breadth 6 Feet, the Area or Surface will be 180 square Feet, and $4 \times 180 = 720$ square Feet, the Area of the 4 Sails; then admitting the Velocity of the Wind the same as before, *viz.* 20 Feet *per* Second, the Force on each square Foot is 0,452, and therefore $0,452 \times 720 = 325,44\,lb.$ This is the absolute or whole Force of the Wind blowing directly on the Sails: But since when the Sails are set right, this Force is diminish'd in the Ratio of 13 to 5, therefore $\frac{5}{13} \times 325,44 = 125,17\,lb.$

16. Suppose the Distance from the Axis Q to each Sail be 5 Feet, then will the Distance of the Center of Gravity PQ be 20 Feet; therefore $20 \times 125,17 = 2503,4\,lb.$ the mechanical Force of the Wind on the Sails *per* Second to produce the Effects within the Mill, which may be computed as in the Example of the Water-Mill, *Annot.* XLIV.

17. To represent these Things more generally, let A = Area of all the Sails, $\mathcal{V}$ = Velocity of the Wind; then $0,00113\,\mathcal{V}^2 A$ = absolute Force, which multiplied by $\frac{5}{13}$ is $0,000435\,\mathcal{V}^2 A$ = Force reduced by the oblique Position of the Sails. Now suppose a Weight W hanging from an uni-

Propagation of the Tremors and Vibrations of the former impreſs'd on the latter, to the *Tym-*

form Axle, whoſe Semidiameter is d, keep the Sails *in Equilibrio* with the Force of the Wind; then D being the Diſtance of the Center of Gravity of the Sails, we have $D : d :: W : 0{,}000435\,V^2 A = F = \frac{W d}{D}$.

18. But becauſe, when the Machine is in its greateſt Perfection, the Weight it is charged with is but $\frac{4}{9}$ of W, (See *Annot.* XL.) therefore $\frac{4}{9} W \times \frac{d}{D} = \frac{4}{9} F = 0{,}000193\,V^2 A = P$; then putting $r = 0{,}000193$, we have $r V^2 A = P$, the reduced Force for the greateſt Effect; and $\frac{P}{r V^2} = A$, the Area or Surface of the Sails; and laſtly, $\sqrt{\frac{P}{r A}} = V$, the Velocity of the Wind, which therefore may be found by having A and P given.

19. Let $\frac{4}{9} W = w$, the Velocity of which Weight let be u; then $\frac{1}{3} V =$ Velocity of the Center of Gravity of the Sails; then $\frac{V}{3} \times P = u \times w$, whence any one of the four Terms may be found, the reſt being given. Alſo $P = \frac{3uw}{v} = r V^2 A$, or $3uw = r V^3 A$; whence again any one of the four Quantities A, V, w, u, may be found, the others being known.

20. Since the Force of the Machine is as $A \times V^2 \times D$, it will be a *Maximum* when $A \times D$ is greateſt, the Velocity of the Wind V remaining the ſame; and if A be given, the *Maximum* will be when D is greateſt of all. Hence it appears, that if we are not confined to a given Diſtance from the Axis Q for adjuſting the Sails, we may diſpoſe the given Surface A into the Form of an *Iſoſceles Triangle* in each Sail,
Plate XXXIII. Fig. 3. *viz.* ſuch as ABCD, inſtead of the equal Parallelogram-Sail $abcd$ in common Uſe; for in the Triangular Sail the Center of Gravity is at P, and its Diſtance is QP; whereas in the Rectangular Sail $abcd$ the Center of Gravity is in the middle Point p, and its Diſtance is Qp, much leſs than before.

21. For Example, let $ab = 6$ Feet, and $bc = 30$; then the Area $A = 30$ Feet ſquare. Let QD equal 5, then is $Qp = 20 = D$, and $A \times D = 600$. But in the Triangu-

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panum or Drum of the Ear, by the Action of whose Membrane they are communicated to the

lar Sail the Distance QP = 35 = D, and therefore A x D = 1050. The Force therefore of the same Wind upon the same Quantity of Sail, at the same Distance QD from the Axis, in the Triangular Sail ABCD, is to that on the common Sail *abcd* as 1050 to 600, that is, QP = 35 to Q*p* = 20, or as 7 to 4, which therefore is nearly twice as great. The Truth of all is evident by Inspection of the Figure, and *Annot*. XXXV. 8, 9.

22. I suppose it was some Consideration of this Kind which led Mr. *Parent* to propose Sails in Form of *Elliptic Sectors*; for the Centers of Gravity in them also are removed to about two Thirds of their Length, and are moreover better adapted to fill a circular Space when placed oblique to the Wind, so that no Wind be lost when you would take in all that falls on a given Space or Area. But for what Reason he should declare the transverse Position of the common Sail to be more advantageous than the longitudinal one, I am at a Loss to guess. However, as I have not seen any thing he has wrote on the Subject, I shall say no more of the Matter.

23. As it would be endless to recount all the various Uses which are or may be made of this most useful universal Element of Air, both for Natural and Mechanical Purposes; I shall content myself with setting before the Reader the Theory of that most useful domestic Instrument the BELLOWS, by whose means the Action of Fire, or Intensity of its Heat, may be increased to a prodigious Degree. And for this Purpose I shall have recourse to the Example of that curious Naturalist Dr. *Hales*, in his *Statical Essays*, Vol. II. Pag. 329. which is as follows.

24. The Doctor measured the upper Surface of a Pair of Smith's Bellows, and also the Space they descended through in a Second of Time; by which he found the Quantity of Air expell'd in that Time was 495 Cubic Inches in its compress'd State. Now to find what Degree of Compression it suffer'd, he fix'd a Mercurial Gage to the Nose of the Bellows, and found the Force of the compress'd Air sufficient to raise the Mercury *one Inch* high, at a Mean. Hence it appear'd, that the Force with which the Bellows impell'd Air into the Fire was $\frac{1}{30}$ of the Weight of the Atmosphere.

25. Hence also it follows, that the Air driven through the Nose of the Bellows in one Second was more than 495 Inches by a $\frac{1}{30}$ Part of that Quantity, *viz*. by 16,5 Inches, which

Air in the internal Cavities of the Ear, where the *Auditory Nerve* receives the Impression, and ex-

added to the former make 511½ Inches of common Air. To find the Velocity with which this Air was impell'd, he measur'd the Area of the Orifice of the Nose, and by that divided the 495 Inches, which gave for the Quotient 825 Inches, or 68,73 Feet, for the Length of the Cylinder of Air which rush'd *per* Second through the Nose of the Bellows; which prodigious Velocity of Air acting constantly on the elastic re-acting Particles of Fire must immensely increase their intestine Motion, and proportionably augment the Heat, which consists therein, and from which all our Sensations of this Kind are derived.

26. The Doctor concludes with a Query, Whether if the Force with which the Air is impell'd by the Bellows into the Organ-Pipes were taken in this Manner, we might not estimate the Velocities of the Undulations of Air required to form the various Notes or Sounds? The Velocity of undulating Air to that of Water being as their Densities inversely, nearly, *viz.* as 860 to 1, as will be shewn farther on.

27. Mr. *Martin Triewald* of *Sweden* has lately exhibited a new Invention for producing a continual Stream of Air, to blow the Fire of great Forges, Founderies, &c. and which may properly be call'd WATER-BELLOWS; for the Contrivance is two hollow Bell-form Vessels, suspended from the Ends of a Lever, which is put into Motion by a Stream of Water running into two Troughs, both uniting or joining rather at the Stream, so that only one at a time can receive the Water; which running to the larger and wider End, laid over the End of the Lever, does by its Weight carry the Lever down on that Part, till by descending the Water all runs out; and then the other Trough (which was filling in the mean time) preponderates, and forces down the other End of the Lever; and thus the Machine is constantly kept in Motion.

28. When one Arm of the Lever is raised, the Bell or Bellows hanging from it will be raised above the Surface of Water (in which the Machine is placed) that it may be fill'd with Air. Upon the Descent of the Lever, the Bell (by Weight affix'd to it) descends into the Water, by which means the included Air is greatly compress'd, and thereby forced to pass through a long small leathern Tube, going from the Top of the Bell to other metalline Tubes, which convey it to the Fire. Thus, by means of these two librating Bells, a constant Blast of Wind is supplied, whose Velocity may be increased

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cites the Senſation in the COMMON SENSORY in the BRAIN (CI).

or diminiſh'd by proper Contrivances, which the Reader may ſee in the *Philoſophical Tranſactions*, together with a Print of the Engine.

(CI) 1. The Structure of the EAR, with its admirable *Apparatus* to conſtitute an *Organ of Hearing*, is well worth the Attention of every Man. The external Part is adapted for taking in a large Portion of the tremulous Air, which is reflected ſtrongly by a fine, elaſtic, tremulous Cartilage, and by this means it is convey'd more denſe and elaſtic to the interior Cavity, or *Concha* of the outward Ear.

2. The free, hollow, elaſtic Aperture of this Cavity, conſtructed with proper Muſcles, is by that means capable of being expanded, contracted, and every way adapted to receive the various Tremors of the Air: And moreover it is ſo diſpoſed, that it is able more firmly to unite and condenſe, or more laxly to diſperſe or rarify, the ſame aerial Rays, ſo as to accommodate itſelf for attemperating a Sound too ſtrong, and augmenting it when too weak, as occaſion requires.

3. The *Meatus Auditorius*, conſiſting partly of a cartilaginous and partly of a bony Pipe, conveys the Sound towards the interior Parts, and the Obliquity of the Canal increaſes the Superficies, and conſequently multiplies the Points of Reflection. Moreover, the triangular cartilaginous Tongue, by its elaſtic tremulous Texture, and erect Poſition in the Hollow of the *Concha*, juſt over the Orifice of the Auditory Paſſage, cauſes, by an egregious Mechaniſm, that all the Rays of Sound which arrive at the Ear ſhall enter the ſaid Paſſage; and prevents their flying out again by any Reflections whatſoever. Its tubulous cylindro-elliptical Figure, by a ſerpentine Progreſs firſt aſcending, then deſcending, and then aſcending again till it terminates in the Membrane of the *Tympanum*, increaſes the Reflection and Sound, and cauſes that all the ſonorific Rays ſhall at laſt fall united upon the central Point of its End; hindering at the ſame time all Senſation of a confuſed and clangorous Sound.

4. The *Membrana Tympani*, or fine Membrane at the End of the *Meatus Auditorius*, is ſo obliquely extended acroſs the Paſſage, as above to make an Obtuſe Angle, and below an Acute one with the ſaid *Meatus*. Hence the Surface is increaſed, and render'd more capable of tremulous Concuſſions, and of concentring the Rays upon its middle Point.

FOR

For the Parts of a *sonorous* Body, being put into Motion by Percussion, do vibrate forwards and backwards through very small Spaces, by their elastic Quality. In this Action, they affect the Particles of Air contiguous to them, and compel them upon the first Impulse to move forwards also; and those propel the next, and so

5. This Membrane being expanded upon and connected with the bony Margin of the *Meatus*, is on the fore Part (towards the *Meatus*) concave, and convex behind or on the internal Part, where it is applied to the Handle-Part of a little Bone call'd the *Malleus* or Hammer, whose Head is moveable in a bony *Sinus* on one Part, and on the other it is articulated with another little Bone call'd the *Incus* or Anvil, which freely moves in that Articulation; and on the other End it is again articulated with a little orbicular Bone, and with the *Stapes* or Stirrop, which on its Base-Part is connected to a Membrane spread over the *Foramen Ovale* or elliptic Hole of another bony Cavity call'd the *Vestibulum*.

6. But as it will be impossible to give an Idea of this wonderful Construction without a Print, therefore let AB be the external Ear; C its *Concha*, or Cavity; DE the *Meatus Auditorius*, which in Length is $7\frac{1}{2}$ Tenths of an Inch, in Breadth 3, and in Depth 4. G is the *Membrana Tympani*, h the Handle of the *Malleus*; k the *Incus*, and *i* the orbicular Bone; *n* the *Stapes*, and r the *Vestibulum* hollow'd out of the *Os Petrosum*, in the Cavity of the Labyrinth.

Plate XXXIV. Fig. 1.

7. In the Vestibule we observe the following particular Construction of Parts. On the larger Part are three semicircular Canals or Conduits O, P, Q, which communicate by five Orifices with the Cavity of the Vestibule; they are of a bony Substance, and of an elliptic Cavity. The lesser Part of the Vestibule communicates with the *Cochlea*, or spiral Fabric S.

8. This wonderful Part merits particular Notice, and is therefore represented by itself in two Figures; wherein is shewn the bony conical Canal ST, making $2\frac{1}{2}$ Revolutions round a bony Cone from the Base to the Apex T. This spiral Cavity is, from the Base S to the Top T, divided by a transverse *Septum*, or Partition, of a triangular Figure, represented by ZX. This on its Base-Part adhering to the Cone, is bony, (which is shewn by *a, a, a,*) and is of an elastic tre-

Fig. 2, 3.

on,

on, to a very considerable Distance, according to the Intensity of the percussive Force. By this means the Particles of Air are compress'd nearer together, than in their natural State.

BUT when the Particles of the sonorous Body make the second Part of the Vibration, by returning back again, the Particles of Air also, by

mulous Texture, and exceeding smooth or polite. The exterior Part *b, b, b,* is of a membranaceous nervous Texture, whose Chords or Fibres lie as represented in the Cut: It is connected with the bony Base on one Part, and with the Canal on the other, so that the Spiral Duct of the *Cochlea* is divided into two equal Cavities without any Communication with each other; though the Orifice of the superior Cavity opens into the *Vestibulum*, and the other is shut close by the Membrane of the *Foramen Ovale*.

9. The Auditory Nerve V enters the Vestibule by several little Holes as at S, and forms a curious Lining or Tapis all over the inside Surface both of the Vestibule and its semicircular Canals O, P, Q. These Nerves also pass into the *Cochlea*, and entering between the two Membranes of the triangular Zone, or *Septum*, Z X, do there divide, and branch themselves out into an exquisite membranous Expansion on each Side the same, which thus becomes the more immediate Organ of Hearing.

10. This Cavity of the Vestibule is always fill'd with an elastic Air, though there appears no visible Way by which it can enter. Also the Labyrinth or Cavity of the Drum is fill'd with common Air, by means of the *Eustachian Duct* or *Tube*, as M N; the Orifice M opening into the Mouth, and N into the Cavity of the Labyrinth.

11. Having thus premised a Description of the several Parts, we shall the better apprehend how Sounds are excited in the Mind. Thus the Pulses of Air entering the *Meatus Auditorius* D E are condensed by various Reflections through the Passage to their Incidence on the *Membrana Tympani* at G, which is render'd more or less concave, or lax and tense, by the Handle h of the *Malleus*, actuated by its proper Muscle. By this means the Air contained in the Labyrinth is admitted, expell'd, compress'd or rarified, according as the *Eustachian Tube* is opened or shut.

12. The *Membrana Tympani* G being thus adapted for re-

their

their repulsive Power, repel each other towards their proper Places, and thus again expand themselves.

Now since Motion once generated in elastic Bodies continues some time before it can be destroy'd by the Resistance and Counteraction of contiguous Bodies, it follows, that the Particles of

ceiving the Sounds of tremulous harmonic Bodies, and modulating the internal Air of the Labyrinth, can easily communicate the Impressions to the *Incus k*, which transmits them to the *Os Orbiculare i*, this to the *Stapes n*, and that to the Membrane of the *Foramen Ovale* of the *Vestibulum r*.

13. This Membrane, by such an *Apparatus* of Parts, may be intended or remitted in infinitely different Degrees, so as to become adapted for the Tremors of every Sort or Degree of Sound; and for communicating them to the internal Air, which affects the Nerves every where expanded over its internal Surface, but more especially the nervous Expansion of the *Cochlea*.

14. For here, as we have shewn, the Fibres of the *Septum Transversale*, *b, b, b*, are contrived like so many Strings of an Harpsichord, of various decreasing Lengths, and different Octaves, that so some or other of them may be of a proper Length to be in Concord with the sounding Body, or to tremble with the same Vibrations, which by means of the Nerves are convey'd to the Common Sensory in the Brain, where the Mind perceives and distinguishes the infinite Differences of harmonious and discording Tones.

15. Thus, though we are admitted to view the amazing Mechanism of the Organ of Hearing, yet can we get but a general Notion of the Manner in which these Sensations are produced, or of the particular Functions perform'd by every Part, and the special Uses to which they are subservient, in the general Execution of this Sense; with respect to which there remain many Things yet to be enquired after, even by the Learned *Boerhaave*, as we find in Page 250 of his *Institutes*, which see.

16. From this Account of the Ear, we have a Solution of some Difficulties; as, Why the Ear is affected with great Pain in going down into the Sea in a Diving-Bell: Why People generally open their Mouths when they listen with great Attention: Why Deafness ensues on a Rupture of the *Membra-*

the

the sonorous Body, and consequently those of the adjacent Air, have for some time a reciprocal vibratory Motion, by going forwards and backwards through very small Spaces in an indefinitely small Particle of Time; which Motion gradually decreases, till it be totally destroy'd (CII).

na Tympani, or from an Obstruction of the *Eustachian* Tube: Why we hear but one Sound with two Ears. But how some People taking Smoke into the Mouth can emit it by their Ears, is not so easy to answer, there being as yet no Perforation of the *Membrana Tympani* discovered; though this seems a plain Demonstration that there is one or more, though not perceptible to the Eye.

(CII) 1. The Doctrine of Sounds is the most intricate and perplex'd of any thing we find in Philosophy; and perhaps this is the only Subject which the greatest of Men has (in his *Principia*) treated in a Manner not quite so physical and mathematical as the Nature of the Thing required. I shall refer the Reader to the Commentaries of Mess. *Le Seur* and *Jacquier* on the *Principia*, where they will find Sir *Isaac Newton*'s Hypothesis relating to the Motion of the Particles of an elastic Medium to be fallacious; and other Methods proposed, by which the *Newtonian* Doctrine of Sound is restored. I shall here add an Explication of such *Phænomena* only, as are of principal Concernment, and at the same time pretty easy to be understood.

2. Let ABC be an elastic String or Chord, fix'd in the Points A and C, and drawn out of its natural right-lined Situation ABC. Such a Chord, in its State of Tension, will, when let go, return by its natural Resort, not only to its natural Situation ADC, but with the Motion it there has will go on to E, so that DE is nearly equal to BD; and from thence it will return again nearly to B; which Motion from B towards E, and from E towards B, will be reciprocated a great Number of Times before the Chord will come to a State of Rest: And each Motion through the Space BE is call'd a *Vibration* of the Chord. Plate XXXIII. Fig. 4.

3. When the Chord begins its Motion at first from B, it strikes the Particle of Air contiguous to it in B, and that will by its Approach towards the next affect it, by means of the repulsive Power, which keeps them all at equal Distances from

FROM

FROM the Nature of a Fluid, whatever Motion is generated in any one Particle, it is by that Particle communicated equally to all around it, as from a Center; consequently the Tremors of

each other; and so on through such a Number of Particles as can receive the Motion while the String moves from B to D. Let A, B, C, D, E, F, G, *&c.* represent such a Series of Particles of Air at an equal Distance, and the first Particle A contiguous to the Middle Point B of such a String, and agitated by it in its Motion.

4. The String beginning to move, all the Particles A, B, C, will begin to move forwards also; and since this Motion is propagated in Time, let E be the remotest Particle moved while the Chord is moving from B to D; during which Time the Chord, having an accelerated Motion, will cause the Particles to approach each other with an accelerated Motion likewise; and because those accelerated Approaches begin at A and reach to E, in the Time the Chord is going from B to D, therefore the Distance AB will be less in BC, and this less than CD, and that less than DE, and the Distance EF will begin to be lessen'd when the String is arrived to the Site ADC, and the Particles A, B, C, D, E, F, *&c.* will have the Arrangement represented in the second Line.

5. But now the Chord, having acquired the Situation ADC, will be no farther accelerated, but on the contrary retarded, as it will now go on from D to E; the Effect of which upon the Particles of Air before it will be as follows. They will all go on forwards till the Chord comes to E, and the Particle A to its Situation in the third Line: But since the Force upon A begins to abate as the String begins to move from D, the elastic Force now between A and B will, by acting both ways, continue to accelerate the Motion of B, and retard that of A. Thus the Distance BC will still diminish till B come to be nearly equidistant between A and C; and C will be accelerated till it be equidistant between B and D; and so on. So that as the Acceleration is continued forwards, the Distances will diminish towards F; and by the Time the Chord is arrived at E, the Particles EE will be at their nearest Distance. And since the Motion of A is continually retarded, it will lose what before it had gain'd in the same Time; and will therefore now be at the same Distance from B as at first nearly. So that the Particles from A to G will have the Situations as represented in the third Line.

the

the founding Body will be propagated all around from the Point of Percuffion, as a Center, in *concentric hollow Superficies or Shells of Air*, which are not improperly call'd *aerial Pulfes*, or *Waves*

6. The Chord now returning from E to D, gives Liberty to the repulfive Power between A and B to feparate them to a greater Diftance than in their natural State; and which they at prefent have. By this means all the other Intervals B C, C D, D E, E F, will alfo increafe; and become fucceffively greater than the natural Diftance; but that Excefs will be leffer in each, till you come to F G, which will be equal to the natural Diftance, at prefent between A and B. The Motion at the fame Time continuing in all the Particles from H to N, they will all move forwards, and the prefent contracted Interval between H and I will fucceed between all the reft, till it arrives to the Particle N, when the Interval M N will be the fame as at prefent is H I. And thofe Particles beyond N to S, will, by the preceding ones, be put into the fame refpective Diftances, but in an inverfe Order, as thofe have between G and N. And the whole Series (now the String is at D) will have the Intervals of the Particles refembling thofe in the 4th Line.

7. The Chord not ftopping at the Situation A D C, but going on towards A B C with a retarded Motion, the Velocity of the contiguous Particle A will alfo be retarded and become lefs than that of B; upon which the Diftance between them will be leffen'd, and the more fo as the String approaches to B. Hence all the Intervals, now dilated beyond their natural State, will, by degrees, contract; but gradually flower, till you come to F, where the prefent largeft Interval between A and B will be found between F and G, and that between A and B will have acquired its natural Extent when the Chord is arrived at B. Then likewife the Particles from G to N will acquire the fame Situation as thofe now have between A and G; and from N to S, the fame as now is feen between G and N; and from S forwards, the fame as is now before the Particle N, the Point S being now the middle Point of Condenfation; all which is clearly feen in the 5th Line of the Figure.

8. Thus the Condenfation which began at A, by the firft Part of the Vibration, was propagated to G by the fecond, from thence to N by the third, and laftly to S by the fourth Part of the whole Motion of the String in going and return-

of

of Air: Analogous to which are the circular Waves generated on the Surface of Water all around the Point where any Impression is made, in any Manner or Direction whatsoever (CIII).

ing; and this Extent of Air, thus agitated by the Chord in going and returning, is call'd by Sir *Isaac Newton* a *Wave or Pulse of Air*. In which Wave the Particles from A to N are in a dilated State, and from N to X in a contracted or condensed State; which two Parts of the Wave answer to the concave and convex, or low and high Part of a *watry Wave*.

9. As the Chord goes on to make another Vibration, it will not only continue to agitate the Air at present in Motion, but will spread the Pulsation of the Air as much farther, and by the same Degrees as before; and the like will happen after every compleat Vibration of the String. Thus the Air being a fluid Body, and the Impression made on any one Part affecting all the Particles alike around it, 'tis plain, those Pulses will be propagated in every Direction all around in concentric Aerial Shells or spherical Waves of Air.

10. That the Motion of the Pulses in an elastic Medium is analogous to that of Waves generated in the Surface of stagnant Water, is evident, when we consider that the Condensation of the Parts of the elastic Medium is in lieu of the Elevation of the Water; the elastic Force effects the same in the Medium as Gravity does in the Water, and the densest Parts of the Pulses correspond to the highest Parts of the Waves. Wherefore as there is so great an Affinity between these two Phænomena, it will be requisite, before we go farther, to explain the Nature and Properties of aqueous Waves, which will therefore be shewn in the next *Annotation*.

(CIII) 1. Sir *Isaac Newton* explains the Nature of Waves in Water after the following Manner. Let A B and C D be the Surface of Water quiescent in the upright Legs K L, M N, of a recurv'd Tube. And if the Water be put into Motion, and ascends in the Leg K L, to E F, it will descend in the Leg M N to G H; so that E F = G H. Again, Let P V be a Pendulum vibrating in the Cycloid R P S; its Length V P, from the Point of Suspension to the Centre of Oscillation, is equal to half the Length of the Water in the Tube; let P be the lowest Point, and P Q an Arch of the Cycloid equal to the Altitude A E.

Plate XXXIII. Fig. 6, 7.

THESE

THESE Pulses or Waves of Air are affected with the following Properties, *viz.*

I. *They are propagated all around, in a spherical undulatory Manner* (as I said but now;) and that not only from the tremulous Body, but from the Holes in any Obstacles they meet with: Whence it comes to pass, that *one and the same Sound may be heard by several Persons*, in any different Situa-

2. The Force by which the Water is alternately accelerated and retarded in its Motion in the Tube, is the Excess of the Weight of Water in either Leg above the Weight in the other; and therefore when the Water in the Leg KL ascends to EF, and in the other Leg descends to GH, that Force is equal to the Weight of the two equal Quantities of Water $AEFB + CGHD = 2AEFB$; and therefore is to the Weight of the whole Water as EA to VP, or as PQ to PR, because the Semi-cycloid PR is equal to the Length of the Pendulum which describes it, from the Nature of the Curve.

3. Also the Power by which the Weight P is in any Point Q accelerated or retarded in the Cycloid, is to its whole Weight as the Distance PQ from the lowest Point P to the Length of the Semi-cycloid PR. Wherefore the moving Forces of the Water and Pendulum, describing equal Spaces AE, PQ, are as the Weights to be moved; and therefore, if the Water and Pendulum are at first quiescent, those Powers will move them equally in equal Times, and cause that they go forwards and backwards together, with a reciprocal Motion. All which is easily deduced from what has been said of the Nature of the Cycloid, the Motion of heavy Bodies, and the Forces of Bodies in Motion.

4. Hence it follows, that whether the Distance AE be great or small, the Reciprocations of the Water will be all perform'd in equal Times. Also it follows, that if the whole Length of the Water be 78,4 Inches, each Reciprocation, or Ascent and Descent of the Water, will be perform'd in *one Second* of Time; because a Pendulum of half that Length vibrates in that Time. Lastly, if the Length of the aqueous Canal be increased or diminished, the Time of each Reciprocation will be increased or diminish'd in the subduplicate Ratio of the Length.

tions with respect to the sounding Body, if not at too great a Distance.

II. *The Density of these aerial Pulses decreases, as the Squares of the Distances from the sounding Body increase:* For since the Force or Motion in each Shell is the same, it must decrease as the Number of Particles increases in each Shell: But this Number of Particles is as the Superficies of

5. When the Nature of Waves in Water is consider'd, it will be found to agree very nearly with the Motion of the Water in the Tube above-mention'd; and consequently their Motion will be similar to that of a Pendulum. For let EFG represent the level Surface of Water when it is not agitated so as to produce Waves; when it is thus agitated, let A, B, C, D, represent the wavy Surface; A, C, the highest Parts of the Waves; and B, D, the lowest or concave Part. Then 'tis evident the Weight of the Water at A above EG will cause it to descend as far below the Level to B; and with the Motion acquired by that Descent, it will again ascend to the same Height C, and so produce a constant Succession of of Waves in the watry Surface, after the same Manner as was shewn in the Tube.

Plate XXXIII. Fig. 8.

6. Hence it follows, that, because the Length of the whole Water to be moved is from the highest Point A to the lowest Point B, if the Length of a Pendulum be $\frac{1}{2}$ AB, it will oscillate once while the Water descends from A to B; and in another Oscillation, it will ascend from B to C, and so on. So that a Wave will pass thro' its whole Length in the Time of two Oscillations; and therefore in the Time of one Oscillation of a Pendulum four times as long, or equal to ABC.

7. Whence because ABC in very large and wide Waves is nearly equal to the Breadth AC; therefore when the Waves are 39,2 Inches broad, they will undulate in one Second of Time; and consequently since the Times of all the Undulations are equal, there will be $39{,}2 \times 60 = 2352$ Inches, or 196 Feet run thro' by a Wave in one Minute, which is 11760 Feet *per* Hour. Hence also the Velocity of greater or lesser Waves will be increased or diminish'd in the subduplicate Proportion of their Breadth; that is, if $V =$ Velocity of the greater Waves ABCD, and $v =$ Velocity of the

the

the Shell, which is as the Squares of the Diameter or Semidiameter of the Sphere, that is, as the Distance from the sounding Body. Hence the Distinction of Sounds into *loud* and *low*, *strong* and *weak*, according as we are nearer to, or farther from, the sounding Body. The utmost Limits of *audible Sounds* are about 180 or 200 Miles. (CIV.)

lesser Waves *a*, *b*, *c*, *d*, *e*, *f*, &c. then it will be $V : v :: \sqrt{AC} : \sqrt{ac}$. Because the Velocities and Times of Bodies moved in any manner by Gravity, are proportional to the Square Roots of the perpendicular Altitudes; and those Altitudes are as the Lengths of Pendulums, and therefore as the Breadth of Waves.

(CIV.) 1. Let ABC represent the sonorous Body; by the tremulous Motion of its Parts, it will agitate the Air contiguous to every Point as A, where it will be condensed to a certain small Distance, and make a Pulse or Wave of Air in the Manner as has been largely shewn (*Annotat.* CII). The first Wave or Pulse will by its elastic Power in expanding itself produce a Second, that a Third, and so on; till the impress'd Motion be diffused thro' too large a Quantity of Air to be any longer sensible. Plate XXXIV. Fig. 4.

2. The Quantity of Motion produced by each Tremor of the sonorous Body, being communicated successively to larger Portions of Air, the Part thereof which each Particle will acquire will constantly decrease. This Decrement of the Motion will be as the Increment of the Number of Particles, which is as the Superficies of the spherical Shell; and since all Superficies are as the Squares of their Diameters or Semidiameters, therefore the Force in the Particles of the Wave or Shell at D is to that in the Particles of the Shell at F as AF^2 to AD^2; that is, the Force of Sound decreases as the Squares of the Distances increase.

3. It is plain, the Distance to which Sounds may be heard will be proportional to the Magnitude or Intensity of the Stroke made on the tremulous Body emitting the Sound; for the greater that Stroke is, the greater will be the Agitation of the Parts of the sonorous Body, and of course the greater will be the Force with which they will strike the Particles of

III. *All the Pulses, whether denser or rarer, move with equal Velocities:* This Sir *Isaac Newton* has demonstrated *a priori*, and also that this Velocity is at the rate of 1142 Feet in one Second of Time; which most exactly agrees with the repeated and most accurate Experiments of the late Reverend Mr. *Derham*. The Velocity of Sound is therefore near thirteen times as great as that of the strongest Wind: And since it must necessarily increase with the Air's Elasticity, it will be greatest in Summer when the Air is most heated, and *vice versa* in Winter: Also, as the Motion of the Wind conspires with, or is contrary to that of Sound, the Velocity of Sound will be in some small Degree augmented or diminished thereby, though not discernible in Experiments.

IV. *The Interval or Distance of the Pulses from each other is the same among all that are excited by the same Stroke:* For since each Pulse is caused by a single Vibration of the sounding Body, and since they all move with equal and uniform Velocities,

Air. Lastly, the greater the Force is upon the Air, the more strongly will it be condensed and expanded; hence the greater will be the Stroke at any given Distance on the Drum of the Ear, and consequently the greater will be the Distance at which the Agitation of the Air will be sensible.

4. The Experiments are numerous by which it has been found, that Sound is audible to the Distance of 50, 60, or 80 Miles: But Dr. *Hearn*, Physician to the King of *Sweden*, tells us, that at the Bombardment at *Holmia*, *A. D.* 1658, the Sound was heard to the Distance of 30 *Swedish* Miles, which make 180 of ours. And in the Fight between *England* and *Holland* *A. D.* 1672, the Noise of the Guns was heard even in *Wales*, which cannot be less than 200 Miles.

locities, 'tis plain they must succeed each other at Intervals proportion'd to the Times of the Vibrations: But the Times of the Vibrations of the same Body are all equal; consequently, the Intervals of the Pulses will be so too. (CV.)

(CV) 1. Sir *Isaac Newton* and other Mathematicians have shewn (in a Method too prolix and intricate to be here repeated) that if a Pendulum were constructed whose Length was equal to the Height of an homogeneal Atmosphere, whose Density is every where the same with that of the Air upon the Surface of the Earth, in the same Time that such a Pendulum makes one whole Oscillation in going forwards and backwards, the Wave or Pulse of Air will pass through a Space equal to the Circumference of a Circle described with a Radius equal to the said Pendulum.

2. Therefore while the Pendulum makes half an Oscillation, or one single Vibration, the Pulse will move through a Space equal to half the Circumference: Whence the Space described by the Pulse in the Time of a Vibration is to the Length of the Pendulum as the Semi-circumference to the Radius, or as the Circumference to the Diameter, that is, as 3,14159 to 1. Now the Length of such a Pendulum is 30100 Feet, (as we have elsewhere shewn) but Sir *Isaac* makes it 29725 Feet, whose Measure we shall here follow. The Circumference of a Circle whose Radius is 29725 is 186768, the Half whereof is 93384 = Space a Pulse passes through in one single Vibration: But since a Pendulum 39,2 oscillates in the Time of one Second, and the Times of Oscillation in different Pendulums are in the Subduplicate Ratio of their Lengths; therefore, since in 29725 Feet we have 356700 Inches, we must say, As $\sqrt{39,2} : \sqrt{356700} :: 1 : 95\frac{3}{8}$ Seconds. But in that Time the Pulse passes over 93384 Feet; consequently, $95\frac{3}{8} : 93384 :: 1 : 979$ Feet = Space pass'd through by a Pulse in one Second of Time.

3. This then would be the Velocity of the Pulses, were the Particles of Air so very small as that their Magnitude should bear no sensible Proportion to the Intervals between them; and also if the Medium had no Admixture of any other Particles but those of pure Air: Neither of which is the Case; for the Particles of Air are so gross that they will not pass through the Pores of Glass any more than Water; and Sir *Isaac Newton* supposes them to be of the same Mag-

V. *The aerial Pulses are propagated together in great Numbers from different Bodies without Disturbance or Confusion*; as is evident from Concerts of Musical Instruments, where divers Sounds, of different Intervals and various Coincidences, strike the Ear at once, yet with Di-

nitude with the Particles of Water or Salt. If this be so, let $D =$ Diameter of the Particles, $S =$ Space or Interval between them; then will $S + D =$ Distance of the Centers of the Particles. Let $N =$ Number of Particles in the Side of a Cube of Air, then will $NS + ND =$ Side of the Cube.

4. Again, let $M =$ Number of Particles of Water in the Side of an equal Cube, and $MD =$ Side of the Cube of Water; whence $NS + ND = MD$. Then if the Density of Air be to that of Water as 1 to A, we shall have $1 : A :: N^3 : M^3$; whence $1 : A^{\frac{1}{3}} :: N : M$; consequently, $M = NA^{\frac{1}{3}}$. Wherefore, since it is $NS + ND = MD = NDA^{\frac{1}{3}}$, it will be $S + D = DA^{\frac{1}{3}}$, and $S = D \times \overline{A^{\frac{1}{3}} - 1}$; therefore $D : S :: 1 : A^{\frac{1}{3}} - 1$; whence $D : S + D :: 1 : A^{\frac{1}{3}}$.

5. If therefore $A = 860$, (as we have shewn) then $A^{\frac{1}{3}} = 9$ nearly; if we put $A = 1000$, then $A^{\frac{1}{3}} = 10$. Whence $D : S + D :: 1 : 9$, or as $1 : 10$; whence the Diameter of a Particle of Air will in such a Case be to the Interval between the Particles as 1 to 8 or 9. And since the Motion is instantaneous through the solid Particles of Air, and they make up $\frac{1}{8}$ or $\frac{1}{9}$ Part of the whole Space 979 Feet pass'd through in one Second by a Pulse, therefore to allow for this we must add $\frac{979}{9}$, or 109 Feet to the former Sum; that is $979 + 109 = 1088$ Feet, for the Velocity of Sound *per* Second.

6. But since the Atmosphere consists not of pure Air, but has an Admixture of Vapours of a different Elasticity and Tone; these Vapours will not participate of the Motion of pure Air, by which Sound is propagated; in like manner as an elastic String, if struck, will not move another very near it, unless it be under the same Degree of Tension, and of the same Tone. Therefore the Quantity of Air producing Sound must be diminish'd in proportion to the Quantity of Vapour,

stinctness

ſtinctneſs and agreeable Conſonance.

VI. *The Particles of Air, and conſequently the Pulſes, ſtriking againſt an Obſtacle, will be reflected back under an Angle equal to that of Incidence*; in the ſame manner as will be ſhewn in regard to the Rays of Light. Hence a *Repetition of the*

in a given Space; in which Sir *Iſaac* ſuppoſes the Air is to the Vapour as 10 to 1. Whence the Air and Vapour together in a given Space is to the pure Air as 11 to 10.

7. But the Velocity of the Pulſes will increaſe in the Subduplicate Ratio of the diminiſh'd Quantity of Matter, that is, in the Subduplicate Ratio of 11 to 10, or in the entire Ratio of 21 to 20, (as he has ſhewn, *Princip*. Prop. 48. Lib. II.) Therefore, if we ſay, As 20 : 21 :: 1088 : 1142; whence the real Velocity of Sound (thus inveſtigated from the Nature of elaſtic Air by our great Author) is at length found to be at the Rate of 1142 Feet *per* Second.

8. The Truth and Accuracy of this noble Theory have been ſufficiently confirm'd by Experiments, particularly thoſe made by the late Rev. Dr. *Derham*, of which I ſhall give ſome Account by and by; but will firſt lay before the Reader a View of the different Eſtimates made of the Velocity of Sound by ſeveral eminent Philoſophers, as in the Table following.

	Feet per *Second*.
The Honourable Mr. *Roberts*,	1300
The Honourable Mr. *Boyle*,	1200
Mr. *Walker*,	1338
Merſennus,	1474
The Academy at *Florence*,	1148
Royal Academy at *Paris*,	1172
Sir *Iſaac Newton*, *Flamſtead*, *Halley*, and *Derham*,	1142

9. As no Man ever had a better Opportunity, ſo none could improve it with greater Diligence, Aſſiduity, and Accuracy, in determining and ſettling the various Phænomena of Sounds, than the ſo often celebrated Philoſopher laſt mention'd. He proved by Experiments made with the Strokes of a Hammer, and the Exploſion of a Gun at the ſame time, at the Diſtance of a Mile, that the Velocity of Sounds produced from different Bodies was the ſame, or came to his Ear in the ſame Time.

Sound, heard by the direct Pulses, will be made by those which are reflected; which is what we call an ECHO.

The LOCUS, *or audible Place of Sound*, will be there where the Particles of Air first begin to diffuse themselves in Form of Waves, Thus a

10. That the Motion of Sound was equable and uniform, or that it pass'd through Spaces proportional to the Times, he found by various Experiments made by the Explosion of Guns at different Distances, as appears by the following Table which he has given us: Where the first Column shews the Places at which the Guns were fired; the second the Number of Vibrations of an Half-Second Pendulum; the third the Distance of the Places in Miles and decimal Parts, as measured by Trigonometry; the fourth the Distances measured by the Velocity of Sound, admitting it to be at the Rate of one Mile every $9\frac{1}{4}$ Half-Seconds.

At *Hornchurch* Church,	9	0,9875	
North Okendon Church,	$18\frac{1}{2}$	2,004	2,009
Upminster Mill,	$22\frac{1}{2}$ / 23	2,4	2,4 / 2,48
Little Warley Church,	$27\frac{1}{2}$	3,0	2,97
Rainham Church,	$33\frac{1}{4}$	3,58	3,59
Aveel Mill,	33	3,58	3,57
Dagenham Mill,	35	3,85	3,78
South Weal Church,	45	4,59	4,86
East Thorndon Church,	$46\frac{1}{2}$	5,09	5,03
Barking Church,	$70\frac{1}{2}$	7,7	7,62
Guns at *Blackheath*,	116	12,5	12,55

11. The great Exactness of measuring Distances by Sounds appears from the above Table, as well as the Equability of the Motion; but to render this Matter still more certain and indisputable, the Doctor took a Journey to *Foulness* Sands on the Coast of *Essex*, which form a smooth large Plain for Miles. On this Plain he measured 6 Miles in a right Line; and causing a Gun to be fired at the End of each Mile, he found that his former Observations were very just and true, and that Sound pass'd the first Mile in $9\frac{1}{4}$ Half-Seconds, two Miles in $18\frac{1}{2}$, three Miles in $27\frac{3}{4}$, and so on to the End of the six.

12. The *Academia del Cimento* made Experiments of this

Person

Person speaking in one End of a Tube, or Trumpet, will be heard as speaking from the other.

Sort, from whence they concluded, that the Velocity of Sounds was so far equable, as not to be accelerated or retarded by conspiring or adverse Winds; but in this they led themselves and many others into a great Mistake, which was owing to their firing Guns at too near a Distance; for in great Distances the Difference is sensible, as will appear by the following Table of many Experiments which the Doctor made on the Guns fired at *Blackheath*, at the Distance of twelve Miles from his House at *Upminster*.

	D.	H.	Vibrat.	Wind.
1704.	*Feb.* 13.	6 to 12 N.	120 / 122	N. E. by E. 1
	21.	$11\frac{1}{2}$ M.	119	E. 2
1705.	*Mar.* 30.	10 M.	113	S. W. 7
	Apr. 2.	$8\frac{1}{2}$ pM.	$114\frac{1}{2}$	S. by W. 1
	3.	10 M.	$116\frac{1}{2}$	S. 4
	5.	1 pM.	111	S. W. by W. 7
	13.	$8\frac{1}{2}$ M.	120	N. by E. 2
	24.	5 pM.	116	S. W. by W. 0
	Sept. 11.	$6\frac{1}{2}$ pM.	115	W. 2
		7	$115\frac{1}{2}$	W. by N. 2
	29.	$10\frac{1}{2}$ M.	112	S. S. W. 6
	Oct. 6.	10 M.	117	E. S. E. 1, 2
	Nov. 30.	Noon	115	S. S. W. 4
	Feb. 15.	11 M.	116	S. by W. 1
1706.	*Nov.* 29.	$11\frac{1}{2}$ M.	116	S. W. 0
		Noon	118	S. W. by S. 1
	Feb. 7.	Noon	113	S. W. by W. 4

13. In the first Column of this Table M denotes the Morning, pM the Afternoon, and N Night: Also the Figures 1, 2, 3, 4, 5, 6, 7, affix'd to the Points of the Wind in the third Column, denote the several Degrees of Strength with which the Wind blew at the Time when the Experiments were made. From this Table it is easy to observe, that in this large Distance (of near 13 Miles) the Velocity of Sound is sensibly affected with the Current of the Air or Wind; for since *Blackheath* lay near S. W. by W. from *Upminster*, we see that on *April* 5. 1705, when there was a strong Wind conspiring with the Sound, it came in 111 Half-Seconds; whereas in *Feb.* 13. 1704, when the Wind was directly contrary, though but a gentle one, the Sound took up no less than 120

And

And as in the Case of Light, we see the Image of an Object always in the Direction of the re-

and 122 Half-Seconds in passing the same Distance. The same is confirmed also by the Experiment on *April* 13, 1705.

14. And it is farther observable, that the Acceleration of Sound depends on the Strength of the Wind; for on *April* 24, 1705, a S. W. by W. Wind in the lowest Degree permitted the Sound to arrive in 116 Half-Seconds; the same Wind on *Feb.* 4, 1706, blowing with 4 Degrees of Strength, brought the Sound in 113 Half-Seconds; and on *April* 5, 1705, the same Wind with 7 Degrees of Strength brought the Sound in 111 Half-Seconds. The Winds which blow transversely (as on *April* 3, *February* 15, 1705.) seem not to affect the Velocity of Sound, it passing then in 116 Half-Seconds, which is the mean Velocity, as appears by the former Table in *Article* 10.

15. The greatest Difference we here observe in the Velocity of Sound, with or against the Wind, is 10 or 11 Half-Seconds, or $5\frac{1}{2}$ Seconds; whence $1142 \times 5,5 = 6281$ Feet, which is somewhat more than a Mile $= 5280$ Feet; and therefore for every 10 Miles we may allow Half a Mile, or 2640 Feet, when the Wind blows strongly against the Sound, and deduct the same when it sets with it; and so in Proportion for any other Distance.

16. The Velocity of Sound being determined, the Intervals of the Pulses are known by finding how many Vibrations the sounding Body performs in one Second. Thus D. *Sauveur* found by Experiments, that an open Pipe, whose Length was about 5 *Paris* Feet, had the same Tone with a String that vibrates forwards and backwards 100 times in a Second; consequently, of the Pulses made by sounding such a Pipe, there are about 100 in the Space of 1142 Feet, or 1070 of *Paris*; and therefore a single Pulse occupies the Space of $11\frac{42}{100}$ Feet *English*, or $10\frac{7}{10}$ Feet of *Paris*; so that the Length of the Pulse was about twice the Length of the Pipe. Whence it is probable, that the Lengths of the Pulses excited by the sounding of open Pipes are in all Cases equal to twice the Length of the Pipes.

17. This was farther confirm'd by the same Gentleman by another Experiment he made afterwards, in which he found that an open Pipe, of about two *Paris* Feet in Length, was in Unison with a String which vibrated forwards and backwards 243 times in a Second; wherefore $\frac{1070}{243} = 4\frac{2}{5}$ nearly;

flected

flected Ray; so in *Echoes*, we hear a Person speak at the Place from whence the reflected Wave comes to the Ear (CVI.)

that is, the Length of a Pulse was about $4\frac{2}{3}$ Feet of *Paris*, or nearly twice the Length of the Pipe.

(CVI) In order to account for the Nature of ECHOES, we must consider, that Sound is perceived as coming from that Place, from which, as a Center, the Pulses are propagated. This is well known by Experience: But to illustrate this Matter, let A be the Center from whence any Sound is directly propagated, and strikes against any plain Obstacle CB, sufficiently large; draw AF perpendicular to BC, and produce it to H, so that it may be $AF = FH$; the Sound reflected will be perceived as coming from the Point H. Plate XXXIII. Fig. 9.

2. For let AB be the incident Ray, impinging against the Obstacle BC in the Point E; from E draw the Ray KD, in such a manner that the Angle CED may be equal to the Angle EFA, or that the Angle of Incidence may be equal to the Angle of Reflection; then will ED be the reflected Ray of Sound, and, if produced, will pass through the Point H; for the Angle $FEH = CED = FEA$. Therefore in the Triangles AFE and EFH, since the Angles of one are respectively equal to the Angles of the other, and the Side FE common to both, the Sides of one Triangle will be respectively equal to the Sides of the other, and therefore $HF = AF$; wherefore the reflex Sound will be heard by a Person at D, as coming from the Point H.

3. As the Place of the Auditor or Point D approaches towards A, the Case will constantly be the same with respect to the Center of Sound H; the Triangles will still be equal, and all their Angles and Sides respectively; therefore when D coincides with A, the reflex Sound, or Echo, will be heard from the Point H; which was to be demonstrated.

4. The same Sound therefore is heard twice by an Auditor at D; first by the direct Ray AD, and secondly by the reflex Ray AED; provided the Difference between AD and AED be sufficiently great, that the direct and reflex Sound do not in the same sensible Moment of Time affect the Ear: For if the reflex Sound arrives at the Ear before the Impression of the direct Sound ceases, the Sound will not be double, only render'd more intense.

5. We know by Experience, if more than 9 or 10 Sylla-

BECAUSE

BECAUSE the Sound is ſtronger in proportion as the Air is denſer, it muſt follow, that the Voice paſſing through a Tube or Trumpet muſt be greatly augmented by the conſtant Reflection and Agitation of the Air through the Length of the Tube, by which it is condenſed, and its Acti-

bles are pronounced in a Second, the Sounds will not be diſtinct and articulate; therefore, that the reflex Sound may not be confounded with the direct Sound, there ought to be at leaſt the 9th Part of a Second between the Times of their Appulſe to the Ear. But in the 9th Part of a Second Sound runs through the Space of $\frac{1142}{9} = 127$ Feet; the Difference therefore between A D and A E D muſt not be leſs than 127 Feet, for the Echo to be diſtinctly heard in D.

6. Hence alſo it follows, that a Perſon ſpeaking or uttering a Sentence in A aloud, in order to obſerve the Echo by Reflection from the Obſtacle B C, ought to ſtand at leaſt 73 or 74 Feet from it, that is, $AF = 74$. And ſince, at the common Rate of Speaking, we pronounce not above $3\frac{1}{2}$ Syllables *per* Second (or read more than 20 Lines of *Engliſh* Poetry *per* Minute) therefore that the Echo may return juſt as ſoon as the 3 Syllables are expreſs'd, we muſt have twice A F equal to about 1000 Feet; or the Speaker muſt ſtand about 500 Feet from the Obſtacle B C; and ſo in Proportion for any other Number of Syllables.

7. In all the Experiments which Dr. *Derham* made with the Guns at *Blackheath*, there was always a Reduplication of the Sound, particularly the firſt in the foregoing Table, on *February* 13, 1704; where the direct Sound came firſt in 120 Half-Seconds, and the reflex Sound or Echo in 122 Half-Seconds. The Difference in Time, being a whole Second, ſhews the Echo paſs'd over 1142 Feet more than the direct Sound; and that therefore the *Phonocamptic Object*, or Obſtacle which reflected the Sound, was very probably near the Guns; ſince after the Pulſes had paſs'd a great way, they would have been too weak, when reflected, to have made an Echo as ſtrong or ſtronger than the direct Sound, as the Doctor always found it was.

8. By ſome Experiments which he made on Guns fired on the River *Thames*, between *Deptford* and *Cuckold's-Point*, he obſerv'd the Sound was not only doubled, but tripled, quaon

on on the external Air greatly increased at its Exit from the Tube; which from hence is call'd the *Stentorophonic Tube*, or *Speaking-Trumpet*.

For the same Reason, those Funnel-like Instruments, which gather the larger and more languid Waves of Air, do greatly condense them,

drupled, and sometimes repeated many more times, and each succeeding Echo was louder and louder; and often when he heard those Fragors of great Guns, he observed a Murmur aloft in the Air, especially if the Heavens were quiet and serene: And those Pulses of Air he has observed to strike against a thin Cloud, and produce in it a Murmur for the Space of 15″. From hence he judged, that those *Murmurs* in the Air proceed from the vaporous Particles suspended in the Atmosphere which resist the Undulations of Sound, and reverberate them to the Ear of the Observer, in the Manner of indefinite Echoes.

9. Among the many pleasant and ludicrous Phænomena of *Echoes*, those which are *Polyphonous*, or repeat divers Syllables or Sounds distinctly, and are therefore call'd *Tautological* or *Prattling Echoes*, afford the most curious Amusement. Of these there are several remarkable in different Parts of the World, and particularly here in *England*; concerning which I refer the Reader to *Harris*'s or *Chambers*'s DICTIONARY under the Word *Echo*, or to my PHILOSOPHICAL GRAMMAR.

10. Nor is this merry Phænomenon of Sound without its Use; for by means of an Echo you may measure inaccessible Distances, the Width of large Rivers, &c. Thus Dr. *Derham* standing upon the Bank of the *Thames*, opposite to *Woolwich*, observed that the Echo of a single Sound was reflected back from the Houses in 6 Half-Seconds, or 3 Seconds; consequently, $1142 \times 3 = 3426$ Feet; the Half of which, *viz.* 1713 Feet, is the Breadth of the River there; which is more tha n a Quarter of a Mile, or) 1320 Feet.

11. After the same Manner we find the Measure of any Depth, as that of a Well for Instance. To do this, let $a =$ Space an heavy Body falls freely in one Second of Time, $b =$ Space through which Sound moves in the same Time, and $c =$ Time given in Seconds from the first Descent of the Stone to the hearing of the Sound, and $x =$ Depth of the Well required.

and

and heighten their Power and Action on the Drum of the Ear; by which means Voices and Sounds are render'd *strong*, *loud*, and *audible*, which were not so before to a deafen'd Ear; and hence these Instruments come to be call'd *Otacoustics*.

12. Then to find how long the Stone is in descending to the Bottom of the Well, say, As $a : x :: 1''^2 : t''^2 = \frac{x}{a} =$ Square of the Time t in which the Descent is made, because the Spaces described by falling Bodies are as the Squares of the Times, (*Annot.* XXVI.) wherefore $t = \sqrt{\frac{x}{a}}$.

13. Again, to find the Time t in which the Sound ascends, say, As $b : x :: 1'' : \mathrm{t}'' = \frac{x}{b} =$ the Time sought in Seconds; but $t + \mathrm{t} = c = \sqrt{\frac{x}{a}} + \frac{x}{b}$; therefore $x + b\sqrt{\frac{x}{a}} = bc$. But $b\sqrt{\frac{x}{a}} = \frac{b}{\sqrt{a}}\sqrt{x}$; and since x is the Square of $\sqrt{x}$, the foregoing is a Quadratic Equation; and, by compleating the Square, we have $x + \frac{b}{\sqrt{a}}\sqrt{x} + \frac{bb}{4a} = \frac{bb}{4a} + bc = \frac{bb + 4abc}{4a} = \frac{ss}{4a}$ (by putting $ss = bb + 4abc$). Whence extracting the Roots on each Side we have $\sqrt{x} + \frac{b}{2\sqrt{a}} = \pm\frac{s}{2\sqrt{a}}$, that is, $\sqrt{x} = \frac{-b \pm s}{2\sqrt{a}}$. But x cannot be a negative Quantity; and therefore it cannot be $\sqrt{x} = \frac{-b-s}{2\sqrt{a}}$, but must be $\sqrt{x} = \frac{-b+s}{2\sqrt{a}}$. Therefore $x = \frac{\overline{s-b}^2}{4a} =$ the Depth of the Well required.

14. Now $a = 16{,}122$ Feet, and $b = 1142$; whence $4a = 64{,}488$, and $bb = 1304164$; also $4ab = 73646$; and

I SHALL only obſerve, in regard of thoſe Inſtruments which magnify Sounds, and aſſiſt the Hearing, that *the longer they are, the greater is their Effect*; and that of all the Forms or Shapes, none is ſo good as *that derived from the Revolution of the* LOGARITHMIC CURVE *about its Axis* (CVII.)

if we ſuppoſe $c = 10''$, then $4abc = 736460$, and $bb + 4abc = ss = 2040624$. Whence $s = 1428,5$; and $s - b = 286,5$; and $\overline{s-b}^2 = 82082,25$. Conſequently $\frac{\overline{s-b}^2}{4a} = 1273 = x$; or the Depth of the Well is 1273 Feet.

15. Since $x = \frac{\overline{s-b}^2}{4a}$, we ſhall have $\sqrt{x} = \frac{s-b}{2\sqrt{a}}$; and therefore $\frac{\sqrt{x}}{\sqrt{a}} = \frac{s-b}{2\sqrt{a}}$ divided by $\sqrt{a}$, that is, $\frac{\sqrt{x}}{\sqrt{a}} = \frac{s-b}{2a} = \frac{286,5}{32,24} = 8,89$ Seconds, the Time of the Stone's Deſcent to the Bottom of the Well. (See *Art.* 12.)

16. The Time of the Sound's Aſcent is $\frac{x}{b} = \frac{\overline{s-b}^2}{4ab} = \frac{82082,25}{73646} = 1,11$ of a Second. But $8,''89 + 1,''11 = 10''$, the whole Time, as it ought to be.

(CVII.) 1. The *Stentorophonic Tube*, or *Speaking-Trumpet*, is uſed for magnifying of Sound, particularly that of Speech, and thus cauſing it to be heard at a great Diſtance, how it does this will be eaſy to underſtand from the Structure thereof. Let ABC be the Tube, BD the Axis, and B the Mouth-Piece for conveying the Voice to the Tube. Plate XXXIV. Fig. 5.

2. Then 'tis evident when a Perſon ſpeaks at B in the Trumpet the whole Force of his Voice is ſpent upon the Air contain'd in the Tube, which will be agitated thro' the whole Length of the Tube; and by various Reflections from the Side of the Tube to the Axis, the Air along the middle Part of the Tube will be greatly condenſed, and its *Momentum* proportionably increaſed, ſo that when it comes to agitate the

FROM

FROM the fourth Property of the *aerial Pulses* we have the Origin of the various Degrees of what we call the NOTE, TONE, or TUNE of Sounds, in regard of which they are distinguish'd into *low* and *high*, or *grave* and *acute*, by Musicians call'd *Flats* and *Sharps*. Now the Tone of a Sound depends on the Time or Duration of the Stroke made on the Drum of the Ear, by a Wave or Pulse of Air; for as that is longer or shorter, the Tone will be move *grave* or *acute*: And since all the Pulses move equally swift, the Duration of a Stroke will be proportional to the Interval between two successive Pulses; and consequently, *a Sound is more or less Grave or Acute in proportion to the Length of that Interval.*

Air at the Orifice of the Tube A C, its Force will be as much greater than what it would have been without the Tube; as the Surface of a Sphere, whose Radius is equal to the Length of the Tube, is greater than the Surface of the Segment of such a Sphere whose Base is the Orifice of the Tube.

3. For a Person speaking at B, without the Tube, will have the Force of his Voice spent in exciting concentric Superficies of Air all around the Point B; and when those Superficies or Pulses of Air are diffused as far as D every way, 'tis plain the Force of the Voice will there be diffused thro' the whole Superficies of a Sphere whose Radius is B D, but in the Trumpet it will be so confined, that at its Exit it will be diffused thro' so much of that spherical Surface of Air as corresponds to the Orifice of the Tube. But since the Force is given, its Intensity will be always inversely as the Number of Particles it has to move; and therefore in the Tube it will be to that without, as the Superficies of such a Sphere to the Area of the large End of the Tube nearly.

4. To make this Matter yet plainer by Calculation; Let B D = 5 Feet, then will the Diameter of the Sphere D E = 10 Feet, the Square of which is 100, which multiplied by 0,7854, gives 78,54 square Feet for the Area of a great Circle

HENCE

HENCE it follows, that all the Sounds from the *loudest to the lowest*, which are excited by the Vibrations of the same Body, are of *one Tone*. It likewise follows, that all those Bodies whose Parts perform their Vibrations in the same or equal Times, have the same Tone: Also, those Bodies which vibrate *slowest* have the *gravest* or *deepest Tone*; as those which vibrate *quickest* have the *sharpest* or *shrillest Tone*.

THE Times of the Vibrations of *Musical Strings*, and consequently the Tones, vary in respect of the *Length*, the *Magnitude*, and the *Tension* of those Strings. For if two Strings A B, CD, are of the same Magnitude, and stretch'd by equal Weights E, F, have their Lengths as Plate XXXV. Fig. 1.

AHEFC. And therefore 4 times that Area, *viz.* $4 \times 78,54 = 314,16 =$ square Feet in the Superficies of the Aerial Sphere. If now the Diameter A C of the End of a Trumpet be one Foot, its Area will be 0,7854; but, $7854 : 314,16 :: 1 : 400$, therefore the Air at the Distance of B D will be agitated by means of the Trumpet, with a Force 400 times greater than by the bare Voice alone.

5. Again, 'tis farther evident how Instruments of this Form assist the Hearing greatly; for the weak and languid Pulses of Air being received by the large End of the Tube, and greatly multiplied and condensed by the tremulous Motion of the Parts of the Tube and Air agitated by them, are conveyed to the Ear by the small End, and strike it with an *Impetus* as much greater than they would have done without it, as the Area of the small End at B is less than the Area of the large End AC.

6. From what has been said, 'tis evident the Effect of the Tube in magnifying Sound, either for Speaking or Hearing, depends principally upon the Length of the Tube. But yet some Advantage may be derived from the particular Form or Shape thereof. Some very eminent Philosophers have proposed the Figure which is made by the Revolution of a *Parabola* about its Axis as the best of any; where the

2 to 1, the Times of their Vibrations will be in the same Ratio. Hence the Number of Vibrations of the two Strings AB, CD, perform'd in the same Time, *will be inversely as their Lengths*; or CD will make two Vibrations, while AB performs one. The Vibrations of two such Strings will therefore co-incide at every second of the lesser.

Fig. 2. AGAIN: If two Strings of the same kind AB, CD, have their Diameters as 2 to 1, and are of equal Length, and tended by equal Weights E, F; *the Times of the Vibrations will be as their Diameters*, *viz.* as 2 to 1; and so the Vibrations in a given Time, and the Co-incidences, as before.

LASTLY: If the Diameters and Lengths of

Mouth-Piece is placed in the Focus of the Parabola, and consequently the sonorous Rays will be reflected parallel to the Axis of the Tube. See the Figure of such a Tube in *Muschenbroek*'s *Essai de Physique*.

7. But this parallel Reflection seems no way essential to the magnifying of Sound; on the contrary, it appears rather to hinder such an Effect, by preventing the infinite Number of Reflections and Reciprocations of Sound, in which, according to Sir *Isaac Newton*, its Augmentation does principally consist. For all reciprocal Motion, in every Return, is augmented by its generating Cause, which is here the tremulous Motion of the Parts of the Tube. Therefore in every Repercussion from the Sides of the Tube, the Agitations and Pulses of the confined Air must necessarily be increased; and consequently this Augmentation of the *Impetus* of the Pulses must be proportional to the Number of such Repercussions, and therefore to the Length of the Tube, and to such a Figure as is most productive of them. Whence it appears, that *the Parabolic Trumpet is of all others the most unfit for this Purpose*, instead of being the best.

8. But there is one Thing more which contributes to the Augmentation of those Agitations of Air in the Tube, and that is the Proportion which the several Portions of Air bear to

the

the Strings be equal, *the Times of the Vibrations will be inversely as the Square Roots of the Weights which stretch them.* If the Weights E and F be as 1 to 4 (the Square Roots of which are 1 and 2) then the Times of Vibration in A B and C D will be as 2 to 1. Hence in *constructing string'd Instruments*, as SPINETS, HARPSICHORDS, *&c.* a skilful Artist will compound these Proportions of the *Length*, *Diameter*, and *Tension* of the Strings to very great Advantage. Fig. 3.

IN *Wind-Instruments*, as the FLUTE, ORGAN, *&c.* where the Sound is made by the *Vibration of a Column of elastic Air* contain'd in the Tube, the Time of Vibration or Tone of the Instrument will also vary with the *Length and Diameter of*

each other when divided by transverse Sections, at very small but equal Distances, from one End of the Tube to the other. Thus let those several Divisions be made at the Points a, b, c, d, e, &c. in which let the Right Lines ak, bl, cm, dn, &c. be taken in Geometrical Proportion. Then will the Portions of Air contained between B and a, a and b, b and c, c and d, &c. be very nearly in the same Proportion, as being in the same Ratio with their Bases when the Points of Division are indefinitely near together. Plate XXXIV. Fig. 6.

9. But it has been shewn already, that when any Quantity of Motion is communicated to a Series of elastic Bodies, it will receive the greatest Augmentation when those Bodies are in Geometrical Proportion. Therefore since the Force of the Voice is impress'd upon and gradually propagated through a Series of elastic Portions of Air in a Geometrical Ratio to each other, it shall receive the greatest Augmentation possible.

10. Now since by Construction it is $Ba = ab = bc = cd$, &c. and also $ak : bl :: bl : cm :: cm : dn$, and so on; therefore the Points k, l, m, n, o, p, q, r, s, A, will in this Case form that *Curve Line* which is call'd the *Logarithmetic Curve*: Consequently, a Trumpet form'd by the Revolution of this Curve about its Axis will augment the Sound in a greater De-

the said Column of Air, and Force of the Voice, which compresses it; as will be easy to observe from Experiments.

IF one Body be made to sound with another, their Vibrations will co-incide after a certain Interval; and the shorter the Interval of the Co-incidence, the more agreeable is the Effect or Consonance to the Ear; consequently, those which are most frequent produce the most perfect Consonance or *Concord*, as it is commonly call'd. When the Times of Vibration, therefore, are equal, the CONCORD is most perfect and more agreeable than any other, and this is call'd UNISON.

IF the Times of Vibration are as 1 to 2, the Co-incidence will be at every second Vibration of the quickest, and so this is the next perfect Concord, and is what we commonly call a DIAPASON, or OCTAVE.

IF the Times of the Vibration be as 2 to 3,

gree than any other figured Tube whatsoever.

11. But to shew the Reason of the *Nature and Name* of this Curve, suppose the following Series of Quantities in Geometrical Progression, viz $a^0 : a^1 : a^2 : a^3 : a^4 : a^5$, &c. then it is plain the Ratio of a^1 to a^0 is 1, the Ratio of a^2 to a^0 is 2, the Ratio of a^3 to a^0 is 3, and so on; whence it appears, that the *Indices* of the several Terms express the Ratios of those Terms severally to the first, and are therefore their *Logarithms*. Now if in the above-mention'd Figure we put the Ordinates $ak = a^0 = 1$, $bl = a^1 = a$, $cm = a^2 = aa$, &c. then will the intercepted Parts of the *Abscissæ* be $Ba = 1$, $Bb = 2$, $Bc = 3$, &c. and therefore the *Logarithms* or *Exponents* of the Ratios of those several Ordinates to the first or Unity. Hence the Curve which connects those Ordinates is call'd the *Logarithmetical* or *Logistical Curve*.

the Co-incidence will be at every third Vibration of the quickeſt; which therefore is in the next Degree of Perfection, and this is call'd a DIAPENTE, or FIFTH. If the Times of Vibration are as 3 to 4, the Co-incidence will be at every 4th of the leſſer; and this is call'd the DIATESSARON, or FOURTH. But this, and the next which follow in Order, are not ſo agreeable and pleaſant to the judicious Ear, and are therefore call'd *Imperfect Concords*. Nor are there above *ſeven Notes* in all the infinite Variety of Tones, which can merit a Place in Muſical Compoſitions, and they are exhibited in *Fig*. IV. which repreſents the Strings in an Octave of a Harpſichord, with the *Semitones* or *Half-Notes*, call'd *Flats* and *Sharps*, by which the natural Notes are made half a Note lower or higher, as the Air of the Song or Muſick requires. And this is call'd the DIATONIC SCALE of Muſic. Plate XXXV.

IN this Scale, the ſeven natural Notes are mark'd on the Keys by the ſeven Letters C, D, E, F, G, A, B. The firſt of which is call'd the *Fundamental* or KEY; the reſt in Order are the *Second Greater*, the *Third Greater*, the *Fourth Greater*, the *Fifth*, the *Sixth Greater*, the *Seventh Greater*, and then the *Eighth*, which begins the next Octave. Between theſe are interpoſed the five *Semitones*, *viz*. the *Second Leſſer*, the *Third Leſſer*, the *Fourth Leſſer*, the *Sixth Leſſer*, the *Seventh Leſſer*. Theſe ſeveral Tones and Semitones have the Lengths of the Strings adjuſted from the Diviſion of the MONOCHORD, or Line

divided into 100 or 1000 equal Parts, as is very easy to apprehend from the Figure.

THE Number of those Divisions are also shewn for each String, by the first Series of Numbers on the Strings; the next Series shew the Proportion of the Length of each String to that of the *Key*, or *Monochord*; and consequently the Number of Vibrations of the Fundamental and each String respectively, perform'd in the same Time.

OF these *twelve Intervals* or Ratios of Musical Sounds, the *Octaves* and *Fifths* are *perfect Concords*; the third Greater, third Lesser, the Greater and Lesser Sixth are *imperfect Concords*; the Greater Fourth, the two *Seconds*, and two *Sevenths* are *Discords*; the *Fourth* is in its own Nature a perfect *Concord*, but lying between the Third and Fifth, it cannot be used as such, but when join'd with the Sixth, to which it stands in the Relation of a Third. All MELODY and HARMONY are compos'd of these twelve Notes; for the Octaves above or below are but the Replications of the same Sounds in a higher or lower Tone. MELODY is the agreeable Succession of several Musical Sounds in any single Piece of Music; as HARMONY is the Effect of several of those Pieces or Parts of Music play'd together (CVIII).

Plate XXXIV. Fig. 7.

(CVIII) 1. In order to account for the Motion and Tone of an elastic String, or *Musical Chord* A B, it will be proper to consider it as tended or stretch'd by a Weight, as F, according to its Length, and drawn out of its right-lined Position A B, into an oblique Position A D B, by another Weight, as E. The former may be call'd the *Tending Force*, and the latter the *Inflecting Force*.

HARMO-

HARMONICAL PROPORTION is that which is between those Numbers which assign the Lengths of Musical Intervals, or the Lengths of Strings sounding Musical Notes; and of *three Numbers* it is, *when the First is to the Third, as the Difference between the First and Second is to the Difference between the Second and Third*, as the Num-

2. Now since the Tending Force F acts upon the String in the Direction DB, it may be represented by the Line CD, which Line or Force may be resolved into two others, *viz.* CB and CD; of which the former draws the String horizontally from D to B, and the other acts in drawing the String directly upwards from D to C. Therefore the Part of the Force which acts in drawing the String perpendicularly upwards is to the whole Force as CD to DB; or, by supposing DC to be indefinitely small, as CD to CB; because in that Case DB = CB nearly. But the Force which acts in drawing the String upwards is equal to the Inflecting Force, because they balance each other. Therefore the Inflecting Force E is to the Tending Force F as CD to CB, or $\frac{F \times CD}{CB} = E$.

3. Therefore, putting CD = S, and 2 CB = L = the Length of the String, we shall have $\frac{F \times S}{L} : E$; hence it follows, that if F and L are given, that is, if the Tending Force and Length of the String remain the same, the Inflecting Force E will be always as the Line CD = S. This is confirm'd by Experiment: For if AB be a Brass Wire 3 Feet long, stretch'd over the Pulley at B by a Weight F = 3 Pounds; if then E be first $\frac{1}{2}$ an Ounce, it will draw the Wire through CD = $\frac{2}{8}$ of an Inch; if E be an Ounce, it will draw it through CD = $\frac{4}{8}$ of an Inch; and so on.

4. The String being drawn into the Position ADB has an Endeavour to return, which is call'd the *Restitutive Force*, and which re-acts against the Inflecting Force; it must therefore be equal to it, and consequently proportional to the Line CD. Wherefore the Point D is carried towards C with a Force every where proportional to the Distance or Space pass'd over. But we have shewn, that the Spaces pass'd by Bodies in Motion are as the Times and Velocities conjointly, that is S : TV;

bers 3, 4, 6. Thus if the Lengths of Strings be as these Numbers, they will sound an *Octave*, 3 to 6; a *Fifth*, 2 to 3; and a *Fourth*, 3 to 4.

AGAIN: *Harmonical Proportion* between four Numbers is, when *the First is to the Fourth as the Difference between the First and Second is to the Difference between the Third and Fourth, as in the*

(See *Annotation* XXII.) also that the Force of moving Bodies is as the Quantity of Matter and Velocity conjointly, *viz.* $M = QV$; therefore $\frac{S}{T} = V = \frac{M}{Q}$, or $TM = SQ$. But in the present Case Q is a given Quantity, therefore TM is as S; and because it has also been shewn that M is as S in the present Case of the String, therefore T, or the Time in which the Vibrations are made, whether through greater or smaller Spaces, is ever the same, or a given Quantity.

5. The Restituent Force of the String, as it acts through very small Spaces, may be look'd upon as uniform; and then the Motion generated in the String will be as the said Force and Time of its acting, that is, $M : ET$. Now in all Cases it is $M : QV$; but here it is $Q : D^2L$, (supposing D = Diameter and L = Length of the String) therefore $M : ET : D^2LV$, and consequently $T : \frac{D^2LV}{E}$; but before we had $E : \frac{FS}{L}$, which substituted in the above Ratio gives $T : \frac{D^2L^2V}{FS}$. But (since $S : TV$) we have $\frac{V}{S} : \frac{1}{T}$, therefore $T : \frac{D^2L^2}{FT}$; that is, $FT^2 : D^2L^2$, therefore $F^{\frac{1}{2}}T : DL$; consequently, $T : \frac{DL}{F^{\frac{1}{2}}}$. That is, *the Time of a Vibration is as the Diameter and Length of the String directly, and as the Square Root of the Tending Force inversely.*

6. Hence if D and F be given, T is as L; that is, if the Diameter of the String and its Tending Force continue the same, the *Time of a Vibration will vary with the Length of the String, or be always proportional to it.* Thus $\frac{1}{2}$ of the Monochord vibrates in $\frac{1}{2}$ of the Time that it does, which is call'd an *Octave*; $\frac{2}{3}$ of the Monochord vibrates in $\frac{2}{3}$ of the Time,

Numbers

Numbers 5, 6, 8, 10: For Strings of ſuch Lengths will ſound an *Octave*, 5 to 10; a Sixth Greater, 6 to 10; a Third Greater, 8 to 10; a Third Leſſer, 5 to 6; a Sixth Leſſer, 5 to 8; a Fourth, 6 to 8.

IT may be here obſerved, *that a Series of Numbers in Harmonical Proportion are reciprocally*

and is call'd a *Fifth*; $\frac{3}{4}$ vibrates in $\frac{3}{4}$ of the Time, and is call'd a *Fourth*; and ſo on.

7. If F and L be given, T is as D; that is, if the Tending Force and Length of the String remain the ſame, *the Time of a Vibration will vary with, and be proportional to the Diameter of the String.*

8. If D and L be given, then T is inverſely as $F^{\frac{1}{2}}$; that is, if the Diameter and Length of the String be given, then *the Time of a Vibration will be as the Square Roots of the Tending Force.*

9. Now as the Tone of a String depends entirely upon the Time of a Vibration, it is eaſy to underſtand, that whatever the ſounding Body be, or how many ſoever there be together, if when they emit a Sound the Vibrations in each are of the ſame Duration, they will all be of the ſame Note, Tone, or Tune, which is call'd *Uniſon*.

10. In a Drinking-Glaſs, if a Perſon paſſes his wetted Finger briſkly round the Brim of the Glaſs, preſſing it at the ſame time, he will by degrees raiſe Tremors or Vibrations in the Parts of the Glaſs, which will produce a Tone or Sound, which will be conſtant ſo long as the Action of the Finger is continued, and more and more intended or heighten'd: So that if the Action be continued long enough, the Agitation will be ſo great as to diſengage the Particles, or break their Continuity, and thus reduce the Glaſs to pieces, if not too ſtrong.

11. The Sound excited in the Glaſs ſeems one entire Effect, whereas it is in reality an Aggregate or Aſſemblage of an indefinite Number of Sounds, each effected by each ſingle Vibration of the Glaſs; but as the Times of the Vibrations are ſo quick and ſhort, their Intervals will be imperceptible, and conſequently the Diſtinction of the particular Sounds, which will therefore be loſt, and the Whole will appear but one entire Sound. After the ſame manner a red-hot Coal whirl'd about makes the Appearance of a fiery Circle, becauſe the

as

as another Series in Arithmetical Progression,

$$\text{As}\begin{cases}\text{Harmonical} & 10:12:15:20:30:60:\\ \text{Arithmetical} & 6:\ \ 5:\ \ 4:\ \ 3:\ \ 2:\ \ 1:\end{cases}$$

for here 10 : 12 :: 5 : 6; and 12 : 15 :: 4 : 5; and so of all the rest. Whence those Series have an obvious Relation to, and Dependence on, each other; which in some Problems of speculative Philosophy will be very useful to know (CIX).

Coal succeeds to every particular Point of the Circle so quick, that a new Impression is made upon the *Retina* before the Effect of the last is obliterated, and so the Coal appears in every Part of the Circle.

12. The Tremors of the Glass are made extremely sensible by putting a little Water into the Glass; for the Agitations of the Glass will by degrees give Motion to the Water, which Motion will continually be increased till it be thrown up from the Surface in Form of a Mist all over the Glass, and to a considerable Height above it every way. It is remarkable that the Motion of the Water is in Form of a *Vortex*, circulating round by the Sides of the Glass, and raging with impetuous Waves like the Sea after a prodigious Tempest.

13. Or otherwise these Vibrations of the Glass are made sensible by adjusting a Screw very near the Rim of the Glass; then upon striking the Glass, it will immediately be heard to strike against the End of the Screw; which will shew not only the Vibration of the Glass, but also that in vibrating the Form is alter'd from circular to elliptical.

(CIX) 1. Let A, B, C, be three Numbers in Musical Proportion; then because we have A : C :: A — B : B — C, therefore AB — AC = AC — BC; whence if any two of the three be given, the other is immediately found by the following Canons, *viz.*

CANON I. If A and B be given, then $C = \frac{AB}{2A - B}$.

CANON II. If A and C be given, then $B = \frac{2AC}{A + C}$.

CANON III. If B and C be given, then $A = \frac{CB}{2C - B}$.

2. Thus, for Example, suppose you would find a Musical

If

IF the three Lines A D, B G, C H, be taken in Musical Proportion, or as the Numbers 6, 4, 3; and in the Line A D we take A E equal to B G, A F equal to C H, *then will the Line* A D *be divided in Harmonical Proportion*, in the Points A, F, E, D; *viz.* A D : A F : : D E : E F. And in this manner is the Axis of a convex and concave Mirrour divided by the *Object*, the *Image*, the *Vertex of the Mirrour*, and the *Centre*, as may be easily shewn by Experiment. Plate XXXV. Fig. 5.

mean Proportional between the *Monochord* 100 = A, and the *Octave* 50 = C; then by Canon II. we have $B = \frac{2AC}{A+C} = \frac{1000}{150} = 66,6$, which is the Length of that Chord which is usually call'd the *Fifth*.

3. Again, If there be four Numbers in Musical Proportion, as A, B, C, D; then, since it is A : D :: A—B : C—D, we have AC — AD = AD — DB. From which Equation we have the following Canons.

CANON I. $A = \frac{DB}{2D - C}$.

CANON II. $B = \overline{2D - C} \times \frac{A}{D}$.

CANON III. $C = \frac{2AD - DB}{A}$.

CANON IV. $D = \frac{AC}{2A - B}$.

4. Hence, when any three of those Numbers are given, the fourth may be found by the above Canons. Thus to the three Numbers 10, 8, 6, we find a fourth Harmonical Proportion, which is 5, the Octave; for thus the Theorem will stand, $\frac{A \times C}{2A - B} = \frac{10 \times 6}{20 - 8} = \frac{60}{12} = 5$.

5. But to carry this *Harmonical Theory* farther, and render it more general:

ALSO

ALSO the Limits of the Colours of Light, as ſeparated by the Priſm, *fall upon the ſeven Muſical Diviſions of the Monochord*; as will be farther taken notice of, and exemplified in the next Lecture.

I SHALL conclude this with taking notice of one ſingular Property of a Muſical Chord, *viz.* that it will be put into a vibratory Motion by the Pulſes of the Air proceeding from the Vibrations of another very near it, and in Concord

Let the Terms of an Harmonic Series be denoted by } A, B, C, D, E, F, *&c.*

And let the Difference between each two be denoted by } M, N, O, P, Q, *&c.*

6. Then will the Product of the two firſt Terms, *viz.* A × B, be to the Product of any other two Terms immediately ſucceeding each other as C × D, in the ſame Ratio with their reſpective Differences M and O. For by the Definition of Muſical Ratio we have $\begin{cases} A : C :: M : N \\ B : D :: N : O \end{cases}$

Therefore $A \times B : C \times D :: M \times N : N \times O :: M : O$.

Alſo $\begin{cases} A : C :: M : N \\ B : D :: N : O \\ C : E :: O : P \end{cases}$

Therefore $A \times B \times C : C \times D \times E$ $(:: A \times B : D \times E) ::$ $M \times N \times O : N \times O \times P :: M : P$. That is, $AB : DE ::$ $M : P$; and ſo on univerſally.

7. Again; the Difference between the two firſt Terms M is to the Difference between any other two, as O, in the Ratio of $B - 2M$ to D; or $M : P :: B - 3M : E$; or $M : Q ::$ $B - 4M : F$; and ſo on continually. For, by the Nature of the Progreſſion, it is $A : C :: M : N$; and it is alſo $A = B - M$, (becauſe $B - A = M$) therefore it is $B - M : C :: M : N$; or, to put it in Form, we have $M : N :: B - 1M : C$. Again; $B - M : M :: C : N$, and by Diviſion $B - 2M : M ::$ $C - N : N :: B : N$; but (by the Definition, *Art.* 1.) it is $B : N :: D : O$; therefore $M : O :: B - 2M : D$. Again; $B - 3M : M :: D - O : O :: C : O :: E : P$; therefore $M :$ $P :: B - 3M : E$. And univerſally, let n = Number of

with

with it: If the vibrating String be *Unison* with it, the other will tremble thro' its *whole Length*; if an *Octave*, it will vibrate by the Half-Lengths only; if the String which communicates the Motion be a *Double-Octave above*, or *one Fourth* of the Length of the other, the Motion will be still correspondent in that other String, for it will vibrate only by the *Fourths of its Length* from one End to the other. Thus if AB be a String four Feet long, and CD another of one Foot; if Fig. 6.

Terms in the Series between the first and the last, and let the last Term be Z, and let the Difference between it and the next preceding Term be S; then will it be $M : S :: B - nM : Z$.

8. Because (by *Art.* 6.) it is $M : S :: A \times B : Y \times Z$, supposing Y, Z, the two last Terms of the Series; therefore $A \times B : Y \times Z :: B - nM : Z$.

9. Because the first Term of the Series is $A = \frac{A \times B}{B}$, and the second Term $B = \frac{A \times B}{A}$, and $A = B - M$; therefore the second Term is $B = \frac{A \times B}{B - M}$. In the same manner it is shewn, that the third Term is $C = \frac{A \times B}{B - 2M}$, the fourth Term $\frac{A \times B}{B - 3M}$; and universally, since $A \times B : Y \times Z :: B - nM : Z$, or, dividing the Consequents by Z, $A \times B : Y :: B - nM : 1$; therefore $Y = \frac{A \times B}{B - nM}$; and since $n =$ Number of Terms between A and Z, it will express the Number or Place which the Term Y holds in the Series. Therefore *any Term* Y *is equal to the Product of the first and second Term* B *of the Series divided by the Difference between that second Term* B, *diminished by so many times its Difference from the first, as is equal to the Number of the Terms from the first to the given Term* Y.

10. *All the Terms in a Musical Progression are among them-*

the

the latter be ſtruck with a Quill, the Vibrations will be communicated to the former in ſuch a manner that it will vibrate only by a *Foot-Length* at the ſame time thro' the whole String; which will be evident by the ſmall Pieces of Paper *b*, *d*, *f*, *h*, hung upon the Middle of every Foot-Length, ſuddenly leaping off; while the other Pieces *a*, *c*, *e*, *g*, *i*, remain unmoved upon the String at the End of every Foot, where the Vibrations ſeverally begin and end, and conſequently where the Line has no Motion at all (CX).

ſelves as Quantities whoſe Reciprocals conſtitute a Series in Arithmetical Progreſſion. Thus the Terms of the firſt Series A, B, C, D, E, &c. are (by *Art.* 9.) as $\frac{A \times B}{B}$, $\frac{A \times B}{B - M}$, $\frac{A \times B}{B - 2M}$, $\frac{A \times B}{B - 3M}$, &c. to $\frac{A \times B}{B - nM}$; which Series divided by $A \times B$ gives the Series $\frac{1}{B}$, $\frac{1}{B - M}$, $\frac{1}{B - 2M}$, $\frac{1}{B - 3M}$, to $\frac{1}{B - nM}$. But the Reciprocals of this Muſical Series are B, $B - M$, $B - 2M$, $B - 3M$, to $B - nM$; which Terms are all in *Arithmetical Progreſſion.* If the *Harmonic Series* had been decreaſing, *viz.* $A - B = M$, $B - C = N$, &c. we ſhould have had $A = M - B$, *viz.* the Signs of M and B changed, but every thing elſe the ſame.

(CX) 1. What is here ſaid relating to Mirrours, and the Colours of Light, will be explain'd and demonſtrated in its proper Place. That one String A ſhould be put into Vibration by another B, by means of the Air, is not ſtrange, becauſe the Air will affect the String A with the ſame Impulſes it receives itſelf from the String B. If therefore the String A be under the ſame Circumſtances with the String B otherwiſe, (*i. e.* if it be of equal Magnitude, and equally tended) it muſt neceſſarily move in a ſimilar Manner, or vibrate in an equal Time.

2. If the String A be twice the Length of B, then (*cæteris paribus*) the Air by its Impulſe received from B cannot ſo affect

fect A as to cause it to vibrate through its whole Length; but it will so affect each Half of A as to produce a similar Effect, or equal Vibrations. Hence the String A will become divided in the middle Point, which will be at Rest.

3. And if the String A were three times as long as B, it would be for the same Reason divided into three Parts, whose Vibrations are synchronous to those of B, with two Points of Rest between; and so on for any other Length. Also, if the Lengths of A and B are as 3 to 2, then if they are Concords, and one be struck, the other will be put into Motion by degrees, and in such a Manner that will alter the Vibrations of the first String B, and each will vibrate by their aliquot Parts, and therefore in equal Times.

LECTURE VIII.

Of the Nature and Properties of LIGHT; *the* VELOCITY *thereof how discover'd and computed. Of the Nature of* HEAT, FIRE, FLAME *and* BURNING. *Of the* IGNES FATUI, NOCTILUCÆ, *natural and artificial* PHOSPHORI. *The* THEORY *of* HEAT *and* COLD. *Of* ASBESTOS. *Of the* Nature *and* Effect *of* BURNING-GLASSES, *whether* Mirrours *or* Lenses. *A Calculation of the* LIGHT *and* HEAT *of the* MOON. *Of the Cause of* TRANSPARENCY *and* OPACITY *in Bodies. Of the* REFLECTION *of* LIGHT; *Of its* INFLECTION; *Of the* REFRACTION *of* Light, *The* Fundamental Laws *thereof demonstrated. The different* REFRACTIVE POWER *of various Substances. The Ratio of the Sines of* INCIDENCE *and* REFRACTION *stated. Of the* TRUE *and* APPARENT PLACES *of Objects. Of the* ANALYSIS *of the* Solar Rays; *the* several KINDS *thereof, their different* REFRANGIBILITY *stated*; EXPERIMENTS *relating thereto by the* PRISM. *Of the various* COLOURS *of* Light *by the* Prism; *the* HARMONIC RATIO *of their Linear Extent in the* Sun's Image. *The* COLOURS *of* Natural Bodies *thence explain'd. Of the different* REFLEXIBILITY *of the* Solar Rays, *and Experiments relating thereto. The Manner*

and

and Cause thereof enquired into. Of RINGS *of* COLOUR'D LIGHT *between* GLASS PLANES, *and* BUBBLES *of* WATER. *The different* ORDERS *and* DEGREES *of the several* Colours *explain'd. The Fits of* EASY REFLECTION *and* TRANSMISSION *explain'd. The* Artificial COMPOSITION *of* COLOURS. *Of the* RAINBOW; *its* Cause *explain'd; Calculations relating thereto. The Phænomena of* HALO's *consider'd and accounted for.*

THAT *Light* is not a *mere Quality* of some Bodies, but is itself a real Body; or distinct Species of Matter, and endued with all the natural Properties thereof, will, I presume, be sufficiently manifest from the following Experiments relating thereto. We shall therefore, at present, take it for granted, *that Light consists of inconceivably small Particles of Matter of different Magnitudes, which are emitted or reflected from every Point in the Surface of a luminous Body in Right Lines, and in all Directions, with an unparallel'd Velocity, and whose Power or Intensity decreases as the Squares of the Distances increase.*

THAT the Particles of Light are refracted thro' the Humours of the Eye to the *Retina*, or fine Expansion of the Optic Nerve over all the interior hinder Part of the Eye; and there, by painting the Images of external Objects, become the immediate Means of Sight, will be fully shewn in the next Lecture.

WE shall now consider'd Light under the various Characters and Qualities of a natural Body, and point out those remarkable Affections and Properties so peculiar to itself, and the Causes of so many very curious and extraordinary Phænomena in Nature.

THAT the Particles of Light are *inconceivably small*, is evident from hence, that the greatest Quantity of Light, in the State of greatest Density, or *Flame*, is found to have scarce any sensible Gravity or Weight, which, we have shewn, is always proportional to the Quantity of Matter in all Bodies: Also, because those Particles pervade the Pores of all transparent Bodies, however hard or heavy, as Glass and Adamant. But we know it more especially from hence, that the Stroke we receive by a Particle of Light has no sensible Force or *Momentum*, which, on account of its prodigious Velocity, would be very great, and insufferable, were it of any assignable or considerable Magnitude.

YET small as they are, we find the Rays consist of different Sorts of Particles in Light emitted from all Bodies; and that this Difference of the Rays of Light arises from the different Magnitude of the Particles, seems most evident from the different Directions the several Sorts of Rays move in, after they have pass'd thro' a Body of Glass, Water, &c. of some special Figure, as that of a *Prism* especially.

THAT the Particles of Light are emitted from every Point in the Surface of a Body, is evident from

from hence; that any given Point in that Surface is visible to the Eye in any Situation, from whence a Right Line can be drawn from the Eye to that Point; which could not be, if the Light were not propagated from that Point in all Directions.

THAT they proceed from the Body in Right Lines, is clearly seen by Experiments on the Sun-Beams, Candle-Light, *&c.* in a darken'd Room; also from the Shadows which Bodies of every Figure cast, being such as would be determined by Right Lines drawn from the luminous Point touching the Extremities of those Bodies (CXI).

(CXI) 1. Before Sir *Isaac Newton*'s Time, scarce any thing of the Nature or Properties of Light was known. It had been esteem'd a mere Quality or Modification of Matter, and was propagated by Pression, and I know not what of such Kind of Stuff and senseless Jargon; than which nothing can be more tiresome to read, or irksome to repeat. Leaving therefore the idle Reveries of the *Cartesians*, we shall contemplate this glorious Phænomenon in the *Newtonian* Manner, which diffuses Lustre over the whole Face of Nature, and adds new Splendor even to Light itself.

2. That Light is a material Substance, and what we properly call *Body*, is not to be doubted; because we find it is something that has Motion, or is propagated in Time; something that acts upon Bodies, and produces great Alteration and Changes in their Natures and Forms. It is something that Bodies act upon, by reflecting, inflecting, and refracting it on their Surfaces, and in their Pores: And it would appear to have Weight, and all other sensible Qualities of common Matter, were it not that the Smallness of its Quantity renders them entirely imperceptible by us.

3. Nor are we to consider Light only as a Body, but as the most active Principle or most general Agent in Nature. I greatly question if it be not the true *Primum Mobile* in Nature, or the Spring of Motion and Action in all other Bodies. Were the Particles of Light to be annihilated, we should see no Marks or Footsteps of Fire or Heat remaining, and there-

THE Velocity of the Rays of Light surpasses that of all other Bodies we know of. By observing the Times of the Eclipses of *Jupiter*'s *Satellites* when the Earth is nearest, and again when it is farthest distant from that Planet, we shall

fore no Power of Motion in Bodies, but all Things would put on the Appearance of lifeless inert Matter, rigid and inflexible, as it would be absolutely cold and dark.

4. The Divine Wisdom and Providence appears perhaps in nothing so remarkably as in the extreme Subtilty of the Particles of Light; without this Qualification it could not have pervaded the Pores of Bodies, and so we could have had none of those which we call *diaphanous* or *transparent* Substances, and every thing but the Surface of a Body would have been concealed from the Sight of Mankind. Again; the Velocity of a Body is always as the Quantity of Matter inversely; and therefore the smaller the Body, the greater Velocity it is susceptible of from the same Force; whence it comes to pass, that Light is thus qualified to be transmitted through immense Distance in a small and insensible Part of Time; which Thing was quite necessary according to the present Frame and State of Nature.

5. But lastly, it was absolutely necessary that the Particles of Light should be so exceeding small, that when compounded with its Velocity it should produce no sensible Force, as it must otherwise have done, and which therefore could not have been born by the tender and delicate Texture of the several Parts of Vegetable and Animal Bodies. To give an Example: The Velocity of a Particle of Light is found to be at the Rate of 897600000 Feet *per* Second; suppose its Matter to be but one Millionth Part of a Grain, then its Force to strike an Object would be as $\frac{897600000}{1000000} = 897,6$ Feet *per* Second for one Grain; or it would strike with the same Force that one Grain Weight would do falling from half that Height, *viz.* through 448,8 Feet; which we should find to be very great, were the Experiment to be made on the sensible Coats of the Eye.

6. Since the Weight of Bodies is proportional to the Quantity of Matter, it follows, that where the latter is diminished indefinitely, the former will be so too; therefore the Weight of Light must be insensible in ever so great a Quantity of it.

find,

find, that in the former Case those Eclipses happen *too soon*, and in the latter *too late*, by the Space of 8 Minutes and 13 Seconds; which shews, that in that Time the Light passes over the Semidiameter of the Earth's Orbit, which is

Dr. *Boerhaave* caused a Globe of Iron 12 Inches in Diameter to be heated red-hot, and suspended at the End of a very exact Balance, and counterpoised by Weights at the other End very nicely, and thus let it hang till all the Particles of Heat or Light were escaped, when he found the Equilibre of the Balance no ways alter'd; which plainly proves the above Thesis.

7. That the Particles of Light have not only Magnitude, but that in *different Degrees* also, is another and perhaps the most subtle Discovery of the *Newtonian* Philosophy. The comparative Terms of *Greater* and *Lesser* are now as applicable to the Particles of Light, as to any other Bodies. This is absolutely proved by the different Refrangibility they are found to have in passing through a Prismatic Figure of Glass or Water; for the Power of the Prism detains the issuing Particle, and draws it a little towards the Surface; and since this Power is the same, it would have the same Effect on all the Particles of Light, if they were all of an equal Magnitude, because they have all an equal Velocity. But since this Effect is different among the Particles, some being detain'd and drawn aside to a greater Distance than others, it follows, they must be less in Magnitude, to become more subject to the Influence of the attracting Surface; in like manner as the electric Effluvia will act upon and agitate very small and light Bodies, much sooner and more easily than they can move those which are larger. But of this more when we come to speak of the Manner in which this Power acts refracting the Rays of Light.

8. If Light were not reflected from every Point in the Surface of a Body in all Directions every Way, there might be assign'd a Point of Space where a Ray of Light from such a Point in the Surface does not come; and there the said Point of the Surface could not be visible, but because the Eye can find no Point of Space in all the visible Hemisphere respecting that Point, but where it is visible, therefore a Ray of Light is reflected from that Point to every Part of Space, from whence a Right Line to that Point can be drawn.

about 82,000,000 Miles; which is at the rate of 170,000 Miles in a Second of Time, and which is therefore nearly 680,000 times greater than the Velocity of Sound (CXII).

9. That the Rays of Light proceed in Right-lined Directions, is evident from hence, that whatever the Figure of the Body be, if it be held perpendicular to the Rays of Light, it will always cast a Shadow of the same Figure against a parallel Plane. Thus a Circle will produce a circular Shadow, a Triangle a triangular one, and so on. Which plainly shews, that the Rays of Light pass by the Extremities of those Bodies in Right-lined Directions, excepting those only which pass contiguous to the Edges of the Body, for they will be a little inflected, which will cause the Extremity of the Shadow to be not so distinct and well defined as it otherwise would be; of which we shall take farther Notice hereafter.

(CXII.) As all the other Affections of Light, so that of Velocity, was utterly unknown to all the ancient, and most of the modern Philosophers, who, before the Time of Mr. *Reaumur*, were of Opinion that the Motion of Light was instantaneous, or that it was propagated thro' immense Spaces in an Instant. But Mr. *Reaumur* and other Philosophers about this time, making frequent Observations on the Eclipses of *Jupiter*'s Moons, found that the Time of those Eclipses did not correspond to the Calculations founded upon the astronomical Tables; where the Times are all calculated for the Distance of the Centre of the Sun, and consequently, where the Eye of the Spectator must be supposed to be in viewing the said Eclipses, Occultations, *&c.* of *Jupiter*'s Moons.

Plate XXXVI. Fig. 1.

2. To illustrate this Matter; let S be the Centre of the *Sun*, A B the Orbit of *Mercury*, C D the Orbit of *Venus*, E F that of the *Earth*, and G H a Part of the Orbit of *Jupiter*. Let I be the Body of *Jupiter*, and K L its Shadow, O M N the Orbit of one of *Jupiter*'s Moons M just entring the Shadow of *Jupiter*. Now a Spectator at S would observe the Moon M to enter the Shadow just at the Time which is calculated from the Tables; but a Spectator at the *Earth* at T always observes it to happen sooner, and when the *Earth* is in the opposite Part of its Orbit R, he will always observe it to happen later, by the Space of about 7 Minutes in both

AGAIN:

AGAIN: Since Light is propagated in Right Lines, its Power or Intensity will decrease as the Squares of the Distances increase; and therefore the Light and Heat of the Sun at the Distances of the six Planets, *Mercury*, *Venus*, *Earth*, *Mars*, *Jupiter* and *Saturn*, will be nearly as 700, 200, 100, 43, 3, 1. supposing their Distances as the

Cases. This Observation gave the first Proof that Light was progressive, and took up about 14 Minutes to pass over the Diameter of the *Earth*'s Orbit from T to R, or 7 Minutes to pass from the *Sun* S to the *Earth* T.

3. But this, tho' a sufficient Discovery or Proof of the progressive Motion of Light, was yet but an Experiment in the Gross, and not accurate enough to determine or define the true Rate of Velocity which did really belong to Light. The Method by which it has been more nicely determined was hit upon in the following Manner: Tho' Sir *Isaac Newton* had demonstrated the Motion of the Earth from the Laws of Gravity, yet as his Book was understood by few, those who could not comprehend his Method were willing to be satisfied of the Truth thereof otherwise, and rightly judged, that if the Earth did move about the Sun, it must necessarily cause an apparent Motion in any fix'd Object at a Distance from it.

4. Thus if ABCD represent the Orbit of the Earth, and A and C the Place of the Earth at two opposite Times of the Year; then a fix'd Object at E will be seen from the Earth at A in the Line AE, which will point out its apparent Place at G in the Concave Expanse of the Sky HI. But at the opposite Time of the Year, it will be seen from the Earth at C in the Line CE, which will project its Place in the Heavens at F. So that while the Earth has pass'd from A by D to C, the Object (tho' in reality fix'd) has appear'd to move thro' the Space GF; and the Angle which measures this apparent Motion of the Object, *viz.* the Angle AEC, is call'd the *Parallactic Angle*, or *Parallax of the Annual Orbit*, because it measures the visible Appearance of the Diameter AC of the Earth's Orbit at the Object E. Fig. 2.

5. This being the Case, it was applied to the fix'd Stars, which they concluded would certainly have an apparent Motion, or Parallax, provided an Instrument could be made sufficiently exact to observe it, and this would be a satisfactory

Numbers 4, 7, 10, 15, 52, 95.

From the stupendous Velocity of luminous Particles arise their prodigious Effects in regard of *Heat*, *Flame*, *Burning*, *Melting*, &c. Thus when they are considerably dense, they act very forcibly on the Parts of an animal Body, and raise the *Sensation of Heat*, by the great intestine

Demonstration of the Earth's Motion. Accordingly several Persons address'd themselves to discover a Parallax of the fix'd Stars; and in the Year 1725, the late Hon. *Samuel Molyneux*, Esq; with an Instrument made by the accurate Mr. *Graham*, began to observe the bright Star in the Head of *Draco* as it pass'd near the Zenith. Professor *Bradley* also observ'd it along with him; and from many Observations made with great Care, it appear'd that the Star was more Northerly 39 Seconds of a Degree in *September* than in *March*, just the contrary Way to what it ought to appear by the annual Parallax of the Stars. That is, the Observers, who in *September* saw the Star at F, did in the *March* following observe it at K, in the Right Line A K parallel to C F, and not at G where it ought to have appear'd by the parallactic Motion.

6. This unexpected Phænomenon perplex'd the Observers very much, and Mr. *Molyneux* died before the true Cause of it was discover'd. After this, Dr. *Bradley*, with another Instrument more exact and accurately adapted for this Purpose, observed the same Appearances, not only in that, but many other Stars; and being by many Trials fully assured that the Phænomenon was not owing to any Error in the Instrument or Observation, applied himself to consider what might be the Cause thereof, and after several Reflections and Hypotheses, which he still found insufficient to account for it, he at last found, that it was really owing to the progressive Motion of Light, and the sensible Proportion which the Velocity thereof bore to the Velocity of the annual Motion of the Earth.

7. This he was fully assured was the true Reason, not only because nothing else could be thought of that would account for it, but because such an Appearance must necessarily result from the above-mention'd Hypothesis, as may be thus shewn. Let A B represent a Part of the Earth's annual Orbit, and let C be a Star observ'd by a Spectator at the Earth at A; when the Earth arrives at B the Star will not be ob- Fig. 3.

Motion

Motion which they produce in every Part. Hence all other Bodies are *hotter* or *colder*, as they contain a greater and lesser Quantity of ignitious Particles, and so have a greater or lesser Degree of intestine Motion of the Parts.

IF these lucific Particles are sufficiently imbibed or generated in any opake Body, they cause it to

serv'd at C, as before, but at D in the Line BD parallel to AC; for let AB be divided into the equal Parts A*a*, *ab*, *bc*, *cd* and *d*B, then thro' those Points draw the Lines *ae*, *bf*, *cg*, *dh*, parallel to AC and DB. Now let the Velocity of the Earth be to that of Light as AB to CB. When the Earth sets out from the Point A, suppose the Ray of Light commences its Motion from the Star at C in the Direction CB perpendicular to AB; then 'tis plain when the Earth is arrived at *a*, the Particle of Light will be got to *i*, the Point where *ae* cuts BC, and the Star will be seen in the Direction *ai*, and appear at *e*. In like manner, when the Earth is at *b*, the Particle of Light will be come to *k*, and will appear at *f*, and so on; when the Earth is at *c*, *d*, B, the Particle will be at *l*, *m*, and B, and the Star will appear at *g*, *h*, and D.

8. If therefore the Line CA represents the Axis of a Telescope, making the Angle BAC with the Direction of the Earth's Motion AB; when he comes to B he will see the Star at D, which he could not do if the Telescope was directed in the perpendicular Line BC; but the Difference of the Positions of the Lines DB and BC, or the Angle DBC, is so very small, as to amount to no more than 20″ 15‴, which gives the Proportion of the Sides BC to CD or AB, as 10210 to 1; which shews that *the Velocity of Light is ten Thousand two Hundred and ten Times greater than the Velocity of the Earth in her Orbit.*

9. But the Velocity of the Earth is known, which is about 500,000,000 Miles in 365 Days, or about 56,000 Miles *per* Hour, whence the Velocity of Light will be found to be such as carries it thro' the Space of 170,000 Miles, or 897,600,000 Feet in one Second; and therefore it will pass from the Sun to us in 8′ and 13″.

10. If a Cannon will throw a Ball 1 Mile perpendicular Height, or 5280 Feet, the Velocity with which it goes from the Mouth of the Cannon is the uniform Velocity of 10,560

shine,

shine, or glow, or become red-hot; and by their prodigious Activity will in Time disunite, dissolve, and destroy its natural Texture, and thus change its Form, and reduce it to another Species of Matter; even the *Asbestos* not excepted (CXIII).

Feet *per* $18\frac{1}{4}''$ (which is the Time of the perpendicular Ascent or Descent,) and therefore the Velocity of the Cannon-Ball is 578 Feet *per* Second. Whence the Velocity of Light is to that of the Cannon-Ball, as 897,600,000 to 578, or as 1,550,000 to 1 nearly.

11. The *Doctor* found that the Parallax of the fix'd Stars, instead of amounting to many Seconds, as many have deduced from their Observations, does not make one Second; and from thence it follows that the above-mention'd Star, in *Draco*, is above 400,000 Times farther from us than the Sun; and consequently, that the Light takes up above $493'' \times 400{,}000 = 197{,}200{,}000''$ Seconds, (which is more than six Years,) in coming from that Star to us. In the mean Time we may reflect how different are the Places of the Sun, Moon, and Planets in the Heavens from those in which they appear. Thus, setting aside the Refraction of the Atmosphere, when the Centre of the Sun is really ascending in the Horizon, it will be $8'\ 13''$ after, that we observe it there; in which Time the Sun will be far advanced in the Heavens.

12. The Motion of the Earth is by this Method absolutely demonstrated, and therefore put beyond all Doubt and Objection; they who deny it now must confess themselves wholly ignorant of one of the finest and most important Discoveries that was ever made in *Astronomy*, and which was finish'd in the Year 1728; concerning which, see Dr. *Bradley*'s own Account in *Phil. Transf.* N°. 406. which we shall farther explain in a future Part of this Work.

(CXIII) 1. That *Heat, Fire, Flame,* &c. are only the different Effects and Modifications of the Particles of Light, is, I think, very evident; and the Particles of Light themselves depend entirely on *Velocity* for their *lucific Quality*; since by many Experiments we know that the Particles of Bodies become lucid, or Particles of Light, by only producing in them a requisite Degree of Velocity; thus the Particles in a Rod of Iron, being hammer'd very nimbly, shine and be-

If

If the ignific Particles of Light are sufficiently condensed, as the Rays of the Sun by a *Lens* or *Burning-Glass*, they become *ardent*, and burn with

come red-hot; thus also the violent Stroke of the Flint against the Steel, in striking Fire, puts the Particles of the Steel which it takes off into such a Motion as causes them to melt, and become red-hot, which makes the Sparks of Fire produced by each Stroke. The same Thing you may observe in many other Cases.

2. As *Fire* consists in the great Velocity of the Particles, so it may be communicated from one Body in which it is to another in which it is not, after the same Manner that one Body in Motion will communicate Motion to another Body that has none. Fire differs from Heat only in this, that Heat is a Motion in the Particles of a Body with a lesser Degree of Velocity; and Fire a Motion with a greater Degree of Velocity, *viz.* such as is sufficient to make the Particles shine, tho' we often call such a Degree of Heat as will burn, Fire, tho' it does not actually shine; and we seldom call those lucid Bodies Fires which only shine and do not burn. These are a Sort of *Phosphori*, which tho' they have no Heat, yet seem to owe their Lucidity to the Motion of the Parts.

3. This I think will appear for the following Reasons; (1.) We observe several of those *Phosphori* are owing to Putrefaction, as rotten Wood, very stale Meat, especially Veal, some Sort of Fish long kept, as *Oysters*, *Lobsters*, *Flounders*, *Whitings*, &c. which Putrefaction is the Effect of a slow and gentle Fermentation, and that consists in the intestine Motion of the Parts as we have formerly shewn. (2.) Most of those *Phosphori* have their Light so very weak as to shine only in the Dark, which seems to indicate a lesser Degree of Velocity in the Parts than what is necessary to produce Heat; for such a Degree of Velocity will cause Bodies to shine in open Day-Light. (3.) Some of those *Noctilucæ*, or Bodies which shine in the Dark, are the Parts of animated Bodies, as in the *Glow Worm*, a small Sort of *Centipede*, &c. but all the Parts of an Animal are undoubtedly in Motion. (4.) Other *Phosphori* put on the Appearance of Flame, as the *Ignis Fatuus*, the Writing of common *Phosphorus* made from Urine, Flashes of Lightning, &c. but all Flame is nothing but a kindled Vapour, whose Parts are all in Motion, but may be too weak to cause Burning. (5.) Several of those innocent lambent Flames may have their Matter so agitated, or the Velocity

an

an Intensity proportional to the Density of the Rays in the *Focus*, or Burning-Point of the Glass; which Density of Rays in the Focus is always

of their Motion so increased, as to produce Heat and burn; thus the Writing of *Phosphorus* on blue Paper, sufficiently rubb'd, will immediately kindle into an ardent Flame, and burn the Paper. (6.) Those *Phosphori* seem to have the essential Nature of Fire, because they are so easily susceptible of a burning Quality from Fire; thus common *Phosphorus* is immediately kindled into a most ardent and inextinguishable Flame by common Fire. (7.) In stroking the Back of a black Horse, or Cat, in the Dark, we produce innumerous *Scintillæ*, or lucid Sparks; in the same Manner as rubbing a black Piece of Cloth, which has hung in the Sun to dry, will cause it to throw out the Particles of Light which it had imbibed from the Sun; whereas a white Piece of Cloth, which reflects most of the Sun's Rays, emits no such lucid Sparks in the Dark. Many other Reasons might be urged to shew that Light of every Kind is owing to one and the same Cause in a greater or lesser Degree, viz. *to the Velocity of the Parts of the lucid Body.*

4. It has been justly observed by some of our modern Philosophers, that *actual* or *absolute Heat* is to *sensible* or *relative Heat* the same as *Motion* is to *Velocity*; for *absolute Heat* is nothing but the whole Motion of all the Parts of the ignited Body, and *sensible or relative Heat* respects only the *comparative Velocity* of the Parts. Thus equal Bulks of Mercury and Water set in a Sand-Heat, where the Heat of the Fire may be uniformly communicated to both, will acquire in equal times equal Degrees of absolute Heat, but the relative Heat of the Water, or that which is sensible to the Finger, will be near 14 times as great as that of the Mercury; because the Water having 14 times a less Quantity of Matter, will admit of Velocity so much in Proportion greater.

5. Again, if Mercury and Water have the same relative or sensible Heat, that is, if both are heated in such a Manner as to cause an equal Ascent in the Thermometer; then a Quantity of Mercury will heat 14 times as much Water as the same Quantity of Water will do; or it will make the same Quantity of cold Water 14 times hotter than the same Quantity of hot Water can. All which is easy to be shewn by Experiment, and abundantly proves the Truth of the foregoing Theory, viz. *That Heat and Fire are wholly owing to*

as

as the Area of the Burning-Glass directly, and the Square of the Focal Distance inversely. Thus suppose the Surface or Area of one Glass contain'd 12

the Velocity of the Parts of the heated or ardent Body.

6. The various Phænomena of *Heat* and *Cold*, *Fire*, *Burning*, &c. are rationally accounted for on this Theory. For first, we are to consider that Cold and Heat are only comparative Terms, or that the same Thing may be either hot or cold according to the Relative Idea, or Standard Degree; thus Ice or Snow is said to be cold with respect to the Finger, but Ice or Snow is warm if compared with a freezing Mixture. So that if (as we commonly do) we make the Hand or any Part of the Body the Standard of Heat or Cold, or the Term of Comparison; then 'tis evident, (1.) If the Parts of any Body applied to the Hand have the same Velocity as the Parts of the Hand, such a Body we naturally pronounce is neither hot nor cold. (2.) If the Particles of the Body have a greater Velocity than those of the Hand, we pronounce it *warm*, if the Excess be small; but *hot*, if it be great. (3.) If the Velocity of the Parts of the Body applied be less than that in the Hand, the Sensation then is what we call *Cold*, which also may be in various Degrees. (4.) Hence it is plain there can be no such Thing as *absolute Cold*, but where the Particles of Matter are absolutely quiescent or at rest. (5.) Hence also there can be no such Thing as absolute Heat, because no Degree of Velocity can be assign'd, but a greater is assignable, till we come to Infinity, where we are quite lost, as having no Idea of infinite Velocity or Heat.

7. From this Theory of Heat and Cold, we may conclude that there is no Body in Nature whose Parts are not in Motion in some Degree, since we have yet been able to discover no ultimate Degree or Limit of Cold; and if any such Thing were to be found in Nature, I believe it would be as impossible to bear or endure the Test as any extreme Degree of Heat; both Heat and Cold naturally tending to destroy the animated Part, or Test, in the extreme Degrees; Cold, by destroying the vital Motion, and fixing the Part rigid and inflexible; but Heat, by putting the Parts into too great an Agitation, causing a greater Velocity in the Fluids and Dissipation, and a Force of Tension in the Solids beyond what the natural State of the Body can bear; and therefore it will inevitably destroy it.

ſquare Inches, and its focal Diſtance were 8 Inches; and the Area of another Glaſs were 9 ſquare Inches, and its focal Diſtance 4 Inches;

8. Whatever be the acting Principle in *Freezing* or *Congelation*, 'tis certain, the *Modus Agendi*, or *Manner of Operation*, muſt be to diminiſh the Velocity of the Parts of the congealable Subſtance to a proper Degree, by which means the Fluidity will be loſt, and the Parts become rigid and fix'd. Thus if the inteſtine Motion of the aqueous Particles be abated by the Admixture of any extraneous Body, the Parts will be no longer fluid, but remain to Appearance fix'd in a Congelation, and become a Body of Ice. Whatever this Principle of Freezing be, it is certainly of a *ſaline Nature*, becauſe 'tis well known Salt will greatly increaſe the Coldneſs of Water, Ice, or Snow; and *freezing Mixtures* are always made therewith, by equal Quantities of each.

9. On the other Hand, fix'd Bodies are render'd fluid by Heat, only by increaſing the Velocity of the Parts; thus Ice becomes Water, thus Metals are put into Fuſion, and a greater Degree of Heat gives a ſtill greater Degree of Velocity to the Parts, and throws them off in the Form of a Steam or Vapour. This Steam or Vapour, if it conſiſts of ſuch Particles as will admit of a proper Increaſe of Velocity, will conceive it very readily, and kindle into a Flame, at the Approach of a Body whoſe Parts are thus in Motion; that is, of Fire or Flame.

10. There ſeems to be no other Difference between *Fire* and *Flame* than this, that *Fire* conſiſts in a glowing Degree of Velocity in the Parts of a Body while yet ſubſiſting together in the Maſs; but *Flame* is the ſame Degree of Velocity in the Particles diſſipated and flying off in Vapour; or to uſe Sir *Iſaac Newton*'s Expreſſion, *Flame is nothing elſe but a red hot Vapour*.

11. The Effect of Fire in burning conſiſts in this, that the Velocity of the Particles of Fire ſo far increaſes the Velocity of the Parts of the Body to which it is applied, as to cauſe a Separation beyond the Sphere of corpuſcular Attraction, by which means the Body will be diſſolved, and the Particles which are volatile will fly off in the Form of Steam, Smoak, Fume, &c. while that which remains appears in the Form of *Coal*, *Calx*, *Aſhes*, *Caput Mortuum*, &c.

12. The Parts of ſome Bodies are extremely volatile, and will moſt of them be diſſipated by the Action of Fire; but

them

then the Effects or Intensity of Burning would be as $\frac{12}{64}$ to $\frac{9}{16}$, or as 12×16 to 9×64, *viz.* as 197 to 576 (CXIV).

others again are to be found whose Parts are of such a Nature, or so fix'd, as not to yield to the Force of Fire, or the Velocity communicated to them will not be able to dissolve the corpuscular Attraction; but when this glowing Velocity of the Parts is abated, or, in other Words, when the Fire in the Body is extinct, the Parts (and of course the whole Body) appear unalter'd. Of which Sort of Substance we have a notable Instance in that Fossil call'd the *Asbestos* or *Amianthus Stone*. This Stone is found in divers Parts of the World; particularly in *Wales* a great deal may be seen adhering to, and growing up with the Stone of many of their Quarries.

(CXIV.) 1. In order to account for the Nature of BURNING-GLASSES, whether *Mirrours* or *Lenses*, we must consider the Area of their Surfaces, and the focal Distance; because both these Quantities enter into the Expression of their Power of Burning. Let A B, and I K, be two Mirrours exposed directly to the Rays of the Sun C D, E F, and L M, N O; then will all the Rays, falling on the Surface of these Mirrours, be reflected to the Focus of the Glasses, where they will be concenter'd, not in a Point of Space, but into a small round circular Area G H, and P Q. Plate XXXVI. Fig. 4, 5.

2. Now this circular Spot is the Image of the Sun inverted, in both Glasses; and the Angle under which the Image of an Object appears from the Centre of the Glass R and S, is equal to the Angle under which the Object appears; all which will be shewn hereafter. Therefore the Angle $GRH = PSQ$, and consequently the Cones G R H and P S Q are similar, and the Areas of their Base G H and P Q will be as the Squares of their Heights R H and S Q, that is, as the Squares of their focal Distances directly.

3. Let A = Area or Surface of the large Glass, a = that of the lesser, F and f the focal Distances, and P and p the Power of Burning in each. Then, since while the focal Distance remains, the Power of Burning (P) will be as the Density of the Rays in the solar Spot H G, and this Density of the Rays will be as the Number of Rays reflected thither by the Glass, which Number of Rays will be as the Surface of the Mirrour (A); therefore P will be as A directly in a Mirrour of the same Concavity, that is, $P : p :: A : a$.

WHEN

WHEN Rays of Light fall on the Surface of an opake Body, part thereof are reflected to the Eye, which render it visible; the other Part is transmitted, and variously reflected thro' the

4. Again, if the Area of each Glass be the same; the same Quantity of Rays will be collected, and converged to the Focus's G H and P Q, and consequently the Density of those Rays will be greater, the less the Spot is in which they are contain'd; consequently the Power of Burning (P) in this Case is inversely as the Area of the solar Spot, or the focal Distance, that is, P will be as $\frac{1}{F^2}$; or $P : p :: \frac{1}{F^2} : \frac{1}{f^2} :: f^2 : F^2$.

5. Consequently when neither the Area of the Glass nor focal Distance is given, we have the Power of Burning compounded of the direct Ratio of the Area and inverse Ratio of the Square of the focal Distance of the Glass; or we have $P : p :: A f^2 : a F^2$; which is the Rule above laid down.

6. It has been shewn (*Annot.* XCIII.) that the Heat of a Wood Fire is about 35 times greater than that of the Summer Sun (because it raises the Fluid in the Thermometer 35 times higher nearly); therefore that a Glass may be able to condense the Rays sufficiently to burn, or to have the Heat of common Fire, the Sun's Image, or solar Spot in the Focus, ought to be at most but $\frac{1}{35}$ Part of the Area of the Glass; and as much as it is less than a $\frac{1}{35}$ Part of the Glass, so much the stronger will it burn. In this Case, if it be desired to know in what Part of the Pencil of Rays the Density is 35 times greater than the common Density, and where the Power of Burning is equal to that of common Fire, it is found as in the following Example. Admit a Glass be 9 Inches in Diameter, and let the Diameter of the required Circle be (a); then since circular Areas are as the Squares of their Diameters, we have $35 : 1 :: 9^2 : a^2$; consequently $\frac{81}{35} = a^2$, and so $a = \sqrt{\frac{81}{35}} = 1{,}5$ nearly; whence that Part of the Cone or Pencil of Rays, whose Diameter is $1\frac{1}{2}$ Inches, has the Density and Power of Burning required; and that this is Fact, and that the Density of the Rays but a little less than that will not burn, I know from repeated Trials with such a Glass, or concave Mirrour.

Pores

Pores of the Body, till it becomes totally suffocated and lost therein; and since none of those Rays come from the interior Parts to the Eye, we can see nothing of the internal Substance of

7. Of *Burning-Glasses* we have some extraordinary Instances and surprizing Accounts of their prodigious Effects. Those made of reflecting Mirrours are more powerful than those made with Lenses, *(cæteris paribus)* because the Rays from a Mirrour are reflected all to one Point nearly, whereas by a Lens they are refracted to different Points and are therefore not so dense or ardent. Also the whiter the Metal or Substance is, of which the Mirrour is made, the stronger will be the Effect; and it is observable, that the great Mr. *Boyle* having made a very large Mirrour of black Marble, it would not so much as set Wood on Fire, tho' exposed a long Time in the Focus, so small a Quantity of Rays are reflected from black Surfaces, the Reason of which we shall hereafter explain.

8. Among a great Number of Mirrours made for *burning*, *melting*, *calcining*, and *vitrifying* Bodies, that of Mr. *Villette* is worth our Notice; it was 3 Feet 11 Inches in Diameter, and its focal Distance was 3 Feet 2 Inches. The following Experiments were made with it by Dr. *Harris* and Dr. *Desaguliers*.

1. A red Piece of *Roman Patera* began to melt in 3″, and was ready to drop in 100″.
2. Another black Piece melted at 4″, and was ready to drop at 64″.
3. Chalk taken out of an *Echinus Spartagus*, fled away in 33″.
4. A Fossile-Shell calcin'd in 7″.
5. A Piece of *Pompey*'s Pillar at *Alexandria* vitrified in the black Part in 50″, and in the white Part in 54″.
6. Copper-Ore vitrified in 8″.
7. Slag, or Cinder of ancient Iron-Work, ready to run in $29\frac{1}{2}$″.
8. Iron-Ore fled at first, but melted in 24″.
9. Talc began to calcine at 40″, and held in the Focus 64″.
10. *Calculus humanus* was calcined in 2″, and only dropp'd off in 60″.
11. A great Fish's Tooth melted in $32\frac{1}{2}$″.
12. The *Asbestos* seem'd a little condensed in 28″, and such

such a Body, which therefore is said to be *opake*.

BUT when Rays of Light fall on *transparent Bodies*, part is reflected at the first Surface, and part is transmitted into the Body, which is re-

Mr. *Villette* says, the Glass usually calcines it.

13. Marcasite of Gold broke to Pieces and began to melt in about 30″.

14. A Silver Six-pence melted in $7\frac{1}{2}''$.

15. A Copper Half-penny (of King *William*'s) melted in 20″, and ran with a Hole in 30″.

16. A King *George*'s, *ditto*, melted in 16″, and ran in 34″.

17. Tin melted in 3″.

18. Cast Iron melted in 16″.

19. Slate melted in 3″, and had a Hole in 6″.

20. Thin Tile melted in 4″, had a Hole and was vitrified in 80″.

21. Bone calcined in 4″, and was vitrified in 33″.

22. A Diamond weighing 4 Grains lost $\frac{7}{8}$ of its Weight.

9. The Power of Burning, in *Villette*'s Mirrour, may be computed, and compared with the Heat of Wood-Fire, as follows: Since the focal Distance R X is 38 Inches, and the Angle under which the Sun's Image in the Focus appears at R, is equal always to 32′ of a Degree; therefore if we say,

As Radius — — — —		90° 00′ =	10,000000
Is to the Tangent of the Angle — —	H R X =	00° 16′ =	7,667849
So is the focal Distance	R X =	38′ =	1,585461
To the Semidiameter of the solar Spot	H X =	0,179′ =	9,253310

Whence 2 H X = 0,358 of an Inch, the Diameter of the solar Focus; but the Diameter of the Mirrour was 47 Inches; now 47 × 47 = 2209, and 0,358 × ,358 = 0,128, *&c.* wherefore 2209 is to 0,128, as the Density of the Rays in the Focus to their common Density; but 70,128)2209(= 17257; which shews that the Mirrour condensed the Rays Seventeen Thousand Two Hundred and Fifty-seven times.

10. Since Rays but 35 times denser than in their natural State with us, have a Power of Burning equal to Wood-Fire, if we divide 17257 by 35, the Quotient will be 493; therefore such a Mirrour will burn with an Intensity of Heat 493 times greater than common Fire. No wonder then that

fracted

fracted in Right Lines to the second or lower Surface, where it is again partly reflected and in part refracted into the Air, and coming to the

Bodies which remain unalter'd by the Force of our greatest common Fires (as that of a Glass-House, where Gold has been found to lie several Days in Fusion, without any sensible Loss of Weight) should immediately become fused, fume away in part, part be dissipated and driven away in large Particles, and part remain in the Form of a *Caput Mortuum*; all which Phænomena have been observed of Gold in the Focus of a large Burning-Glass. And how rudely such a Glass would treat the Principles of the *Chymists*, and what Confusion it would induce in their Arithmetic of Elements, they will be better certified of, when they shall attempt to analyse Nature, and reduce Substances to their original Principles, by more active and effectual Means than Laboratories at present afford.

11. Notwithstanding the prodigious Density of the Rays in the Focus of those large Burning-Glasses, yet it has been always observed, that the Rays reflected to us by the Moon when at Full, and concenter'd in the Focus of those Glasses, produce no Heat that is sensible in the least Degree, as is demonstrated by holding a Thermometer in the Focus of lunar Rays, which always remains without the least Appearance of Motion. The Reason of this will appear by the following Calculation.

12. Let ABD be the Earth, C its Centre, MO the Moon, N the Centre, N*e* the Semidiameter of the Moon, which is equal to 1087,5 *English* Miles; the Semidiameter of the Earth DC = 4000 Miles; the Distances of the Centres of the Earth and Moon NC = 240000 Miles. Then since the Rays of the Sun's Light at the Moon are of the same Density as with us (as being parallel); and since the lunar Rays are only the solar Rays reflected to us by the convex Surface of the Moon; and lastly, since parallel Rays are reflected by a spherical convex Surface, in such a Manner as to go after Reflection diverging from a Point which is $\frac{1}{2}$ the Semidiameter of the Sphere distant from the Vertex (as will be shewn hereafter); therefore supposing the Surface of the Moon to be perfectly spherical and polished, we may compute the Density of the solar Rays reflected from the Moon to the Earth as follows. Fig. 6.

13. Let *ab*, *cd*, be two parallel solar Rays falling on

Eye, renders the internal Parts of those Bodies visible, which for that Reason are said to be *diaphanous* or *transparent* (CXV).

the Surface of the Full Moon, these Rays will be reflected to the Earth in the Directions $b g$ and $d h$ diverging from a Point f in the Radius N e, half way between N and e. Now the Density of the Rays falling on the Moon will be to those reflected at the Earth's Surface, as the Square of $g h$ to the Square of $b d$, or as the Square of f D to the Square of $f e$; but $f e = 544$ Miles, and fD ($=$ NC—CD—Nf $=$ $240000 - 4544 =$)235456; and the Square of 235456 is to the Square of 544, as 187400 to 1 nearly; consequently the Density of the lunar Rays is to that of the solar Rays at the Earth's Surface as 1 to 187400 nearly; therefore a Burning-Glass must condense the lunar Rays 187400 times to make them have the Heat of the common Sun-Beams. But this is 10 times more than *Villette*'s Mirrour can effect.

14. Now this is all upon Supposition that the Moon is a Sphere, and its Surface a perfect Polish, whereas neither of these Things have Place in Nature; for the Moon is not a Sphere but a Spheroid, and her Surface very unequal or uneven, on both which Accounts the Reflection of Light must be many times weaker than we have supposed it; and accordingly Mr. *Bouguer*, by Experiments, has found that it is about 17 times less, or that the Density of the lunar Rays is to that of the solar as 3000000 to 1; wherefore a Burning-Glass must condense the Rays of the Moon near 3000000, *i. e.* three Millions of times, to make them warm enough to raise the Liquor of the common Thermometer; which is an Effect almost 200 times greater than *Villette*'s Mirrour can produce.

Plate XXXVII. Fig. 1.

(CXV.) 1. The Opacity and Transparency of Bodies in general is thus occasion'd: Let A B be the Surface of an opake Body A B C D, a Ray of Light G H falling thereon in the Point H will in part be reflected into the Ray H I, and by this reflected Ray the Point H becomes visible to the Eye at I; and thus all the Points, and consequently the whole Surface, is made visible by that Part of the Light which it reflects.

2. But the other Part of the Ray entring into the Body, being irregularly refracted and reflected thro' its internal Substance of Particles and Pores, becomes divided, dissipated,

WHEN a Ray of Light HC falls on any plain, convex, or concave Surface, as AB, DE, FG, in the Point C, the Angle HCK, made by the incident Ray HC and the Perpendicular KC, is always equal to the Angle KCI, made by the said Perpendicular and the reflected Ray CI: Or the Angle of Incidence is equal to Plate XXXVIII.

abſorb'd and loſt therein; and therefore as none of the Rays can come from the internal Parts to the Eye, ſo none of thoſe Parts can be viſible, and the Body is in that caſe ſaid to be *opake*.

3. In order to this we muſt conſider, that tho' the whole Body be opake, yet the Particles of ſuch a Body are not ſingly opake, but freely tranſmit the Light without reflecting any Part between the Surfaces, and are therefore in themſelves tranſparent; and were thoſe Particles contiguous to each other, the Light would paſs from one to another (and ſo thro' the whole) without Reflection, as we find by Experiment it will paſs thro' ſeveral contiguous Pieces of poliſh'd Glaſs, and thus produce Tranſparency.

4. But if the Particles do not touch in ſuch manner as to leave the Interſtices or Pores exceeding ſmall, there will be a Reflection of Light at every Pore from the Air which it there meets with, as being a Medium of different Denſity. For it is known by Experiment, that tho' a Ray of Light will paſs from one Piece of Glaſs to another, that is contiguous without Reflection, yet will it not paſs from the Glaſs thro' the contiguous Air without being in part reflected; conſequently where the Pores are large and very numerous, there the Reflection of the Light will be ſo great upon the whole, as to cauſe a total Diſſipation and Loſs of the Light that enter'd the Body, and ſo render it opake.

5. This is confirm'd by taking ten Pieces of clear Glaſs, and laying them one upon another over a Leaf of Print, quite dry, and having only Air between them; then taking ten other Pieces of the ſame Glaſs, and putting them into Water, ſo that it may fill all their Interſtices, and then laying them on the ſame printed Paper by the other, a Perſon looking thro' each will ſee the Print or Reading much more diſtinct, clear, and bright, thro' the latter Pieces than thro' the former; the Rays being more regularly tranſmitted thro' them where the Denſity of the Parts is not ſo unequal, and alſo

the Angle of Reflection in every Inclination of the Ray of Light. This is evidently shewn by Experiment; and it is very well worth our Observation, that in this Case only, the said Ray

with much less Reflection, than thro' the other, where the Light undergoes a considerable Reflection at every Interstice or Plate of Air between the Glasses.

6. 'Tis hence also that transparent Bodies are render'd opake by separating their Parts and rendering them more porous; thus Beer before it is raised into Froth is transparent, but the Froth, by reason of its Pores, becomes opake; thus dry Paper is more opake than that which is wetted with Water or Oil, because more porous. Thus the *Oculus Mundi* Stone is more opake when dry than when steep'd in Water; and Glass reduced to Powder is no longer transparent.

7. Hence it follows, that the Parts of Bodies and their Pores must not be less than a certain definite Bigness to render them opake. For the opakest Bodies, if their Parts be subtilly divided, become perfectly transparent. Thus Copper dissolved in Aqua-fortis has all its Particles pellucid, and the whole Solution is transparent. Thus a Bubble blown of Soap-Water may become so thin on the Top as to reflect no Light, but will transmit the whole. Thus Water, Salts, Glass, Stones, &c. tho' they are as porous as other Bodies, yet their Parts and Interstices are too small to cause Reflections in their common Surfaces.

8. Therefore in all transparent Bodies, as B E F C, a Ray of Light, as K L, falling on its Surface in the Point L, will
Fig. 2. there be in part reflected (as before) into the Ray L M; the other Part will go regularly on in a rectilineal Direction from the upper to the lower Surface at N, where meeting with the Air (a Medium of a different Density) it will be in part reflected again into the Ray N O; the other Part goes out to the Eye at P, by which means all the internal Parts from whence that Ray comes will be render'd visible to the Eye; and since this may be conceived of every Point in the Body, it is easy to understand how the Whole becomes transparent.

9. I have often found Gentlemen reflect with great Surprize on the exceeding great Porosity of Bodies necessarily required for the Transmission of Light, and yet at the same time on the Hardness and Firmness of the Parts of such Bodies, as Glass, for Instance, and others. But Sir *Isaac Newton* takes

takes the *shortest Way possible* from any Point H, to any other Point I, if it must, in its Passage, touch any of those Surfaces (CXVI).

has put us into a Method by which we may conceive this with as much Ease as it produces Surprize; and it is this: Suppose a Body be composed of such Particles, and of such a Figure, that when laid together, the Pores or Interstices may be equal to the Particles themselves; how this may be done, and the Body hard and firm, is not difficult to conceive; such a Body then will be half solid and half porous.

10. Now if each of these constituent Particles, instead of being solid, should be supposed to consist of other Particles, equal in Bulk to their Pores between them, then would the solid Part of the whole Body be but half what it was before supposed to be, that is, it will be but $\frac{1}{4}$ Part of the whole Bulk. In like manner if these Parts are supposed not solid, but to consist of other Parts with equal Pores between them, 'tis then manifest the solid Matter will be but $\frac{1}{8}$ of the whole Bulk of the Body. And thus by continuing this Subdivision of the Parts, you diminish the Quantity of the solid Parts, and increase that of the Pores, till it shall be in any Proportion greater than that of the solid Matter, and yet the Parts, and consequently the whole Body, shall be every where compact and hard.

11. Hence it follows that the least assignable Particle of Matter may be conceived to be so minutely divided, that it shall be diffused thro' any assignable Space, how great soever, in such a manner, as to be in Contact, and to constitute a hard and compact Body, whose Pores shall be less in Diameter than any assignable Length; or, in other Words inversely, the solid Matter in the Globe of our Earth, yea of all Bodies in the Universe, may be no more than what may be reduced within the Compass of a cubic Inch, or be contain'd in a *Lady's Thimble*. They who would see a Mathematical Demonstration of this, may consult Dr. *Keill*'s Introduction to *Natural Philosophy*.

12. Hence we see the Possibility of Bodies being so exceeding porous, as to be rare enough to transmit Light with all that freedom pellucid Bodies are found to do. Tho' what their real Structure or inward Frame may be, is yet unknown to us.

(CXVI.) 1. The Demonstration of this is as follows: Let A C be the incident Ray, and C B the reflected one; from Fig. 3.

THE Rays of Light reflected from the first Surface of a Glass are in a much less Quantity than those reflected from the second Surface, as is evident from hence, that the Image form'd in the first Case is less bright and splendid than that of the latter; and if the second Surface be contiguous to any transparent *Medium*, as Air, Water, *&c.* the Rays will be reflected from thence in greater Plenty, as the *Medium* is more rare; whence the Image by Reflection from the second Surface is brighter when that Surface is contiguous to Air, than when it touches Water; and most bright when it is contiguous to a *Vacuum*.

IF the second Surface of Glass be cover'd with an opake Body impervious to the Rays of Light,

A and B let fall the Perpendiculars AE, BD, and let $AE = a$, $BD = b$, $ED = c$, and $EC = x$; then $CD = c - x$, and $AC = \sqrt{aa + xx}$, and also $CB = \sqrt{bb + cc - 2cx + xx}$. Then since $AC + CB$ is to be a *Minimum*, we must make the Fluxion of its Expression $\sqrt{aa + xx} + \sqrt{bb + cc - 2cx + xx}$ equal to nothing, *viz* $\frac{x\dot{x}}{\sqrt{aa + xx}} + \frac{x\dot{x} - c\dot{x}}{\sqrt{bb + cc - 2cx + xx}} = 0$; whence dividing by $\dot{x}$, and multiplying cross-wise, we have $x \times \sqrt{bb + cc - 2cx + xx} + \overline{x - c} \times \sqrt{aa + xx}$, consequently $x \times \sqrt{bb + cc - 2cx + xx} = \overline{c - x} \times \sqrt{aa + xx}$, that is, $EC \times CB = CD \times AC$; and so we have $EC : AC :: CD : CB$. Consequently (by *Euclid*, 6 and 7.) the Triangles AEC and BDC are equiangular, and therefore the Angle of Incidence $ACE = BCD$ the Angle of Reflection.

2. Since the concave Plane FCG, and convex Plane DCE, do both touch the Plane AB in the same single Point C on which the Ray of Light is suppos'd to fall; the same Law of Reflection must hold with respect to all the Planes equally; because the Situation of any other Particles have nothing to do in the Cause of Reflection of Light, but that on which the Ray immediately impinges.

they

they will then be reflected in much greater abundance from the second than from the first Surface, and the Image will be proportionally more bright than that form'd by Reflection from the first Surface; which is the Case of all Glasses foliated or quicksilver'd. Whence it appears, that the Light reflected from the first Surface bears a very small Proportion to that which is tramitted into the Substance of the Glass.

WHEN a Ray of Light, as HC, passes out of Air into a denser Medium, as ABFO, it will be strongly attracted by the Particles of the Surface of the Medium AB, a little way on each Side; the Consequence whereof is, that its Motion will be accelerated at the Entrance of the Medium, and its Direction somewhat alter'd; for since the Attraction of the Medium is perpendicular to its Surface, it will deflect or bend the Ray out of its first Direction HF, into a new one CE, (thro' the *Medium*) which lies nearer to the Perpendicular KD, drawn thro' the Point of Incidence C: And this is call'd the REFRACTION *of a Ray of Light*; HCK is the Angle of Incidence, and DCE the Angle of Refraction. Plate XXXVIII.

IF on the Point C be described a Circle DHKG, and from the Points H and G (where the Circle cuts the incident and refracted Ray) be drawn the Lines HL, GI, at Right Angles to the Perpendicular KD, they will be the *Sines of the Angles of Incidence and Refraction.* And it is several ways demonstrable, that in every Inclination of the Ray of Light HC to the Surface

of the Medium A B, those two Sines H L and G I will always have one certain or constant Ratio or Proportion to each other: And that H L: G I :: 4 : 3, if the Refraction be out of *Air* into *Water*; but H L : G I :: 17 : 11, or 3 : 2 nearly, if out of *Air* into *Glass*; and in general, the denser the Medium, the greater its refractive Power, or Disproportion of the Sines; all which Particulars will be very evident by Experiments.

IF a Ray of Light, as E C, pass out of a denser Medium into a rarer, as a *Water* or *Glass* into *Air*, it will, upon entering the rarer Medium at C, be refracted from its first Direction E N into a new one C H, which will be farther off from the Perpendicular K C D; and in this Case, I G will be the Sine of the Angle of Incidence, and H L that of the Angle of Refraction; and all other Particulars just the reverse of what they were before under the same Names.

HENCE it follows, that if any Object be placed at E, and cover'd with Water to the Height C D, it will be seen by an Eye placed any where above the Surface A B, in a Situation lower than would be otherwise possible; and thus Objects which are invisible may be render'd visible by the Interposition of a denser Medium, as is well known by a common Experiment. On this Account it is that we see the Sun, and other Luminaries, while they are yet below the Horizon, in a Morning before they rise, and in the Evening after they are set, by the Refraction of the Atmosphere. Hence also the Difference in the Diameters

meters of the *horizontal Sun and Moon*, and their *elliptic Figure*, by the greater Refraction of the Rays coming from the lower Limb.

AGAIN; it follows, that if an Object be view'd which is part in one Medium and part in another, as a Staff represented by N E, it will not appear *strait*, but *crooked*; for if the Eye be in the rarer Medium, the Part of the Staff in the denser, C E, will be refracted into the Line C F, and the whole Staff will appear in the crooked Form N C F.

HENCE also all Objects in a denser Medium appear raised or elevated above their real Situations: Thus the Part of the Staff C E is raised into the Situation C F; and the Bottom of all Vessels, if cover'd with Water, appear raised, or higher by a fourth Part of the Depth of the Water, than what they really are (CXVII).

(CXVII.) 1. If Bodies, on which Light falls, were supposed to affect it no other ways than by giving Admission to the Rays, or permitting them to pass thro' their Substance, they would then persevere in the same Right Line after their Immersion, as before; and of course there could be no such thing as the Refraction above defined. But Bodies are not passive to the Rays of Light, but act upon them with a real and determinate Force, as is evidently proved by Experiments. Thus if a very small round Hole be made in a thin Piece of Metal, and the Light of the Sun transmitted thro' it into a dark Room; if the Metal acted not on the Ray passing thro' the Hole, the Spot of Light would always be of the same Size with the Hole at all Distances from it; but because we always observe the luminous Spot is larger than the Hole, and the more so as it is farther distant, is a plain Proof that the Particles of the Metal in the Periphery of the Hole act with an attracting Force on the Rays of Light, and inflect them in such a manner as to cause them to proceed

THE Sun's Rays, as I have said, are not homogeneous, but of different Kinds; and each Sort has a different Degree of Refrangibility; that is, in passing through a dense Medium, they

diverging from each other.

2. In like manner, if the Rays of Light are made to pass between the parallel Edges of two Knives placed at the Distance of $\frac{1}{10}$ of an Inch, we shall observe on each Side the transmitted Beam a Glare of Light like that of the Tail of a Comet, if the Beam be received on a Sheet of Paper, at the Distance of about 4 or 5 Foot from the Knives. And if the Knives are placed with their Edges about $\frac{1}{100}$ of an Inch apart, instead of the Light above-mention'd, you'll observe on each Side the Beam of Light, three Fringes of colour'd Light parallel to the Edges of the Knives, which are more distinct as the Hole of the Window or Beam of Light is less.

3. If the Edges of the Knives be brought within $\frac{1}{400}$ of an Inch, no Light will appear on the Paper between the said Fringes, so that all the Light which passes between the Edges is inflected on either Side, which plainly shews that Steel acts at the Distance of $\frac{1}{800}$ Part of an Inch upon the Rays of Light, by an *attractive Force* which is increased as the Distance of the Knives is diminish'd.

4. On the other Hand, the Shadows of all Bodies placed in the Beam of Light in the dark Room are larger than they ought to be, were the Rays of Light to pass by them unaffected by any Power from them; for then the Shadow would be at all Distances of one and the same Bigness, *viz.* equal to that of the Body; but since we observe the Shadow always larger than the Body, it follows, that the Rays must proceed diverging from the Surface of the Body, which they could not do but by virtue of a *repellent Power*, which causes them to separate to a greater Distance after they have pass'd by the Surface of the Body; thus the Shadow of a Hair has been observed 35 times bigger than the Hair itself.

5. This attracting and repelling Power in the Particles of Bodies, by which they inflect the Rays of Light, is the Cause of all Reflection and Refraction of Light, of which we shall now treat more particularly. Let there be two Mediums (suppose of *Air* and *Water*) and a Ray of Light HG in the
Fig. 4. rarer Medium (*Air*) tend towards a Point K in the Surface of the denser Medium (*Water*) AB; the attracting Power

are

are differently disposed to be refracted, being bent or turn'd out of their first Course to different Distances from the Perpendicular: And these several Sorts of Rays have each a peculiar

of the Particles in the Surface of the denser Medium extends to a certain small Distance, as to the Line E F; as soon then as the Ray is arrived at the Line E F, it gets into the Attraction of the Medium, which acts perpendicular to the Surface.

6. The Particle of Light in the Point G, begins to be acted upon by two Forces; one derived from its natural Velocity in the Direction G K, the other derived from the attracting Medium in the Direction G I; let then the Parallelogram G K M I be compleated, and 'tis manifest (from what we have shewn already) that the Ray will move in the Diagonal of this Parallelogram, *viz.* in the Direction G M, and impinge on the Surface at L.

7. Now since the Ray of Light, after it comes to G, is influenced by the attracting Virtue of a Number of Particles continually increasing till it comes to L, the Force therefore by which it is urged in the Direction G I, is a Force uniformly increasing, like that of Gravity; its Motion therefore will be constantly accelerated, and its Direction G L not a Right Line, but a Curve. But since the Distance G I is indefinitely small, the Curvature of its Path for so short a Space is not sensible, and may therefore be represented by a Right Line.

8. Let N O be drawn parallel to the Surface A B, at the same Depth below, as E F is above it; and then it is evident, that since the Particle of Light is attracted every way equally within the Distance of I G all round, the Attraction will be greater towards the Line N O as it approaches nearer to it; consequently its Motion will still be accelerated from L to the said Line, and will also be a Curve; therefore the Particle will not go on to M in the Diagonal G M, but will go to a Point P in the Curve L P, nearer to the Perpendicular Line L Q.

9. After it is arrived to the Line N O in the Point P, the Attraction will be on all Sides equal, its Motion or Velocity uniform, and its Direction a Right Line, till it comes within the same Distance G I of the under Surface of the Medium C D, where its Path will again begin to be incurvated into R S, and every thing will be the Reverse of what we have now observed at its Immersion, that is, R S will be similar

Colour,

Colour, *viz.* those which are least refrangible are *Red*; the second Sort, *Orange*; the third Sort, *Yellow*; the fourth Sort *Green*; the fifth Sort, *Blue*; the sixth Sort, *Indigo*; and the seventh Sort, *Violet*, which are most refrangible, or refracted to the greatest Distance from the Perpendicular.

To illustrate this Matter, let GF represent a

to GL, and SV parallel to HG, or the Angle HGX = VSY.

10. The denser any Medium is, the greater will be the Number of attracting Particles in a given Space, and so the greater will be the Force GI, or the refractive Power of the Medium; thus Water is less dense, and therefore has a less refractive Power than Glass, and Glass less than Diamond. But *Oils*, though less dense than Water, have yet a greater refractive Power, as containing a greater Proportion of *Sulphur* than other Bodies; for since Action and Re-action are mutual and equal between all Bodies, and since we see that Rays of Light congregated by a Burning-Glass act most upon sulphureous Bodies in turning them into Fire and Flame, so on the contrary, Sulphurs, Oils, Spirits, *&c.* ought to act most upon Light, as we constantly find they do; and Sir *Isaac Newton* thought it reasonable to attribute the refractive Power of Bodies chiefly, if not wholly, to the sulphureous Parts with which they abound.

Plate XXXVIII. 11. Since the Velocity of Light in different Mediums is different, let its Velocity in the rarer Medium from H to C be to that in the denser Medium from C to E, as m to n; and since the Spaces described are as the Rectangles under the Times and Velocities, the Times will be as the Spaces directly, and the Velocities inversely; whence the Time of describing the Line HC will be to the Time of describing the Line CG, as $n \times HC$ to $m \times CG$. Let $CI = a$, $CL = b$, $HL + IG = c$, and $IG = x$; then will $HL = c - x$, and consequently $CG = \sqrt{aa + xx}$, and $HC = \sqrt{bb + cc - 2cx + xx}$; whence the Time in which HC + CG is moved through is $m\sqrt{aa + xx} + n\sqrt{bb + cc - 2cx + xx}$.

12. Now admitting that Nature does every thing *in the shortest Way*, we have the foregoing Expression of the Time

Parcel

Parcel of the solar Rays entering through the Hole H of a Window-Shutter, into a darken'd Room; and there let them fall on the Prism ABC, in the Point F: In passing through the Plate L. Fig. 1.

a *Minimum*, and so its Fluxion equal to Nothing, *viz.* $\frac{mx\dot{x}}{\sqrt{aa+xx}} + \frac{nx\dot{x} - nc\dot{x}}{\sqrt{bb+cc-2xc+xx}} = 0$; whence we have $\frac{mx}{\sqrt{aa+xx}} = \frac{n \times \overline{c-x}}{\sqrt{bb+cc-2cx+xx}}$; that is, $\frac{m \times IG}{CG} = \frac{n \times HL}{HC}$. Hence, making $HC = CG$, we have $m \times IG = n \times HL$; and consequently, $m : n :: HL : IG$.

13. But the Ratio of m to n, that is, of the Velocity before and during the Refraction, is constant, or always the same in the same *Media*; therefore the Lines HL and IG are in a given or constant Ratio. Hence we have this fundamental Law of Refraction, *That the Sine of the Angle of Incidence is always in a constant Ratio to the Sine of the Angle of Refraction*, in all Inclinations of the incident Ray whatsoever.

14. Since the Proportion of these Sines is constant, it remains that we determine what that Ratio is in different *Media*; and for that Purpose there are various Methods, one of the best of which I shall here describe, but must first premise the following *Lemma*. Let GHD be an equilateral Triangle, and let the Angle D be bisected by the Right Line DO; let AKMC be drawn parallel to the Side GH, and through the Point K draw IKN cutting OD in N; then is the Angle AKI = NKB, as being vertical to each other. Also the Triangle NKD is divided into two similar and equiangular Triangles NKB and BKD, by the Perpendicular KB; and therefore the Angle NKB is equal to the Angle KDB. All which is evident from *Euclid*'s Elements. Plate XXXVII. Fig. 5.

15. Suppose now that GHD be the Section of a Prism of Water or Glass, or any pellucid Medium, and KM a Ray of Light passing through it parallel to the Side GH; and let it go out of the Prism and be refracted into the Air on each Side into the Directions KF and ME; upon the Point K describe the Semicircle PIQ; then is NKB (= KDB) = AKI, the Angle of Incidence out of the Prism into Air, and FKI is the Angle of Refraction; consequently, AR and FS are the

Prism

Prism they will be severally refracted in a different Degree, and thus separated from each other, so that at their Exit on the other Side at

Sines of the Angles of Incidence and Refraction out of the Prism into Air.

16. On the contrary, we may consider FK as the incident Ray falling upon the Prism in the Point K, and refracted in the Direction KM parallel to the Side GH, which at the Point M emerges again into the Air in the Direction ME, making the Angle EML with the Perpendicular ML equal to the Angle FKI. In this Case the Angle FKI is the Angle of Incidence, and NKB is the Angle of Refraction in the Prism; which Angle of Refraction is therefore given, or constant, as it is always equal to the Angle KDB, or half the Angle of the Prism.

Plate XXXVII. Fig. 6.

17. The Angle of Incidence FKI consists of two Parts, *viz.* of the given Angle AKI (= KDB) and the additional Angle AKF. Now the Angle AKI is known, as being equal to half the Angle of the Prism; and the Angle FKA is known by placing the Prism by the Center of a graduated Semicircle, as ABC, carrying an Index, whose two Arms FK and KE are equally elevated above the horizontal Line AC, and correspond to the incident and emergent Ray FK and ME in the other Figure. For here 'tis evident, if an Object be placed on the End of the Arm F, it will be seen by an Eye looking through the Sights at the other End of the Index E; and when the Object is thus seen, the Angle AKF is known by the Number of Degrees which each Arm cuts upon the Limb of the Semicircle.

18. This Number of Degrees, added to the constant Number 30°, which is equal to half the Angle of the Prism, gives the whole Angle of Incidence FKI; and thus the Angles of Incidence and Refraction being found, the Proportion of the Sines FS and AR will be discover'd, which Ratio is always the same while the Matter of the Prism remains the same, as was before shewn from the Theory, and may by this Instrument be proved by Experiment. For Example, Let the Prism be of *Water*, it will be necessary to elevate each Arm 12 Degrees upon the Limb, before the Image of the Object at F can be seen by the Eye at E; then $12 + 30 = 42° =$ $FKA + AKI = FKI$, the Angle of Incidence. But the Sine FS of 42° is to the Sine AR of 30° as 4 to 3 very nearly.

E, they will proceed at different Distances from the Perpendicular E P to the other Side of the Room, where they will make a long and various-

19. Now it is plain, if the Ratio of the Sines A R and F S were not fix'd, since F S might be in any Ratio greater or less than A R, the incident Ray F K may make an Angle F K I greater or less than 42°, and yet the Object at F be seen by the Eye at E; but this we find by Experiment to be impossible, because there is no other Elevation of the Arms of the Index that will exhibit the Appearance of the Object, but the one above-mentioned.

20. If G H D were a Prism of Glass, as that is a denser Body than Water, so its refractive Power will be greater, and consequently it will act more strongly upon the Ray K M at it Exit into the Air, and cause it to be refracted farther from the Perpendicular I K or M L. Therefore the Angle of Incidence out of Air into Glass, *viz.* the Angle F K I, ought to be greater, and so to require a greater Elevation of the Legs of the Index than before in the Prism of Water: And this we find by Experiment is the Case; for then the Elevation, instead of 12°, must be about 22° or 23°.

21. Hence 'tis plain, the Sine of Incidence F S must be in a constant Ratio to the Sine of Refraction A R; because, since the Angle A K L is invariable, (being always equal to G D O) and in the same Medium G D H, the Angle F K I must always be the same, because the refractive Power is every where so; therefore, the Angles being constant, the Sines will be so too, or their Ratio to each other always the same.

22. As by this Instrument the Angles of Incidence and Refraction are discover'd, the Ratio of their Sines will be known of course, for each respective Medium. Thus in *Water* the Sine of 42° is to the Sine of 30° as 4 to 3 very nearly; and in *Glass* the Sine of 46° is to the Sine of 30° as 3 to 2, or more nearly as 17 to 11. By some Experiments it has been found, that the Sine of Incidence is to the Sine of Refraction in *Diamond* as 5 to 2.

23. But since in Physical Matters we have no Authority comparable to Sir *Isaac Newton*, I shall here give a Table (from his *Optics*) of the Proportion of the Sines of Incidence and Refraction of Yellow Light (that being nearly a Mean between the greatest and least refrangible Rays, as we shall see farther on). This will be contain'd in the first Column; the second expresses the Densities of the Bodies estimated by their

colour'd Image of the Sun XY, which is, perhaps, one of the most surprizing and agreeable Spectacles of Nature.

Specific Gravities; and the third the refractive Power of each Body in respect of its Density.

24. *The Refracting Body.*	*Proportion of the Sines.*	*The Density.*	*Ref. Power*
Air —— ——	3201 to 3200	0,0012	5208
Glass of Antimony	17 to 9	5,2800	4864
A Pseudo-Topaz	23 to 14	4,2700	3979
A Selenites ——	61 to 41	2,2520	5386
Common Glass	31 to 20	2,5800	5436
Crystal of the Rock	25 to 16	2,6500	5450
Island Crystal	5 to 3	2,7200	6536
Sal Gemmæ ——	17 to 11	2,1430	6477
Alum —— ——	35 to 24	1,7140	6570
Borax —— ——	22 to 15	1,7140	6716
Nitre —— ——	32 to 21	1,9000	7079
Dantzick Vitriol	303 to 200	1,7150	7551
Oil of Vitriol	10 to 7	1,7000	6124
Rain-Water ——	529 to 396	1,000	7854
Gum *Arabic*	31 to 21	1,3750	8574
Spirit of Wine rectified	100 to 73	0,8660	10121
Camphire ——	3 to 2	0,9960	12551
Oil Olive ——	22 to 15	0,9130	12607
Linseed Oil ——	40 to 27	0,9320	12819
Spirit of Turpentine	25 to 17	0,8740	13222
Amber ——	14 to 9	1,0400	13654
A Diamond ——	100 to 41	3,4000	14556

25. The Refraction of the Air in this Table is determined by that of the Atmosphere observed by Astronomers; for if Light pass thro' many refracting Substances, or Mediums, gradually denser and denser, and terminated with parallel Surfaces, the Sum of all the Refractions will be equal to the single Refraction it would have suffer'd in passing immediately out of the first Medium into the last; because the emergent Ray will be parallel to the incident one in every Medium singly (by *Art.* 9.) if they were separated; and their being contiguous can make no Alteration. Hence, if AA be the Medium of Air interceding two different Media, as BB of Water, and CC of Glass; then the emergent Ray *ei* out

Plate XXXVI. Fig. 7.

THE

THE several Sorts of Rays, after they are refracted, appear in their own proper Colours in Order as follows, *viz.* Those which are least re-

of the Water is parallel to the incident Ray *ac*, and the emergent Ray *lo* out of Glass is parallel to the incident Ray *ei*; whence 'tis plain, the Refraction of the Ray *il* is the same as if the two Media BB and CC were contiguous, the Ray *ei* in that case being lost, which makes no Difference.

26. Hence, if the Sine of Incidence out of Air into Water be as $(ab:de)$ I : R, and that of Incidence to the Sine of Refraction out of Air into Glass as $(no:ik)$ $I:R$; then $no = \frac{ik \times I}{R} = ab = \frac{de \times I}{R}$; whence $R \times I \times ik = R \times I \times de$; but when the two Media BB and CC are contiguous, $de = cf$ will be the Sine of Incidence out of Water into Glass, and $ik = ml$ the Sine of Refraction; therefore $cf : ml :: R \times I : R \times I ::$ the Sine of Incidence out of Water : to the Sine of Refraction in Glass.

27. I cannot here omit to mention the accurate Method which was made use of by Mr. *Hawksbee*, at the Appointment of the Royal Society, to determine the refractive Power of the Air, which was thus: He made choice of a distinct erect Object P, at the Distance of 2588 Feet; a Prism ABC was exhausted of its Air, and applied to the End of a 10 Foot Telescope with a Hair in its Focus. The Object was then view'd thro' the Vacuum by the Ray PES; then admitting the Air into the Prism, the Object was seen to rise above the Hair gradually, as the Air entered; in the End, the Hair was found to hide a Mark in the Object $10\frac{1}{4}$ Feet below the Mark, as at P, so that PM $= 10\frac{1}{4}$ Feet. Fig. 8.

28. This done, the Condenser was applied, and one Atmosphere injected into the Prism, which was applied to the Telescope, as before, and letting out the Air, the Object was seen to descend thro' the same Space of $10\frac{1}{4}$ Feet. Now since the Radius PI $= 2588$, and PM $= 10{,}25$, we shall find the Angle PIM $= 68''$; the half of which gives $34''$ for the Angle QDI, which taken from the QDK or QBD ($= 32° =$ half the Angle of the Prism) gives the Angle KDI or LDS $= 31°\ 59'\ 26''$, and so the Sine of the Angle of Incidence in *Vacuo* (32°) is to the Sine of the Angle of Refraction into Air (31° 59′ 26″) as 1000000 to 999736. (See Mr. *Hawksbee*'s own Figures in *Plate* 36. *Fig.* 7, 8, 9.)

29. In order to understand the Difference between the true

fracted, or fall nearest the Perpendicular P E, are *Red*, and make the red Part of the *Spectrum* at R; the next are the *Orange* at O, the *Yellow* at

and apparent Places of Objects, seen thro' a Medium of different Density from the Air, let the Scheme be constructed as in the Figure, where the Sines of Incidence and Refraction are H L and G I; and these are in a given Ratio of A to B, that is, H L : G I :: A : B; but because of parallel Lines N Q K C, we have H L = N K; therefore N K : I G :: A : B :: N C : I C = H C; but N C : H C :: C E : C M, because P E is parallel to K D, therefore C E : C M :: A : B.

Plate XXXVIII.

30. Now since the Ray E C coming from an Object at E is refracted in the Air into the Ray H C; if H C be continued to F, the apparent Place of the Object will be in the refracted Ray at M in the Perpendicular E P, and projected to F on the horizontal Plane O R, but the Point M will always be the visible Place of the Image; therefore when the Angle C E O is indefinitely small, or the Point C coincides with O, the Lines C E and C M will become O E and O M; and in that case, O E : O M :: A : B :: 4 : 3, in Water. Whence 'tis evident that the apparent Place of an Object immersed in Water, and view'd in the Perpendicular, will be at $\frac{3}{4}$ of the Depth of the Water.

31. But if the Medium be Glass, then O E : O M :: 3 : 2, or more nearly as 17 to 11; so that $\frac{11}{17}$ O E = O M, or the apparent Place of an Object seen thro' a Medium of Glass, will be at the Distance of $\frac{11}{17}$ of the Thickness of the Glass O E. In Diamond, it would be at the Depth of $\frac{2}{5}$ of the Thickness, and so on for all the other Bodies mention'd in the foregoing Table.

32. On the other Hand, as the Point C recedes from the Point O, the Angle C E O which is equal to the Angle of Incidence E C D, becomes greater, and therefore also the Angle of Refraction H C K, or the refracted Ray H C will have a greater Inclination to the horizontal Line A B, and therefore also C F; on which Account 'tis evident the Distance of the apparent Place of the Object, *viz.* the Line O M, will decrease, and of Course the Object will seem to rise in the Perpendicular. And when the Angle C E O is so great, that the Ray C H is refracted parallel to the Horizon, or becomes coincident with A C, then C M will become C O, and the Object at the Bottom at E will appear on the Surface of the Medium at O.

Y,

Y, the *Green* at G, the *Blue* at B, the *Indigo* at I, and the *Violet* at V: And these Seven are all the *original simple Colours in Nature*; and of

33. In this Case, if the Medium be Water, we have CE : CO :: 4 : 3, whence we shall find OE = 2,65 nearly; therefore in any Vessel, whose Width is 2 CO = 6, and Depth OE = 2,65, when fill'd with Water, any Object placed at the Bottom, when view'd in the Perpendicular, will appear raised from E to M, $\frac{1}{4}$ of the Depth; and as the Eye recedes from the Perpendicular to the horizontal Line AC, the Object will appear to rise from M to the Surface of the Fluid at O; all which may be confirm'd by pouring Water into a common Tea-Dish, or Bason, and viewing the Flower, &c. painted at the Bottom.

34. Hence appears the Reason why a strait Stick, as NCE, when placed with one Part CE in Water, will always appear crooked, *viz.* in the Form NCM, the Part CE being raised by Refraction into the apparent Situation CM; and the Part under Water will always appear shorter, for EC will be contracted into CM. All which is known by common Experience.

35. Also, since EM the Difference between the true and apparent Place of Objects, seen thro' a Medium, is always greater in Proportion to the Depth OE, and the Obliquity of the Rays refracted to the Eye, it will follow, that any circular Body immersed in Water, in a Position perpendicular or inclining to the Horizon, will suffer a greater Refraction of Rays from the lower Parts, than from those above; and consequently the lowermost Semicircle will put on the Appearance of a Semi-ellipsis; and also the upper one, but not so much so, the Refraction being less than below. The Consequence of which is, that the Circle thus view'd in the Medium will appear elliptical, as having its vertical Diameter shorten'd by the Refraction; whereas the horizontal Diameter will remain of the same Length, being only raised apparently above its real Situation, whence the Reason of the Figure of the *horizontal Sun and Moon* above-mention'd.

36. From what has been said, 'tis easy to understand, that when the Ray EC in the Medium is refracted into the Air nearly parallel or coincident with the Horizon AC, in which Case (if the Medium be Water) the Line CE = AC being Radius, we have the following Analogy; As 4 is to 3, so is Radius AC or CE to the Sine of the Angle of Refraction

 which,

which, by various Mixtures, all others are compounded, in the common Refractions and Reflections from natural Bodies. (CXVIII).

CO or DE, which Angle is therefore nearly 48°; I say, 'tis easy to understand, that if the Ray of Light EC fall on the Surface of the Medium with a greater Obliquity than what is here specified, that is, so as to make the Angle ECD greater than 48°, the Ray will be wholly reflected back again to the lower Surface, and none will go out into the Air at either Surface of the Medium.

37. Again, if the Medium be Glass, since the Sines of Incidence and Refraction in that Case are as 11 to 17, the Angle ECD will be about 41°, when the refracted Ray CH becomes coincident with the horizontal Line AC; and therefore when the Angle is greater, the Light will be wholly reflected from one Surface of the Glass to the other; and never let out into the Air; whence it follows, that tho' the Particles of Matter in Bodies be in themselves transparent, yet if they are so disposed one among another as to reflect the Light very obliquely, 'tis plain, the Light in such a Case will be lost by various Reflections within the Body, and thus prove a Cause of the Body's Opacity.

(CXVIII.) 1. This different Refrangibility of the Sun's Light proceeds from hence, that the Particles of Light are of different Degrees of Magnitude; for if any Power act upon a Body, so as to give it a particular Determination or Direction of Motion, that Determination or Direction of the Body's Motion will always be the same, while the Energy of the Power and the Quantity of Matter remain the same, and will be variable in Proportion as either of these is so.

2. But the refracting Power of the Medium will be always the same while it is homogeneous or all of one Sort of Matter, therefore when a Ray of Light passes thro' a Substance of Water, Glass, Crystal, &c. and a different Direction of Motion is thereby communicated to different Parts of the Ray, it follows, that the Particles which constitute those Rays, which have a different Direction, must be among themselves unequal in Quantity of Matter, and consequently in Bulk; and since the Quantity of Motion is in the Ratio of the Bulk and Velocity, (in this Case) 'tis plain, the greater the Velocity is, the less will be the Bulk; and therefore those Rays of Light which suffer the greatest Refraction are less in Bulk or Magnitude than others which

SINCE

Since a Lens does, in the manner of a Prism, more or less separate the Rays of Light passing through it, it follows, that all the several Sorts of Rays will have their proper *Focus's*, or be convened to so many different Points in the Axis of

are not so much refracted, the greater Particles being not so much subject to the Power of the Glass; as a large Needle is not so easily moved by a Loadstone, nor at so great a Distance.

3. This being the Case, 'tis easy to be understood, that when a Beam of Light, as H F, is let into a dark Room, thro' a Hole in the Window-Shutter, and is made to fall on a Prism A C B at F, it will be attracted by the Surface of the Glass at F in a perpendicular Direction, and cause the several Particles to deviate from their right-lined Course to T, (which they before had) and decline towards the Pependicular *ab*, that is, towards the Part F *a* within the Glass; which Deviation or Refraction will be greater in Proportion as the Particles of Light are smaller.

4. Hence the several Particles of Light will proceed from the Side A C to the Side D C in different Directions; where, when they arrive, and go out again into the Air, they will be again affected by the same attracting Power of the Glass, which will here produce the same Effect as before, that is, it will cause each Sort of Ray to incline towards the Side of the Glass, and consequently to be refracted from the Directions they severally had in the Glass, and from the Perpendicular P E.

5. Thus those Rays, whose Particles are largest, will deviate least from the Perpendicular, and will therefore go to R, and make the lowest Part of the colour'd *Spectrum*, and these will appear of a *Red Colour*. The Particles next less in Magnitude will be somewhat more refracted, and will go to O, and be of an *Orange Colour*; the next Size less will be still more refracted, and appear *Yellow* at Y, and thus the Refraction will proceed in the *Green* at G, the *Blue* at B, the *Indigo* at I, and the *Violet-colour'd Rays* at V; which as they are most refracted, are thereby proved to be the least of all in Magnitude.

6. I shall now proceed to shew, since the Sun's Light is variously refrangible, what the particular Degree of Refraction is which every Species of Rays undergoes, and the Sines of those Angles respectively. In order to this it must be con-

the Lens, and not all to one Point only, as is necessary for a perfect and uniform Representation of the Image of any Object: For the Red Rays proceeding from the Object will be converged to a *Focus* at a greater Distance from the Lens, than

sider'd, that the Sine of Incidence is the same in all; and that when the Incidence is such as that the Ray FK, upon the first Refraction, shall pass in the Direction parallel to the upper Side of the Prism GH, the Refractions made at each Side of the Prism are equal, and equal to the refracting Angle of the Prism GDH; all which is evident from what was demonstrated in *Annot*. CXVI.

Plate XXXVII. Fig. 5.

7. Also it is known by Experience, that when the Prism ABC is held with its Axis perpendicular to the Sun-Beam, and then turn'd round upon its Axis, the Image or colour'd *Spectrum* will first descend to a certain Limit, where it will become stationary, and then ascend to the same Place as at first; whence it appears plain, that since the Altitude of the Image above the Place where the Sun-Beam would fall, were the Prism away, is owing to the Sum of the Refractions made at each Side of the Prism, while the Image descends the Sum of these Refractions decrease, and when the Image ascends the said Sum must increase.

8. Consequently, since the Image falls twice upon the same Place in one Rotation of the Prism, there are two Positions of the Prism wherein the Sum of the Refractions at its Sides are equal; and these happen when the Angles of the incident Beam HDL and ODL are such as will cause the refracted Parts DG and DF to be equally inclined to the Sides of the Prism, but contrary Ways; that is, so as to make the Angle DGB = BDF, and GDB = DFB, and therefore the Triangles DBG and DBF equi-angular. For in the Position of the Ray HDG the Refractions at the Angles D and G are respectively equal to the Angles E and D in the other Situation of the Ray QDE; and therefore the Sum of the Refractions on each Side in each Case must be equal, and cause the Image to appear twice in the same Place.

Fig. 9.

9. While the unequal Refractions at each Side the Prism, at D and G, or D and F, are approaching towards Equality, the refracted Ray DG or DF is continually approximating to the Situation DE; where when it arrives, the Angles at D and E being then equal, the Refractions at each Side will be

the

the Indigo or Violet Rays; and so the Image will be colour'd and confused in every Point between those Extremes, except just in the middle Point, where the several Sorts of Rays all intersect each other, and exhibit the Image tolerably distinct

equal also, and the Image in that Case be brought to its Limit or lowest Site. Then R D will be the incident Ray, and E P the emergent one.

10. Produce R D and E P till they intersect each at I, and any horizontal Line in M and N; then let the Angle R M N be the Altitude of the Sun, and P N M that of the *Spectrum* at P; which Angles are easily measured with a Quadrant. Their Sum is equal to the external Angle P I M, which is again equal to the two internal Angles of Refraction I D Q and I E Q; and, by what has been now shewn, $IDQ = IEQ = QDK$; wherefore $QDK = DBK = \frac{1}{2}\overline{N+M}$. Hence $\frac{1}{2}\overline{N+M} + IDQ = IDK$ or R D L, the Angle of Incidence.

11. We shall give Sir *Isaac Newton*'s Example in this Affair. The refracting Angle of his Prism was $ABC = 62°\ 30'$, the Half of which is 31° 15', whose Sine is 5188, the Radius being 10000. When the *Spectrum* was in its Limit, or stationary, he observed with a Quadrant the Angle P N M of a mean refrangible Ray E P, that is, of one that went to the Middle of the colour'd Image at P; and by adding this to the Angle R M N of the Sun's Altitude taken at the same Time, he obtain'd the Angle P I M to be 44° 40'; whose Half 22° 20', added to Half the Angle of Refraction 31° 15', makes the Angle of Incidence $RDL = 53°\ 35'$, whose Sine is 8047. The Sine of Incidence, therefore, is to the Sine of Refraction of a mean refrangible Ray, or that of Yellow Light, as 8047 to 5188, which is as 31 to 20. (See the Table, *Annot.* CXVII. 24.)

12. If there were but one Sort of Light, it then would be equally refracted, and the Image of the Sun would not then be long, but round; and if the Rays were first received by a Convex Lens, they would all pass to its Focus, and there represent the Sun's Image very distinctly in a circular Spot, which Image would subtend the same Angle at the Lens as the Sun itself does, or half a Degree, at a Mean. All this will be demonstrated hereafter.

and

and colourless. To this different Refrangibility of the Rays is owing the Imperfection of the common *refracting* Telescope, as will be but too easy to experiment.

13. If these Rays, after having pass'd through the Lens, were received by a Prism, since the Sum of Refractions at the Sides of the Prism are equal, (as we have shewn they are when the Image is stationary, *Art.* 8.) the Rays will have the same Inclination to each other after Refraction through the Prism as before; whence the Angle is not changed, but gives the Image of the Sun still equal to 30′. But to illustrate this, let M N be the Section of the Window-Shutter in a dark Room, in which, through a Hole O, a Pencil of Rays KOL is transmitted to the Lens KL; which would converge them to a Focus at H, were they not intercepted by the Interposition of the Prism ABC, by which means they are refracted to I. And since the Sum of the Refractions at E and D is equal to that at F and G, the Angle FIG will be equal to the Angle FHG; and if the Sun were but a Point, its Image at H and I would be a Point also.

Plate XXXIX. Fig. 1.

14. But since the Sun has the apparent Magnitude of 30′, let the Angle MQN be the Angle under which the Sun appears; that is, let MQ be a Ray coming from the upper Limb of the Sun, and NQ another from the lower Limb. These crossing each other in the Center of the Lens KL, at Q, make the Angle DQE = MQN; nor is this Angle alter'd by the Refractions through the Prism, as being equal on each Side; therefore the Image at I will be subtended under an Angle of 30 Minutes.

Fig. 2.

15. And since this will be the Case of every Sort of Rays contain'd in the Sun's Light, if that which we have been considering be a mean refrangible Ray, then the least refrangible Rays will form an Image in like manner at R, the most refrangible Rays another at P, and the intermediate Rays their several Images respectively. So that the colour'd *Spectrum* PR consists of as many circular Areas as there are different Sorts of Rays; and is every where of an equal Breadth, *viz.* half a Degree.

16. Now 'tis evident, that if the Sun be supposed a Point, each of those Circles, being the Images of the Sun and similar to it, must also be contracted into a Point, and so the colour'd *Spectrum* PIR would in that Case have no Breadth; and its Length would decrease at each End by the Semidia-

HENCE

HENCE also Objects of any of the simple Colours, though contiguous to each other, yet, if view'd through a Prism, appear separated, and at a distance from one another: And those Objects

meter of the Circles P and R, and therefore would subtend an Angle of 30′ less than it now does.

17. In order to determine the Angles of Refraction of the least and most refrangible Rays, we must first determine the Angle PIR, which the Image PR subtends at the Distance it is form'd from the Prism ABC. The Sun being supposed a Point, let SD be the incident Ray, which continue out to V; and let LDK be perpendicular to the Side AB in the Point of Incidence D. The Ray SD at its first Refraction is diffused through the Space GDF within the Prism; DG is the least refrangible Ray, DF the greatest, and DE (parallel to AC) the mean refrangible Ray. The Sine of the Angle of mean Refraction EDK to that of Incidence IDK has been already shewn to be as 5188 to 8047, or as 20 to 31. Plate XXXVII. Fig. 10.

18. We are now to find the Quantity of the Angles GDK and FDK. Since each Ray will suffer the same Degree of Refraction at the second Surface as at the first, very nearly; let the refracted Rays FP, ET, GR, be produced, and they will intersect each other in the Point I, making IF, IE, IG severally very nearly equal to DI. and therefore the Angles $IFD = IDF$, and $IGD = IDG$; therefore $PIV = 2FDV$, and $RIV = 2GDV$. Hence $PIV - RIV = PIR = 2FDV - 2GDV$; consequently, $\frac{1}{2}PIR = FDV - GDV = FDG$.

19. The Angle PIR is discover'd by measuring the Length of the Image PR, and its Distance from the Prism ABC. This Sir *Isaac Newton* has done with great Exactness. The refracting Angle of his Prism was $ABC = 62° 30'$, the Distance of the *Spectrum* $18\frac{1}{2}$ Feet, the Length $9\frac{3}{4}$ or 10 Inches, the Breadth $2\frac{1}{8}$ Inches. This subducted from the Length leaves $7\frac{3}{4}$ for the Length of the Image were the Sun but a Point, and therefore subtends the Angle which the most and least refrangible Rays PF and RG do contain with one another after their Emergence from the Prism.

20. But at the Distance of 18,5 Feet, the Length $7\frac{3}{4}$ Inches is the Chord of an Arch equal to $2° 0' 7'' = PIR$; therefore $\frac{1}{2}PIR = FDG = 1° 0' 3\frac{1}{2}''$; whence EDG $(= \frac{1}{2}FDG) = 0° 30' 2'' = FDE$. But the Angle $EDK = DBK = 31° 15' 00''$. Wherefore $EDK + EDG =$

will

will have their Images form'd by a Lens at very different Distances in its Axis, especially in Ex-

$31^\circ\ 45'\ 2'' = $ GDK, the Angle of Refraction of the least refrangible Rays; and EDK — EDF $= 30^\circ\ 44'\ 58'' =$ EDK, the Angle of Refraction of the most refrangible Rays.

21. The natural Sine of $31^\circ\ 45'\ 2''$ is 5262, (as *per* Table, *Annot.* XLVI.) also the Sine of $30^\circ\ 44'\ 58''$ is 5112. The common Sine of Incidence being IDK or SDL $= 53^\circ\ 35'$, and Sine 8047; this compared with the Sines of Refraction of the most, mean, and least refrangible Rays will stand as follows.

The Sine of Incidence is to the Sine of	The most refrangible Rays FD, as 8047 to 5112.
	The mean refrangible Rays ED, as 8047 to 5188.
	The least refrangible Rays GD, as 8047 to 5262.

22. I have hitherto consider'd the Refraction made out of Air into Glass, after the common Way. But as Sir *Isaac Newton* has proceeded in a contrary Method, and stated the Proportions of the Sines of Refraction (as they are out of Glass into Air) to the common Sine of Incidence in Glass, I shall for the future follow his Steps; and therefore supposing a Beam of common Light within the Prism, as DE, shall consider its Refraction into the Air at the Side BC in the Point E. Plate XXXVII. Fig. 4. The common Sine of the Angle of Incidence KED or IEL, $= 31^\circ\ 15'$, was found to be 5188; and the Angle PER, $= 1^\circ\ 0'\ 3\frac{1}{2}''$, the same as before. Also the Angle of the mean refrangible Rays TEL being $53^\circ\ 35'$, we have the Angle of the least refrangible Rays REL $= 53^\circ\ 4'\ 58''$, and the Angle of the most refrangible Rays PEL $= 54^\circ\ 5'\ 2''$. The Sines of these Angles are 7995 and 8099; the Sine of Incidence, therefore, and of Refraction into Air, in the least and most refrangible Rays, are in the least round Numbers as 50 to 77 and 78.

23. Now if you subduct the common Sine of Incidence 50 from the Sines of Refraction 77 and 78, the Remainders 27 and 28 shew, that in small Refractions, the Refraction of the least refrangible Rays is to that of the most refrangible as 27 to 28 very nearly; and that the Difference of the Refractions of the least and most refrangible Rays is about the $27\frac{1}{2}$ Part of the Refraction of the mean refrangible Rays.

periments

periments of the deepest *Red*, and *Violet*, or *Blue Colours*; as a Card painted half with *Carmine*, and

24. Now in order to define the Refrangibility of the several intermediate Rays of Light, Sir *Isaac* took the following Method. He caused the *Spectrum* to be well defined, and delineated upon Paper its Perimeter, as FAPGMT; this he held in such a Manner that the *Spectrum* might fall upon and exactly agree with the delineated Figure; this done, an Assistant drew the Lines *ab*, *cd*, *ef*, &c. across the Figure very nicely upon the Confines of the several Colours, that is, of the Red *M ab* F, of the Orange *abcd*, of the Yellow *cdef*, and so of the rest; which Operation being divers times repeated, he found the Observations agreed very well, and that the Divisions made by the cross Lines were those of a *Musical Chord*. Plate XXXIX. Fig. 3.

25. That is, if GM be produced to X, so that it be GM = MX, then the Line XM = $\frac{1}{2}$ XG will be the *Octave*. The Line *a*X : XG :: 9 : 16; therefore *a*X will be the *Lesser Seventh*. The Line *c*X will be $\frac{3}{5}$ of XG, and therefore the *Sixth Greater*. *e*X will be $\frac{2}{3}$ of XG, which is the *Fifth*. *g*X is $\frac{3}{4}$ of XG, a *Fourth*. *i*X is $\frac{5}{6}$ of XG, a *Third Lesser*. *l*X is $\frac{8}{9}$ of XG, the *Second Greater*. So nicely has Nature observ'd an harmonical Distribution of Colours in the *Solar Spectrum*.

26. If then the Difference between the Sines of 77 and 78 be in like manner divided, that is, as the Line MG is divided, we shall have the Sines of Refraction in the several Red Rays extend from M to *a*, or from 77 to $77\frac{1}{8}$; those of the Orange Colour from *a* to *c*, or from $77\frac{1}{8}$ to $77\frac{1}{5}$; those of the Yellow from $77\frac{1}{5}$ at *c*, to $77\frac{1}{3}$ at *e*; those of the Green from $77\frac{1}{3}$ to $77\frac{1}{2}$ at *g*; those of the Blue from $77\frac{1}{2}$ to $77\frac{2}{3}$ at *i*; those of the Indigo from $77\frac{2}{3}$ to $77\frac{2}{9}$ at *l*; and from thence the Violet to 78 at G.

27. This Discovery of the *Harmonic Proportion* of Colours in the Sun's Light has suggested the curious Hint or Idea of a *Visual Music* by means of an *Ocular Harpsichord*, which shall entertain the Eye with the Succession of harmonic Colours, as the common Harpsichord does the Ear with musical Sounds. Yea, some have carried this Matter so far, as actually to attempt the making of such a Harpsichord, with full Assurance of being able to play Tunes to the Eyes. It were greatly to be wish'd this Chromatic Music could be made as effectual to give Pleasure to our Eyes, as common Music does to the Ears. We should then have Harmony the Subject of two of

half with *Ultramarine*, made deeper with a little *Indigo*. (CXIX).

our Senses: And who can tell but we may have *musical Eyes*, as well as *musical Ears*, could they be exercised by proper Objects? Nay, who can tell what may be the Consequence of this Discovery in regard of our other Senses in the Ages to come?

Plate XXXIX. Fig. 4. (CXIX) 1. Let DEKI be a double Convex Lens, N its Center, and ND the Radius of Convexity at D; FV its Axis; and HE a Beam of the Sun's Light incident on the Lens parallel to its Axis in the Point F. Let ABC be a Prism touching the Lens in the Points E and D, and it is evident the Law and Manner of Refraction of the Beam at E will be the same, whether we consider it as made through the solid Glass Prism ABC, or through the Lens DEK, because the Point of Incidence E is the same in or common to them both.

2. The Beam being refracted to D, it is plain the Refraction will be there also made into the Air in the same Manner from the Lens as from the Prism, supposing them to touch in the Point D. Let ND be continued to L, then will LD be perpendicular to the Lens in D; and the Refraction being made into the Air, (a rarer Medium) the refracted Rays will tend towards the Axis, and meet it sooner or later as they are more or less refrangible. Thus the most refrangible Rays DW will cut the Axis in G, the least refrangible Rays DT in Q, and the mean refrangible Rays in O; and the others in the intermediate Space between O and G, and O and Q. The same is to be understood of the Beam IK on the other Side the Axis.

3. Hence we see, that in the Axis of the Lens the Images of an Object will be form'd in several Parts from G to Q; by which means the Object will appear *Red* at G, *Violet-colour'd* at Q, and of other Hues in the Parts between. Nor are we to understand that seven Images only are form'd by the seven Sorts of Rays; but each particular Sort of Ray, according to the Intensity of the Colour, from the strongest to the faintest Part, consists of an indefinite Number of differently refrangible Rays, each of which will form an Image of the Object in its proper Focus: And therefore we may conceive as many Images form'd in the Space from G to Q, as there are Points in the Line GQ.

4. The Object seen by such a Refraction of Rays, in such an Infinity of Images, must necessarily appear very indistinct,

SIR *Isaac Newton* found, by a very curious and convincing Experiment, that the Rays of Light were as variously reflexible as refrangible; and that those which were most or least refrangible were also most or least reflexible: And farther, that Rays of Light were not reflected by impinging on the solid Parts or Corpuscles of Bodies,

confused, colour'd, and obscure; and the Object-Glass of every common Dioptric Telescope being of this Sort, is the Occasion why they will not bear an Eye-Glass of so deep a Charge, or so short a focal Distance as is requisite for great Degrees of magnifying. This put Sir *Isaac* upon inventing another Sort of Telescope by Reflection, of which we shall speak largely hereafter.

5. Suppose DE parallel to the Axis of the Lens, and produced to Z; then is $ZDL = EDN$ the Angle of Incidence, and PDL, ODL, MDL, the Angles of Refraction in the least, mean, and most refrangible Rays; and consequently the Angles ZDP, ZDO, and ZDM will shew the Quantity of Deviation or respective Refraction of those Rays from the first Direction EZ. Whence $ZP : ZN :: 27 : 28$; and $ZP : ZO :: 27 : 27\frac{1}{2}$. Also $PM = \frac{1}{27\frac{1}{2}} ZO$, the whole Refraction of the mean refrangible Rays.

6. But the Angle $ZDP = DQX$, and $ZDO = DOX$, and $ZDM = DGX$. Now the Sines of the Angles DQX, and DOX or DOQ, are as their opposite Sides OD and QD, that is, nearly as XO and XQ. For the same Reason the Sines of the Angles DOX and DGX are nearly as GX and OX. Wherefore ZP, ZO, ZM, are as GX, OX, and QX; whence $PM : ZM :: QG : QX$; therefore $GQ = \frac{1}{28} QX$. But QO is nearly equal to OG, when QX is very great; and $QO : QX :: PM : DY :: 1 : 56$, because $QX = 28\, GQ$, or $56\, QO$.

7. Or thus more accurately, without regard to the focal Distance QX or OX. Let I, L, G, be as the Sines of Incidence, and of the least and greatest Refraction, or as the Numbers 50, 77, 78; (See *Annotat.* CXVII. 22.) then will $ZP = L - I$, $ZM = G - I$, and $PM = G - L$; whence $PM : ZP :: G - L : L - I$; and doubling the Consequents, we have $PM : 2\,ZP\ (= DY - PM) :: G - L : 2L - 2I$.

and

and rebounding from thence like a *Tennis-Ball*, but from ſome other Principle depending on the Size of the Particles of Light, and the Thickneſs or Denſity of the Particles of the Body reflecting it, which are all of them, in the moſt opake Bodies, tranſparent in themſelves, as is eaſy to be ſhewn in the thin *Lamellæ* or Plates, of which an

Then conjointly, PM : DY :: G — L : G + L — 2I :: 78 — 77 : 78 + 77 — 100 :: 1 : 55; or PM $= \frac{1}{55}$ DY, the Aperture of the Glaſs.

8. From hence it appears, that the Ratio between PM and DY is conſtant, or always the ſame, whatever be the focal Diſtance of the Lens. It is alſo very evident, that PM is the Diameter of a Circle, in which will be a Mixture of every Sort of Rays, from the leaſt to the moſt refrangible. This Circle therefore is that in which the Light is *white*, or not tinctur'd with the Colour of any particular Sort of Rays; for the Rays being here promiſcuouſly thrown together, the Light compounded of them muſt be nearly the ſame with that of the Beam before Refraction.

9. By the ſame Rule we may find the Diameter of the leaſt Circle that receives the Rays of any ſingle Colour, or of any contiguous Colours. Thus all the *Yellow* is contained in a Circle whoſe Diameter is a 409th Part of the Breadth of the Aperture of the Glaſs, (which we ſuppoſe a *Plano-Convex*, becauſe of DE parallel to the Axis) for in this Caſe G $= 77\frac{1}{5}$, L $= 77\frac{1}{3}$, and I $= 50$. (See *Annot*. CXVII. 26.) Whence by the Analogy we have PM : DY :: G — L : G + L — 2I :: 0,133 : 54,533 :: 1 : 409. Thus for two contiguous Colours, the *Orange* and *Yellow*; the Sines on each Side being $77\frac{1}{8}$, $77\frac{1}{5}$, give the Diameter of the Circle in which both theſe colour'd Rays are contain'd, a 260th Part of DY.

10. From hence it is plain, that when the Sun's Rays are received upon a large and very convex Lens, the conic Superficies of the converging Rays DPMY will conſiſt of the Red-colour'd Rays; and if received on a white Paper, held perpendicular to the Axis, the Circumference of the circular Section or Area will be remarkably tinged with a reddiſh Colour inclining to Orange, by the leaſt refrangible Rays DP and YM. On the contrary, the diverging Rays will, in the conic Surface RPMW, have all the Violet and Indigo Rays NPR and DMW, and will therefore exhibit ſuch a colour'd Circle

Oyſter-

Oyster-Shell doth consist.

IT will be thought very strange to assert, that a rare Medium is more impervious to the Rays of Light than a denser one; and yet nothing is more certain, or easier proved by Experiment: For Example, a Beam of Light is much more copiously reflected from the second Surface of a Piece of Glass when contiguous to the *Air*, than when

about the Light received on the Paper held any where in that Cone of Rays.

11. Since G is the Focus of Violet Rays, that is the Place where any Body of a Violet Colour will be seen distinctly, because the Rays of that Kind, passing from that Point to the Lens, will after Refraction pass parallel to the Eye; which is a Condition absolutely necessary to distinct Vision, as will appear hereafter. For the same Reason Q will be the Focus or Place where Objects of a red Colour will be most distinctly seen. Whence it appears, that in viewing Objects through Glasses (as *Spectacles* for Instance) the Distance of the Glass from the Object will be variable according to its different Colour.

12. Hence a various-colour'd Object ABEF will have its Image form'd in Parts by the Lens HI. Thus suppose ABCD be a Red Part, and DCEF a deep Blue; if this Object be well illumined, and black Threads or Silks laid across those Colours, they will appear distinctly in their respective Focus's, *viz* the red Part ABCD will have its Image distinctly form'd at K, and the blue Part at L; the former will be represented by *abcd*, the latter by *dcef*; and these Images will be at very different Distances from the Lens. Thus if the Lens HI be of 3 Feet *focal Distance*, and the Object be placed at the Distance of 6 Feet from it, the Images on the other Side, at the Distance of 6 Feet, will be form'd *one Inch and a half* from each other; that is, the Red at K will be $1\frac{1}{2}$ Inch beyond the Blue at L. Plate XXXIX. Fig. 5.

13 Another Consequence of this different Refrangibility of the Rays of Light is, that if two Objects of different Colours, as *Red* and *Blue*, be view'd through a *Prism*, they will be refracted to different Heights; and though they were contiguous before, or Parts of *one and the same Object*, yet will they appear separate, or as two *distinct and distant Objects*.

it touches *Water*; and ſtill more, if contiguous to *Water*, than when it is contiguous to *Glaſs*; in which Caſe the Rays are totally tranſmitted.

HENCE, wonderful as it may ſeem, 'tis neceſſary, in order that a Body may be tranſparent, that its Subſtance ſhould be very denſe, and its Pores very ſmall; and that Opacity reſults chiefly from the Largeneſs of the Pores of a Body, oc-

Plate XXXIX. Fig. 6. Thus ſuppoſe DHEI be an Object whoſe Part DG is intenſely *blue*, and the other Part FE intenſely *red*; if this be view'd by a Priſm BAC*bac*, with the refracting Angle or Edge A*a* upwards and parallel to the Horizonand the Sides DI and HE of the Object, the Image of this Object will appear at de, with the *blue* Part dg refracted higher than the *red* fe. On the contrary, if the refracting Angle of the Priſm be turn'd downwards, the Image will be refracted downwards to *de*, the *blue* Part lower to *dg*, and the *red* higher at *fe*.

14. We alſo ſee the Reaſon why Objects appear differently colour'd when the Eye is held near the Priſm, as at D, to view them; *viz.* becauſe the Rays of every Colour are there ſo very near together, that they can be all received by the Pupil of the Eye, and will therefore paint the Image in all its proper Colours on the *Retina*. Whereas if the Eye be removed to a greater Diſtance from the Priſm, as to *a*, *b*, *c*; there, becauſe the Rays ſpread through ſo wide a Space, but few can enter the Pupil, perhaps only one particular Sort, and then the Object will appear of that particular Colour only; as *Violet-colour'd* at *a*, *Green* or *Yellow* at *b*, and *Red* at *c*.

Fig. 7. 15. The Rays of the Sun's Light, once refracted, undergo no farther Refraction by a ſecond Priſm, and of courſe exhibit no other Colours: For let an Hole be made at *g* in the Board *de*, on which the colour'd *Spectrum* is made in the dark Room, by Rays which come through a Hole G in a Board DE placed juſt before the Priſm ABC; by turning the Priſm ABC ſlowly about its Axis, the Image will be made to move up and down on the Board *de*, by which means each colour'd Ray will paſs ſingly through the Hole *g* ſucceſſively; and if theſe Rays be refracted a ſecond time through the Priſm *abc* placed juſt behind the Hole *g*, they will go from thence to the oppoſite Wall at M or N, and there appear juſt as before in their proper ſimple Colour; the *Blue* will appear *Blue*, the

caſion'd

casion'd by its Particles touching in but very few Points: Because, if the Pores of such a Body be fill'd with a Substance nearly of the same Density, it becomes in some Degree transparent, as *Paper wetted with Water or Oil:* And on the contrary, *Water blown up into small Bubbles* has its Density diminished, and its Porosity increased, and thus becomes opake (CXX).

Red will be still *Red*, and the *Violet* the same *Violet* as before.

16. But though the Rays are not any farther refrangible by the second Prism *abc*; yet it appears that those Rays which were least and most refrangible by the first Prism are likewise so by the second; for the Boards DE and *de* being fix'd, cause the Incidence of Light on the second Prism to be always the same: Yet by moving the first Prism ABC about its Axis, the *Red* Light would go by a second Refraction to M, but the *Violet* Light would go higher to N. Which plainly and undeniably shews that some Sort of Rays will always be more refracted, and are therefore more refrangible than others. And hence this decisive Experiment has gain'd the Title of *Experimentum Crucis*.

(CXX) 1. The same great Author of the Doctrine of the *different Refrangibility* of the Sun's Rays, (as deliver'd in the last *Annotation*) found also by other Experiments, that they were in the same manner *differently reflexible*; or that those Rays which were least and most refrangible were also least and most reflexible. This he proved in the following Manner.

2. From a Hole F in the Window-Shutter EG, a Beam of the Sun's Light FM pass'd to the Base BC of a Prism ABC, whose Angles B and C were equal and half right ones, and the Angle A a right one. The Light was first refracted at M into the diverging Beam MGH, of which MG was the least refrangible Part, and MH that which was most so. MN is the Light reflected from the Base through the other Side to a second Prism VXY, by which the reflected Beam is refracted to *t* and *p*; N*t* being the less, and N*p* the more refracted Part. Plate XXXIX. Fig. 8.

3. When the first Prism ABC is turn'd about its Axis according to the Order of the Letters ABC, the Rays MH

IF the Object-Glaſs of a large Teleſcope be laid with its convex Surface on a plain Glaſs, the Light falling on the thin Portion or Plate of Air contain'd between the Glaſſes will be, at ſeveral

emerge more and more obliquely out of the Priſm, till at length they become reflected towards N. And it was evidently obſerved, that as the Priſm ABC was ſlowly moved about its Axis, all the Rays from MH to MG became ſucceſſively reflected towards N.

4. The Conſequence of this was, that the Violet Colour *p* received an Addition to its Strength and Brightneſs upon the firſt Reflection of the Rays MH, beyond any of the other Colours towards *t*; but as the Priſm ABC continued its Motion, and the other Ray between H and G became reflected, ſo the other Colours from *p* to *t* became more intenſe and vivid, one after another, by the new Acceſſion of Light to the Beam MN.

5. In this Experiment no Notice has been taken of any Refraction made at the Sides of the firſt Priſm ABC, becauſe the Experiment was made in ſuch Circumſtances that the Beam FM enters it perpendicularly at the firſt Side AC, and goes out ſo at the ſecond AB; and therefore can ſuffer no Refraction, or ſo little that the Angles of Incidence at the Baſe are not ſenſibly alter'd by it. In order to this, the Angle FMC ſhould be about 45 Degrees; and then a ſmall Motion of the Priſm, to make the Angle FMC $=$ 49°, will cauſe the Beam FM to begin its Refraction. Or if the Angles B and C were each of them 41°, the Sun-Beam FM making an Angle FMC $=$ 49° will begin to be reflected at the ſame Time that it contains a Right Angle with the Side AC (by *Annot.* CXVII. 37.).

6. The Reaſon of the different Reflexibility of the Rays of Light is the ſame as was before aſſign'd for their different Refrangibility, *viz.* the different Sizes or Magnitudes of the ſeveral Rays; for when the refracted Beam MGH approaches very near the Baſe of the Priſm BC, the attracting Power of the ſaid Baſe will ſooner affect the Particles of a leſſer Size than thoſe of a larger, even though they were at an equal Diſtance from it; and therefore the moſt refrangible Rays MH will be firſt within the reflexive Power of the Surface BC, on account of the greater Tenuity of its Particles, as well as on account of its greater Proximity than the other Rays MG; on both which Accounts therefore the Ray HM

Diſtances

Diſtances from the Centre, alternately tranſmitted and reflected. In the Centre of the Lens, where it touches the Glaſs, it will be tranſmitted, and ſo cauſe a *dark Spot* to appear: At a ſmall Di-

will be firſt and moſt eaſily refracted.

7. Now though in Refraction the Sine of the Angle of Incidence is different from the Sine of the Angle of Refraction, on account of a ſmaller Particle being attracted more out of its Way towards the Perpendicular than a larger one, whereby a Separation of the Rays is produced; yet becauſe in Reflections every Particle whether great or ſmall muſt neceſſarily be reflected under an Angle equal to that of Incidence, it follows, that all the Rays after Reflection will have the ſame Inclination to each other as before, and ſo no Separation can be made among them, and conſequently no different colour'd Light will be produced by a total Reflection of the Sun's Rays.

8. What has been ſaid of the Manner in which Light is reflected is in the groſs only, and true but in part; for though in Reflections the Angle of Incidence be ever equal to the Angle of Reflection, yet the Reflection of the Particles of Light is not made by their impinging on the ſolid or impervious Parts of Bodies, as is commonly believed. This our great Author proves by the following Reaſons.

9. *Firſt*, That in the Paſſage of Light out of Glaſs into Air, there is a Reflection as ſtrong as in its Paſſage out of Air into Glaſs, or rather a little ſtronger, and by many Degrees ſtronger than in its Paſſage out of Glaſs into Water. And it ſeems not probable that Air ſhould have more ſtrongly reflecting Parts than Water or Glaſs: But if that ſhould be ſuppoſed, it will avail nothing; for the Reflection is as ſtrong or ſtronger when the Air is drawn away from the Glaſs by an Air-Pump, as when it is adjacent to it.

10. *Secondly*, If Light in its Paſſage out of Glaſs into Air be incident more obliquely than at an Angle of 40 or 41 Degrees, it is wholly reflected; if leſs obliquely, it is in a great meaſure tranſmitted. Now it is not to be imagined that Light at one Degree of Obliquity ſhould meet with Pores enough in the Air to tranſmit the greater Part of it, and at another Degree of Obliquity ſhould meet with nothing but Parts to reflect it wholly; eſpecially conſidering, that in its Paſſage out of Air into Glaſs, how oblique ſoever be its Incidence, it finds Pores enough in the Glaſs to tranſmit a great Part of it.

ſtance from thence, all around, the Light will be reflected in various-colour'd Rings: In the next Diſtance it will be tranſmitted, and in the next to that reflected; and ſo on alternately to a con-

11. If any Man ſuppoſe that it is not reflected by the Air, but by the outmoſt ſuperficial Parts of the Glaſs, there is ſtill the ſame Difficulty; beſides that ſuch a Suppoſition is unintelligible, and will alſo appear to be falſe by applying Water behind ſome Part of the Glaſs inſtead of Air: For ſo in a convenient Obliquity of the Rays, as of 45 or 46 Degrees, (at which they are all reflected where the Air is adjacent to the Glaſs) they ſhall be in great meaſure tranſmitted where the Water is adjacent to it; which argues that their Reflection or Tranſmiſſion depends on the Conſtitution of the Air and Water behind the Glaſs, and not on the ſtriking of the Rays on the Parts of the Glaſs.

12. *Thirdly*, If the Colours made by a Priſm placed at the Entrance of a Beam of Light into a darken'd Room be ſucceſſively caſt upon a ſecond Priſm placed at a greater Diſtance from the former, in ſuch a manner that they are all alike incident upon it, (as they will be when trajected through the Holes G and *g* in the two Boards mention'd in *Art.* 15. of the laſt Note) the ſecond Priſm may be ſo inclined to the incident Rays, that thoſe which are of a *Blue* Colour ſhall be all reflected by it, and yet thoſe of a *Red* Colour pretty copiouſly tranſmitted. Now if the Reflection be cauſed by the Parts of Air or Glaſs, I would aſk why, at the ſame Obliquity of Incidence, the *Blue* ſhould wholly impinge on thoſe Parts ſo as to be all reflected, and yet the *Red* find Pores enough to be in a great meaſure tranſmitted?

13. *Fourthly*, Where two Glaſſes touch one another, there is no ſenſible Reflection, (as will be ſhewn *Annot.* CXXI. 7.) yet I ſee no Reaſon why the Rays ſhould not impinge on the Parts of Glaſs as much when contiguous to other Glaſs, as when contiguous to Air.

14. *Fifthly*, When the Top of a Water-Bubble, (as will be ſhewn *Annot.* CXXI. 24.) by the continual ſubſiding and exhaling of the Water, grows very thin, there is ſuch a little and almoſt inſenſible Quantity of Light reflected back from it, that it appears intenſely black; whereas round about the black Spot, where the Water is thicker, the Reflection is ſo ſtrong as to make the Water ſeem very white. Nor is it only at the leaſt Thickneſs of thin Plates or Bubbles that there

siderable Distance from the central Spot. If we take the Distances as the Numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. 10, &c. then at the Distances 0, 2,

is no manifest Reflection, but at many other Thicknesses continually greater and greater. And yet in the Superficies of the thinned Body, where it is of any one Thickness, and the Rays are transmitted, there are as many Parts for them to impinge on, as where it is of any other Thickness where the Rays are reflected.

15. *Sixthly*, If Reflection were caused by the Parts of reflecting Bodies, it would be impossible for thin Plates or Bubbles at one and the same Place to reflect the Rays of one Colour, and transmit those of another, as is known by Experiment they do: For it is not to be imagined, that at one Place the Rays, which, for Instance, exhibit a *Blue* Colour, should have the Fortune to dash upon the *Parts*, and those which exhibit a *Red* to hit upon the *Pores* of the Body; and then at another Place, where the Body is a little thicker or a little thinner, that (on the contrary) the *Blue* should hit upon its *Pores*, and the *Red* upon its *Parts*.

16. *Seventhly*, and lastly, Were the Rays of Light reflected by impinging on the solid Parts of Bodies, their Reflections from polish'd Bodies could not be so regular as they are: For in polishing Glass with *Sand*, *Putty*, or *Tripoli*, it is not to be imagined that those Substances can, by grating and fretting the Glass, bring all its least Particles to an accurate Polish, so that all their Surfaces shall be truly plain or truly spherical, and look all the same Way, so as together to compose one even Surface. The smaller the Particles of those Substances are, the smaller will be the Scratches by which they continually fret and wear away the Glass until it be polished; but be they never so small, they can wear away the Glass no otherwise than by grating and scratching it, and breaking the Protuberances, and therefore polish it no otherwise than by bringing its Roughness to a very fine Grain, so that the Scratches and Frettings of the Surface become too small to be visible; And therefore if Light were reflected by impinging on the solid Parts of Glass, it would be scatter'd as much by the most polish'd Glass as by the roughest. So then it remains a Problem, *How Glass polished by fretting Substances can reflect Light so regularly as it does?*

17. And this Problem is scarce otherwise to be solved than by saying, *That the Reflection of the Ray is effected, not by a*

4, 6, 8, 10, &c. the Light will be transmitted; and at the Distances 1, 3, 5, 7, 9, &c. it will be reflected in colour'd Rings: And this alternate

single Point of the reflecting Body, but by some Power of the Body which is evenly diffused over all its Surface, and by which it acts upon the Ray, without immediate Contact. For that the Parts of Bodies do act upon Light at a distance, has been already observed, and may be seen more at large in the Third Part of our Author's admirable Treatise of *Optics*.

18. Now (continues Sir *Isaac*) if Light be reflected, not by impinging on the solid Parts of Bodies, but by some other Principle, it is probable that as many of its Rays as impinge on the solid Parts of Bodies are not reflected, but stifled and lost in the Bodies; for otherwise we must allow two Sorts of Reflections. Should all the Rays be reflected which impinge on the internal Parts of clear Water or Crystal, those Substances would rather have a cloudy Colour than a clear Transparency.

19. Concerning this Power, by which Light is reflected and refracted, Sir *Isaac* understands it to be of an attractive and repulsive Nature; for he reasons thus: Since Metals dissolved in Acids attract but a small Quantity of the Acid, their attractive Force can reach to but a small Distance from them. And as in *Algebra*, where Affirmative Quantities vanish and cease, there Negative ones begin; so in *Mechanics*, where Attraction ceases, there a repulsive Virtue ought to take place.

20. And that there is such a Virtue seems to follow, (1.) From the Reflections and Inflections of Light, as before observed. (2.) From the Emission of Light; the Ray, so soon as it is shaken off from the shining Body by the vibrating Motion of the Parts of the Body, and gets beyond the Reach of Attraction, being driven away with exceeding great Velocity. For that Force which is sufficient to turn it back in Reflection may be sufficient to emit it. (3.) It seems also to follow from the Production of Air and Vapours; the Particles when they are shaken off from Bodies by Heat or Fermentation, so soon as they are beyond the Reach of the Attraction of the Body, receding from it, and from one another, with great Strength, and keeping at a distance, so as sometimes to take up a Million of times more Space than they did before in the Form of a dense Body.

21. To this repulsive Power he ascribes the *Reflection* of

Disposition

Disposition of Light to be reflected and transmitted, Sir *Isaac* calls the *Fits of easy Reflection*, and *Fits of easy Transmission* (CXXI.)

Rays, and to the attractive Power the *Refraction*; as has been before described. But how the Light is partly reflected and partly refracted at the Surfaces of Bodies, and what Phænomena do thence arise, we shall shew from the same illustrious Author in the following *Annotation*.

(CXXI.) 1. Concerning the particular Manner in which Light is reflected from natural Bodies, whether it be by a repulsive Power before it arrives at the Surface, or by an undulating Virtue every where diffused over the Surface, and causing a Reflection by the rising Wave, and a Transmission by the subsiding Wave; or lastly, whether the Reflection be occasion'd by the Vibrations of the Parts of Bodies, or the Mediums next the reflecting or refracting Surfaces, it will not be worth while here to spend Time in examining, since Sir *Isaac Newton* has confess'd himself unable to determine the *Modus agendi*, which Nature makes use of in this Affair.

2. Nor is his Doctrine of the *Fits of easy Reflection and easy Transmission* to be esteem'd a meer Hypothesis, or so much *clogg'd with Suppositions*, as to be dissonant from that Simplicity, Uniformity, and Regularity with which Nature is every where observed to act; since nothing can be more certain than that Light is at one Distance reflected, at another refracted, and that this is by a continual Alternation at exceeding small Intervals thro' the Substance of various *Media* or Bodies; and the Experiments which he made were many, and most convincing Proofs of the Thing.

3. And his Vibrations in the Parts of Bodies, and the elastic Medium which every where surrounds them, arising from thence, is very consonant to the Process of Nature, in propagating Sounds by the Undulations of the Air arising from the Vibration of the Parts of Bodies agitated by Percussion. Nature in each Case seems very consistent with her self, and to act with a wonderful Uuiformity, and equal Simplicity. Nor can I see any reason to hope for (much less to promise) a Solution of this Phænomenon from the ambigeneous Principle of Attraction, whose Action is well known to be always the same to a certain Distance or Limit one way, and beyond that as constantly the reverse; such a Circumstance little favours the Prediction of an easy and simple Solution. See *Rowning's Compendious System*, Part III. pag. 167.

As

As Light, falling upon this thin Plate of Air between the Glasses, is variously disposed to be *reflected* or *transmitted*, according to the several

4. I shall therefore proceed to give an Idea of one of the most beautiful, delicate, and importing Discoveries that was ever made; and that as nearly as may be after the Manner, and in the Words of the Author, by the Experiments which he made, and his Observations and Reasonings thereupon.

5. The first Experiment he mentions is the Compression of two Prisms hard together, whose Sides were a little convex, by which means they touched by a small Part of their Surfaces, and contain'd every where else a thin Plate of Air, as it may be properly call'd, whose Thickness did every where gradually increase from the touching Parts. He observed the Place where they touched became absolutely transparent, as if they had there been one continued Piece of Glass.

6. For when the Light fell so obliquely on the Plate of Air between the Prisms as to be all reflected, it seem'd in that Place of Contact to be wholly transmitted, insomuch, that when look'd upon it appear'd like a black or dark Spot, by reason that little or no sensible Light was reflected from thence, as from other Places.

7. When he look'd through the Prisms, this Place of Contact seem'd (as it were) a Hole in the Plate of Air, and through this Hole Objects that were beyond might be seen distinctly, which could not be seen through other Parts of the Glasses where the Air was interjacent. By harder Compression, the Spot was dilated by the yielding inwards of the Parts of the Glasses.

8. When the Plate of Air, by turning the Prisms about their common Axis, became so little inclined to the incident Rays, that some of them began to be transmitted, there arose in it many slender colour'd Arches, which at first were
Plate XL. shaped almost like the Conchoid, as in *Fig.* 1. and by continuing the Motion of the Prisms, these Arches increased and bended more and more about the said transparent Spot, till they were compleated into Circles or Rings encompassing it; and afterwards continually grew more and more contracted.

9. These Arches and Rings became tinged with various Colours, as the Motion of the Prisms was continued, being at first of a *Violet* and *Blue*; afterwards of a *White*, *Blue*, *Violet*; *Black*, *Red*, *Orange*, *Yellow*, *White*, *Blue*, *Violet*, &c. After this, the colour'd Rings contracted, and became only *black* and *white*: The Prisms being farther moved about, the

Degrees

Degrees of Thickneſs; ſo when it falls on the Surface of natural Bodies, it is as variouſly reflected from the Pores of Air of different Thick-

Colours all began to emerge out of the Whiteneſs, and in a contrary Order to what they had before.

10. But to obſerve more nicely the Order of the Colours which aroſe out of the white Circles, as the Rays became leſs and leſs inclined to the Plate of Air, Sir *Iſaac Newton* made uſe of two Object-Glaſſes, one a Plano-Convex, and the other a Double-Convex, of the ſame Sphericity on both Sides, of 51 Foot focal Diſtance; and upon this he laid the plane Side of the other, preſſing them ſlowly together to make the Colours ſucceſſively emerge in the Middle of the Circles, and then ſlowly lifted the upper Glaſs from the lower to make them ſucceſſively vaniſh again in the ſame Place.

11. Upon Compreſſion of the Glaſſes, various Colours would emerge and ſpread into concentric Circles or Rings of different Breadths and Tints encompaſſing the central Spot. Their Form, when the Glaſſes were moſt compreſſed, is delineated in the 2d Figure, where *a* is the central black Spot, and the Circuits of Colours from thence outwards as follows. Plate XL.

1. { *b*, Blue. *c*, White. *d*, Yellow. *e*, Red. }
2. { *f*, Violet. *g*, Blue. *h*, Green. *i*, Yellow. *k*, Red. }
3. { *l*, Purple. *m*, Blue. *n*, Green. *o*, Yellow. *p*, Red. }
4. { *q*, Green. *r*, Red. }
5. { *s*, Greeniſh Blue. *t*, Red. }
6. { *u*, Greeniſh Blue. *x*, Pale Red. }
7. { *y*, Greeniſh Blue. *z*, Reddiſh White. }

12. To determine the Thickneſs of the Plate of Air, where each of the Colours was produced, he meaſured the Diameter of the firſt ſix Rings at the moſt lucid Part of their Orbits, and ſquaring them found thoſe Squares to be in the Arithmetical Progreſſion of the odd Numbers 1, 3, 5, 7, 9, 11; and ſince one of thoſe Glaſſes was plane, and the other ſpherical, their Intervals at thoſe Rings muſt be in the ſame Progreſſion. Alſo he meaſured the Diameters of the dark or faint Rings between the more lucid Colours, and found their Squares to be in the Arithmetical Progreſſion of the even Numbers 2, 4, 6, 8, 10, 12.

13. All this follows from the Nature of the Circle; for

neſſes

nesses in those Bodies; and according to the different Texture of Bodies, and Magnitude of the Particles of Light, it will be either transmitted

Plate XL. Fig. 3. let the Circle EFG be the Section of the Sphere whose Convexity is equal to that of the Double-Convex above-mention'd, and the Line AB a Section of the plane Surface of the Plano-Convex touching the other in the Point D; then supposing De, Df, the Semidiameters of two Rings, the Thickness of the Air between the Glasses at those Rings will be ec and fd, which are equal to Da and Db respectively. If therefore, as usual, we put $DG = a$, $Da = x$, $Db = X$, $ac = (De =) y$, and $bd = (Df =) Y$; then by the Property of the Circle we have $yy = ax - xx$, and $YY = aX - XX$; and therefore $y^2 : Y^2 :: ax - xx : aX - XX :: x : \frac{a - X}{a - x} \times X$. But when a or Dg is very great with respect to x and X, or Da, Db, then $\frac{a - x}{a - X} = 1$ nearly; consequently, in the present Case $y^2 : Y^2 :: x : X$; or the Squares of the Semidiameters of the Rings De, Df, are as the Intervals ec, fd, or Thicknesses of the Plates of Air in those Places; and therefore the Squares of the Whole Diameters are in the same Ratio.

14. Sir *Isaac* measured the Diameter of the 5th dark Circle, (suppose $2Df$) and found it equal to $\frac{1}{5}$ of an Inch; but then viewing it through a Glass $\frac{1}{8}$ of an Inch thick, and nearly in the Perpendicular, it must by Refraction appear diminish'd nearly in the Proportion of 79 to 80; so that, As $78 : 80 :: \frac{1}{5} : \frac{16}{79} = 2Df$ the real Diameter between the Glasses. Whence $Df = \frac{8}{79}$, and in this Experiment $DG = 182$ Inches, we have $DG : bf\ (= Df) :: bf : Db = fd$; or, in Numbers, As $182 : \frac{8}{79} :: \frac{8}{79} : \frac{32}{567931}$ or $\frac{100}{1774784} = fd$; and since the Thickness of the Air at the 5th Ring is to that at the first as 10 to 2, or 5 to 1, (by *Art.* 12.) therefore $\frac{1}{5}$ of $\frac{100}{1774784} = \frac{1}{88739}$ Part of an Inch, for the Thickness of the Air at the first dark Ring.

15. By another Object-Glass of a Sphere whose Diameter $DG = 184$ Inches, he found the Dimension or Thickness of Air

wholly,

wholly, or in part; and that which is reflected will be *all of one Sort of Rays*, or of *several Sorts promiscuously and unequally*, or of *all Sorts equally*.

at the same dark Circle to be $\frac{1}{88850}$ Part of an Inch: But the Eye in both these Observations was not quite perpendicularly over the Glass, and the Rays were inclined to the Glass in an Angle of 4 Degrees; therefore (as *per* next Article) had the Rays been perpendicular to the Glasses, the Thickness of the Air at these Rings would have been less, and that in Proportion of the Radius 10000 to the Secant of 4 Degrees 10024. The Thicknesses found diminished in this Ratio will be $\frac{1}{88952}$ and $\frac{1}{89063}$, or in the nearest round Numbers $\frac{1}{89000}$ Part of an Inch. Now half of this, *viz.* $\frac{1}{178000}$, is the Thickness of the first colour'd Ring; and of the rest as follows, $\frac{3}{178000}$, $\frac{5}{178000}$, $\frac{7}{178000}$, *&c.* And $\frac{2}{178000}$, $\frac{4}{178000}$, $\frac{6}{178000}$, *&c.* are the Thicknesses at the several dark Rings.

16. The Rings were observed to be least when the Eye was held perpendicularly over the Glasses in the Axis of the Rings; and when they were view'd obliquely, they became bigger continually, swelling as the Eye was removed farther from the Axis. And by measuring the Diameters of the same Circle at several Obliquities of the Eye, and by some other Methods, Sir *Isaac* found its Diameter, and consequently the Thickness of the Air at its Periphery in all those Obliquities, to be very nearly in the Proportions expressed in the following Table; where the first Column expresses the Angles of Incidence which the Rays of Light make with the Perpendicular in the Glass; the second Column expresses the Angle of Refraction into the Plate of Air; the third Column shews the Diameter of any colour'd Ring at those Obliquities expressed in Parts, of which ten constitute the Diameter when the Rays are perpendicular; and the fourth Column shews the Thickness of the Air at the Periphery of that Ring expressed in Parts, of which the Diameter consists of ten also when the Rays are perpendicular.

Whence

Whence it will follow, (1.) If the Light be wholly transmitted, the Body will appear *black*;

17.

Angle of Incidence on the Plate of Air		*Angle of Refraction into the Pl. of Air.*		*Diameter of the Ring.*	*Thickness of the Air.*
Deg.	Min.	Deg.	Min.		
00	00	00	00	10	10
06	26	10	00	$10\frac{1}{13}$	$10\frac{2}{13}$
12	45	20	00	$10\frac{1}{3}$	$10\frac{2}{3}$
18	49	30	00	$10\frac{3}{4}$	$11\frac{1}{2}$
24	30	40	00	$11\frac{2}{5}$	13
29	37	50	00	$12\frac{1}{2}$	$15\frac{1}{2}$
33	58	60	00	14	20
35	47	65	00	$15\frac{1}{4}$	$23\frac{1}{4}$
37	19	70	00	$16\frac{4}{5}$	$28\frac{1}{4}$
38	33	75	00	$19\frac{1}{4}$	37
39	27	80	00	$22\frac{6}{7}$	$52\frac{1}{4}$
40	00	85	00	29	$84\frac{1}{12}$
40	11	90	00	35	$122\frac{1}{2}$

18. By looking through the two contiguous Object-Glasses or Prisms, it was observed that the Rings of Colours appear'd as well by transmitted as by reflected Light. The central Spot now became white and transparent. The Order of the Colours was *Yellowish Red*; *Black*, *Violet*, *Blue*, *White*, *Yellow*, *Red*; *Violet*, *Blue*, *Green*; *Yellow*, *Red*, &c. as they are written in the 4th Figure below, those above being the Colours by Reflection; AB and CD being the Surfaces of the Glasses contiguous at E, with Lines between shewing the Intervals or Thicknesses of Air in Arithmetical Progression. Where comparing the Colours, you observe that White is opposite to Black, Red to Blue, Yellow to Violet, Green to Red, &c. in reflected and refracted Light: But the Colours by refracted Light were very faint and diluted, except when view'd very obliquely, for then they became pretty vivid.

Plate XL.

19. By wetting the Glasses round their Edges, the Water crept in slowly between them, and the Circles thereby became less, and the Colours more faint. Their Diameters being measured were found in Proportion to those of the Rings made in Air as 7 to 8, and therefore the Thickness of Air at like Circles as $7 \times 7 = 49$ to $8 \times 8 = 64$, or as 3 to 4 very nearly, which is the Ratio of the Sines of Incidence and Refraction out of Water into Air. And this perhaps (says Sir *I-*

which

which is the Abſence of all colour'd Light. (2.) If the Light reflected from Bodies be all of one

ſaac) may be a general Rule for any other Medium interceding the Glaſſes more or leſs denſe than Water.

20. The colour'd Rings made in Air became much more diſtinct, and viſible to a far greater Number, when view'd in a dark Room by the Reflection of the colour'd Light of the Priſm. The Rings made by Reflection of *Red* Light were manifeſtly bigger than thoſe made by the *Blue* and *Violet*; and it was very pleaſant to ſee them gradually ſwell and contract according as the Colour of the Light was changed. The Motion was quickeſt in the *Red*, and ſloweſt in the *Violet*; and by an Eſtimation made of the Diameters of the Rings, the Thickneſſes of Air in the Places where the Rings are made by the Limits of the ſeven Colours, *Red, Orange, Yellow, Green, Blue, Indigo, Violet*, ſucceſſively in Order, were to one another as the Cube Roots of the Squares of the 8 Lengths of a Chord which ſound the Notes of an *Octave*, that is, of the Numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$.

21. Theſe Rings were not of various Colours, as thoſe made in the open Air, but appear'd all over of that Priſmatic Colour only with which it was illumin'd; and by throwing the colour'd Light directly on the Glaſſes, that which fell on the dark Spaces between the Rings was tranſmitted through the Glaſſes without any Variation of Colour. This appear'd by placing a white Paper behind, on which the Rings were painted of the ſame Colour as thoſe by reflected Light, and of the Bigneſs of their immediate Spaces.

22. Hence the Origin of theſe Rings is manifeſt; namely, that the Air between the Glaſſes, according to its various Thickneſs, is diſpoſed in ſome Places to reflect, in others to tranſmit the Light of any one Colour; and in the ſame Place to reflect that of one Colour, where it tranſmits that of another; in the Manner as you ſee repreſented in the 5th Figure: Where AB, CD, are the Glaſſes, as before; and *a, c, e, g, i, l, n, p*, the Parts of the Beam tranſmitted; and *b, d, f, h, k, m, o*, the Parts of the Beam reflected, making the colour'd Rings. Plate XL.

23. The Squares of the Diameters of theſe Rings made by any Priſmatic Colour, and conſequently the Thickneſſes of the Air at each, were in Arithmetical Progreſſion, as in the Rings of common Light; and the Dimenſion of the Rings made by Yellow Light the ſame as ſpecified in *Article* 14.

Sort

Sort, that Body will appear all of one Colour; which will be most simple and intensely deep.

These Observations were made with a rarer thin Medium terminated by a denser, *viz.* Air and Water between Glasses. In those which follow are set down the Phænomena of a denser Medium thinn'd within a rarer, as Plates of *Muscovy* Glass; Bubbles of Water, &c. bounded on all Sides with Air.

24. In the Experiment made with a Bubble of Soap-Water cover'd by clear Glass, and exposed to the white Light of the Sky, it was observed, that as the Bubble grew thinner by the continual subsiding of the Water, it exhibited Rings of Colours slowly dilating, till they overspread the whole Bubble, and vanish'd at the Bottom successively. The Bubble was black at Top, and this central Spot was surrounded with Rings of the same Colours, and in the same Order as those of Air in *Art.* 11, but much more extended and lively.

25. As the Thickness of the aqueous Shell diminished, the Colours of the several Rings by Dilatation were succeeded by others in Order from the Red to the Purple. Thus the Red of the second Ring from the Top (or sixth from the Bottom) was at first a fair and lively Scarlet, then became of a brighter Colour, being very pure and brisk, and the best of all the Reds. Then after follow'd a lively Orange, which was succeeded by the best of Yellows, which soon changed into a greenish Yellow, and then into a greenish Blue. Afterwards a very good Blue, of an azure Tint, appear'd; which was succeeded by an intense and deep Violet. And so it happen'd in all the other Orders of Colours, only not in so regular and perfect a Manner, the Colours in them being more compounded and less distinct.

26. These Rings of Colours, view'd in various Positions of the Eye, were found to dilate according as the Obliquity of the Eye increased, but not so much as those of Air in *Art.* 16. For by the Table, *Art.* 17, it appears they expanded to a Part where the Thickness of the Air was to that where they appear'd when view'd perpendicularly as $122\frac{1}{2}$ to 10, or more than 12 to 1; whereas Sir *Isaac* found, by measuring the Thickness of the Bubble at the several Rings, as they appear'd at the several Degrees of Obliquity mention'd in the Table below, that the greatest was to the least only as $15\frac{1}{2}$ to 10; which Increase is but about a 24th Part of the former in Air.

(3.) If

(3.) If the Rays are promiscuously reflected, but one Sort more than the rest, the Body will ap-

27. The Angles of Incidence on the Water, and the Refraction into the Water, are shewn in the two first Columns, and in the third the Thicknesses of the aqueous Shell corresponding thereto.

Incidence on the Water.		*Refraction in to the Water*		*Thickness of the Shell.*
Deg.	Min.	Deg.	Min.	
00	00	00	00	10
15	00	11	11	$10\frac{1}{4}$
30	00	22	01	$10\frac{4}{5}$
45	00	32	02	$11\frac{4}{5}$
60	00	40	30	13
75	00	46	25	$14\frac{1}{2}$
90	00	48	35	$15\frac{1}{5}$

28. The Sines of these Angles out of Water into Air are assumed as 3 to 4; and Sir *Isaac* has collected, (with a prodigious Sagacity) that *the Thickness of the Plate of Air or Shell of Water, requisite to exhibit one and the same Colour at several Obliquities of the Eye, is proportional to the Secant of an Angle whose Sine is the first of 106 mean Proportionals between the Sines of Incidence and Refraction.*

29. As in *Art.* 18, so here the Bubble by transmitted Light appear'd of a contrary Colour to that which it exhibited by Reflection: Thus that Part which look'd *Red* by reflected Light look'd *Blue* by refracted, and the Part which was *Blue* by reflected Light was *Red* by Rays transmitted. These Rings appear much more numerous, and more dilated, when view'd through a Prism than to the naked Eye; and by means of the Prism several Rings may be discover'd between the Glasses or in the Bubble, when none appear to the bare Eye.

30. The colour'd Rings now described appear also in thin Pieces of *Muscovy* Glass; which when they were wetted on the Side opposite to the Eye exhibited still the same Colours, but more languid and faint. Whence, and by *Art.* 19, it evidently appears, that the Thickness of a Plate requisite to produce any Colour depends only on the Density of the Plate, and not on that of the ambient Medium. And upon the Whole, if the Plate be denser than the ambient Medium, it exhibits more brisk and lively Colours than that which is so much rarer.

pear of the Colour proper to that Sort of Ray, but it will be not so pure and strong as before.

31. The Colours which arise on polish'd Steel being heated are of the same Kind with those in the Rings of the Bubble, emerging one after another from Red to Blue or Purple successively; and like the others will change in being view'd at different Obliquities of the Eye, but not in so great a Degree.

32. That we may be able to shew how the Colours in the several Rings are produced, we shall a little illustrate Sir *Isaac*'s Invention for that Purpose. In order to this, Let there

Plate XLI. Fig. 1.

be taken, in any Right Line YH, the Lengths YA, YB, YC, YD, YE, YF, YG, YH, in Proportion to each other as the Cube Roots of the Squares of the Numbers $\frac{1}{2}$, $\frac{9}{16}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, 1; that is, in the Proportion of the Numbers 6300, 6814, 7114, 7631, 8255, 8855, 9243, 10000. See *Article* 20.

33. In the Points A, B, C, D, E, F, G, H, erect the Perpendiculars A*a*, B*b*, C*c*, *&c.* by whose Intervals the Extent of the Colours wrought by them will be represented. For if at the Thickness YA the Violet Colour begins, and the Indigo at B, the Extent AB will represent the Breadth of the Violet; and so of the rest.

34. Then let the Line A*a* be divided into equal Parts, and number'd as in the Figure to 43; and through those Divisions from Y draw the Lines 1 I, 2 K, 3 L, 5 M, 6 N, 7 O, *&c.* Then will the Parts A 2, A 6, A 10, A 14, *&c.* be in Proportion to the odd Numbers 1, 3, 5, 7, 9, 11, *&c.* or as the Thicknesses of the Air at the several Rings. See *Art.* 12.

35. Therefore since A 2 represents the Thickness of any thin transparent Body, at which the Violet of the first Order or Ring is most copiously reflected; then will HK represent its Thickness where the Red of that Order is most copiously reflected: Because, in the similar Triangles AY 2 and HYK, we have YA : YH :: A 2 : HK. But YA and YH are as the Thicknesses of the Plate of Air at these Colours, and therefore also A 2 and HK. See *Art.* 32.

36. Again; because (by *Art.* 12.) A 6 is the Thickness where the Violet of the 2d Ring is most copiously reflected, and (by *Art.* 20.) the Ratio of the Thickness of the Air where Violet and Red are reflected is the same as of YA to YH; therefore since YA : YH :: A 6 : HN, the Line HN will represent the Thickness of the Plate where the Red of the second Order is reflected most copiously. Thus also A 10

(4.) If three or four Sorts of Rays are promiscuously reflected more than the rest, the Colour

and HQ will represent the same for the Violet and Red of the third Order, and so on.

37. And the Thicknesses at which the intermediate Colours will be reflected most copiously will be defined by the Distance of the Line AH from the intermediate Parts of the Line 2K, 6N, 10Q, *&c.* against which the Names of the Colours are written; which is easy to understand.

38. But farther to define the Latitude or Breadth of the Colours in each Ring, let A 1 denote the least Thickness, and A 3 the greatest, at which the extreme Violet in the first Series or Ring is reflected; then shall HI and HL be the like Limits for the extreme Red, and the intermediate Colours will be limited by the intermediate Parts of the Lines 1 I and 3 L, against which the Names of those Colours stand; and so on. *Note,* The same Latitude is assign'd to every Series of Colours, AHL3, 5MO7, 9PR11, *&c.* because the Difference of the Breadths of the Rings in the Plates of Air and Water were insensible to the Eye in the Experiment.

39. From hence it is easy to observe, that the Spaces A1IH, 35ML, 79PO, *&c.* are those at which the Rays are transmitted, and the dark Circles appear. And therefore we may know from this Scheme what Colour must be exhibited (in the open Air) at any Thickness of a transparent thin Body: For if a Ruler be applied parallel to AH, at the Distance from it by which the Thickness of the Body is represented, the alternate Spaces 1IL3, 5MO7, *&c.* which it crosses, will denote the reflected original Colours, of which the Colour exhibited in the open Air is compounded.

40. Thus, for Example, if it be required to find what is the Constitution or component Colours of the Green of the third Order or Series, apply the Ruler as you see at *rstuw*, (parallel to AH) and by its Passage through some of the Blue at *s*, and Yellow at *u*, as well as through the Green at *t*, you may conclude that the Green exhibited at that Thickness of the Body is principally constituted of original Green, with a Mixture of some Blue and Yellow.

41. By this means also you may know how the Colours from the Center of the Rings outward ought to succeed in the Order as they have been described in *Art.* 11. For if you move the Ruler gradually from AH through all the Distances, having pass'd over the first Space A1, which denotes little or no Reflection to be made by thinnest Substances, it will first

of the Body will be a Mix'd or Compound, inclining to the Tint of the most predominant Co-

arrive at 1 the Violet, and then quickly at the Blue and Green, which together with the Violet compound Blue; and then at the Yellow and Red, by whose farther Addition that Blue is converted into Whiteness, which continues during the Transit of the Ruler from I to 3; and after that, by the successive Deficience of its component Colours, turns first to compound Yellow, and that to Red, which ceases at L. Thus are the Colours of the first Series generated.

42. Then begin the Colours of the second Series, which succeed in Order during the Transit of the Edge of the Ruler from 5 to O, and are more lively than before, because more expanded and severed: And here, because the Ruler arrives to and passes over the Point 7 before it comes to M, there cannot be a Reflection of all the Colours at the same Time, and therefore no Whiteness between the Blue and Yellow, as before; but there will be a Reflection of original Green, with Yellow and Orange on one Side, and Blue and Indigo on the other, which together make a compound Green. The Violet will here first appear at 5, before it comes to be reflected with Indigo and Blue.

43. So the Colours of the third Series happen in Order; first the Violet at 9, which as it interferes with the Red of the second Order, is thereby inclined to a reddish Purple. Then the Blue and Green, which here are less mix'd with other Colours, and consequently are more lively than before, especially the Green. Then follows the Yellow, some of which towards the Green is distinct and good, but that Part towards the succeeding Red, as also that Red, is mix'd with the Violet and Blue of the fourth Order; whereby various Degrees of Red, very much inclining to Purple, are compounded.

44. Hence the Violet and Blue, which should succeed and begin the fourth Series, being mix'd with and hidden in the Red of the third Order, there succeeds a Green, which at first is much inclined to Blue, but soon becomes a good Green, being the only unmix'd and lively Colour of this fourth Order: For as it verges towards the Yellow, it begins to interfere with the Colours of the fifth Series, by whose Mixture the succeeding Yellow and Red are very much diluted and made dirty, especially the Yellow, which being the weaker Colour is scarce able to shew itself; so that this Order consists of Green and Red only.

lour.

lour. (5.) When all Sorts of Rays are equally reflected from Bodies, those Bodies appear *white*,

45. After this, by passing the Edge of the Ruler along parallel to AH, it will cut the Colours of the second, third, and fourth Series at once; which will shew those Colours become more and more intermix'd, till after three or four more Revolutions (in which the Red and Blue predominate by turns, making the fifth, sixth, and seventh Rings) all Sorts of Colours are in all Places pretty equally mix'd, and compound an even Whiteness. Thus the Line xy passing through the Red of the 7th Series, the Yellow and Green of the 8th, the Blue of the 9th, and the Purple of the 10th, shews Whiteness at the Thickness Ax or $H4$ must necessarily result from the Mixture of so many original Colours.

46. Since (by *Art.* 20, 21.) the Rays of one Colour are transmitted where those of another Colour are reflected, the Reason of the colour'd Rings made by transmitted Light is from hence manifest; because what has been said with respect to the Colours made by Reflection from the Spaces 1 L, 5 O, 9 R, *&c.* is equally applicable to account for the Colours made by Refraction through the Spaces AI, 3 M, 7 P, 11 S, *&c.*

47. Not only the Order and Species, but also the precise Thickness of the Plate at which any of those Colours are exhibited in Parts of an Inch, may be obtain'd as follows. Since (by *Art.* 14, 15, and 23.) we have the Thickness of the Plate where Yellow Light is reflected already measured, *viz.* $Fl = \frac{1}{178000}$, $Fm = \frac{3}{178000}$, $Fn = \frac{5}{178000}$, $Fo = \frac{7}{178000}$, *&c.* Pl. XLI. Fig. 2.
and since $\frac{1}{178000} = 0{,}0000056$, or 56 Parts of Ten Million of an Inch; if the Scale of equal Parts be constructed such of which $Fl = 56$, it is plain any other Thickness of Air may be immediately measured thereon by means of a Pair of Compasses, or by a parallel Ruler. Thus $Gw = 0{,}0000254$; $A2 = 0{,}0000040$; $HK = 0{,}0000065$; $A6 = 0{,}0000119$; $HN = 0{,}0000194$. And thus any other Thickness for any proposed Colour or Series is evident almost by Inspection, to the Ten Millionth Part of an Inch.

48. Since by *Art.* 19. it appears, that the Thicknesses of Air and Water, exhibiting the same Colour, are as 4 to 3; if the Thicknesses in Air are known for the several Rings, you'll have the Thickness of the Bubble of course where the several Colours appear; and thus the Table in *Art.* 27. was made. Also hence the Thicknesses of thin Plates of Glass producing the Rings of Colours will be known, being to those of Air as 20 to 31, *viz.* in the Proportion of the Sines of Incidence

 or

or of the Colour of the Sun's Light. (6.) Where there is no Light at all incident on Bodies, those

to Refraction out of Glass into Air for Yellow Light; and the Difference of the Proportion of the Sines for the other Rays is not considerable.

49. These are the Measures nearly, which Sir *Isaac* has express'd in the following Table, where the Numbers are so many Millionth Parts of an Inch for the Thicknesses of the Plates of Air, Water, and Glass, which exhibit the various Colours of the several Orders.

		Air.	*Water.*	*Glass.*
COLOURS of the First Order.	Very Black,	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{3}$
	Black,	1	$1\frac{1}{2}$	$1\frac{2}{7}$
	Blue,	$2\frac{2}{5}$	$1\frac{4}{5}$	$1\frac{1}{2}$
	White,	$5\frac{1}{4}$	$3\frac{7}{8}$	$3\frac{2}{5}$
	Yellow,	$7\frac{1}{9}$	$5\frac{1}{3}$	$4\frac{3}{5}$
	Orange,	8	6	$5\frac{1}{6}$
	Red,	9	$6\frac{3}{4}$	$5\frac{4}{5}$
Of the Second Order.	Violet,	$11\frac{1}{6}$	$8\frac{3}{8}$	$7\frac{1}{5}$
	Indigo,	$12\frac{5}{6}$	$9\frac{5}{8}$	$8\frac{2}{11}$
	Blue,	14	$10\frac{1}{2}$	9
	Green,	$15\frac{1}{8}$	$11\frac{1}{3}$	$9\frac{5}{7}$
	Yellow,	$16\frac{2}{7}$	$12\frac{1}{5}$	$10\frac{2}{5}$
	Orange,	$17\frac{2}{9}$	13	$11\frac{1}{9}$
	Bright Red,	$18\frac{1}{3}$	$13\frac{3}{4}$	$11\frac{5}{6}$
	Scarlet,	$19\frac{2}{3}$	$14\frac{3}{4}$	$12\frac{2}{3}$
Of the Third Order.	Purple,	21	$15\frac{3}{4}$	$13\frac{1}{2}$
	Indigo,	$22\frac{1}{10}$	$16\frac{4}{7}$	$14\frac{1}{4}$
	Blue,	$23\frac{2}{5}$	$17\frac{1}{2}$	$15\frac{1}{10}$
	Green,	$25\frac{1}{5}$	$18\frac{9}{10}$	$16\frac{1}{4}$
	Yellow,	$27\frac{1}{7}$	$20\frac{1}{3}$	$17\frac{1}{2}$
	Red,	29	$21\frac{3}{4}$	$18\frac{5}{7}$
	Blueish Red,	32	24	$20\frac{2}{3}$
Of the Fourth Order.	Green,	$35\frac{2}{7}$	$26\frac{1}{2}$	$22\frac{3}{4}$
	Blue,	$40\frac{1}{3}$	$30\frac{1}{4}$	26
Of the Fifth Order.	Greenish Blue,	46	$34\frac{1}{2}$	$29\frac{2}{3}$
	Red,	$52\frac{1}{2}$	$39\frac{3}{8}$	34
Of the Sixth Order.	Greenish Blue,	$58\frac{3}{4}$	44	38
	Red,	65	$48\frac{3}{4}$	42
Of the Seventh Order	Greenish Blue,	71	$53\frac{1}{4}$	$45\frac{4}{5}$
	Ruddy White,	77	$57\frac{3}{4}$	$49\frac{2}{3}$

Bodies

Bodies can have no Colour, which is a Property of the Rays of Light only (CXXII).

50. These are the principal Phænomena of thin Plates or Bubbles, which follow from the Properties of Light by a mathematical Way of Reasoning; whence it follows, that the colorific Disposition of Rays is connate with them, and immutable, there being always a constant Relation between Colours and the Refrangibility and Reflexibility of the Rays. In this respect the Science of Colours becomes a Speculation as truly Mathematical as any other Part of *Optics*; and consists of two Parts, one *Theoretical*, which delivers the Properties of Light, and the Principles on which the various Phænomena of Colours depend. This Part we have hitherto been treating of: The other is *Practical*, and consists in applying these Principles to account for the permanent Colours of Natural Bodies; to which we shall now proceed in the following Note.

(CXXII) 1. As I here intend to deliver the whole *Newtonian* Doctrine of Colours, it will be necessary to begin and proceed with the Definitions and Precautions which Sir *Isaac Newton* himself has made use of, and which are as follows.

2. His general Position is, *That if the Sun's Light consisted but of one Sort of Rays, there would be but one Colour in the whole World; nor would it be possible to produce any new Colour by Reflections and Refractions; and by Consequence that the Variety of Colours depends upon the Composition of Light.* All which is evident from the Subject of the foregoing *Annotations* on the Properties and Phænomena of Light by Reflection and Refraction.

3. His Definition of Light is as follows: The Light whose Rays are all alike refrangible he calls *Simple, Homogeneal*, and *Similar*; and that whose Rays are some more refrangible than others he calls *Compound, Heterogeneal*, and *Dissimilar*.

4. The Colours of *Homogeneal Lights* he calls *Primary, Homogeneal*, and *Simple*; and those of *Heterogeneal Lights* he calls *Heterogeneal* and *Compound*, because these are all compounded of the Colours of Homogeneal Lights; as hath been in part already, and will be farther shewn in the Sequel of this *Annotation*.

5. The Homogeneal Light and Rays which appear *Red*, or rather make Objects appear so, he calls *Rubrific* or *Red-making* Rays; those which give Objects a *Yellow, Green, Blue*, or *Violet* Colour, he calls *Yellow-making, Green-making, Blue-*

Plate L. Fig. 2. LET BNFG be a spherical Drop of falling Rain, and AN a Ray of the Sun falling upon it in the Point N, which Ray suppose refracted to F, from thence reflected to G, and there again refracted in the Direction GR to the

making, *Violet-making* Rays; and so of the rest. And therefore whenever he speaks of Light and Rays as colour'd, or endued with Colours, he would be understood to speak not philosophically and properly, but grosly, and according to the vulgar Notion of Common People.

6. For the Rays, to speak properly, are not colour'd; in them there is nothing but a certain Disposition and Power to excite a Sensation of this or that Colour. For as Sound in a Bell or musical String is nothing but a tremulous Motion, and in the Air nothing but that Motion propagated from the Object in aerial Undulations; and in the *Sensorium* 'tis a Sense of Motion under the Notion of Sound: So Colours in the Object are nothing but a Disposition to reflect this or that Sort of Rays more copiously than the rest; in the Rays they are nothing but their Disposition to propagate this or that Motion to the *Sensorium* by the Optic Nerve; and in the *Sensorium* they are Sensations or Ideas of those Motions under the Forms or Notions of Colours.

7. Every Ray of Light in its Passage through any refracting Surface is put into a certain transient Constitution or State, which in the Progress of the Ray returns at equal Intervals, and disposes the Ray at every Return to be easily transmitted through the next refracting Surface, and between the Returns to be easily reflected by it. This is manifest from *Art.* 21, 22, of the last *Note*. These Returns of the Disposition of any Ray to be reflected he calls its *Fits of easy Reflection*, and those of its Disposition to be transmitted its *Fits of easy Transmission*; and the Space it passes between every Return he calls the *Interval of the Fits*.

8. This Alternation of its Fits depends on both the Surfaces of every thin Plate or Particle, because it depends on its Thickness; and also because, if either Surface be wetted, the Colours caused both by Reflection and Refraction grow faint, which shews it to be affected at both. It is therefore perform'd at the second Surface, for if it were perform'd at the first, it could not depend on the second; and it is influenced by some Action or Disposition propagated from the first to the

Eye

Eye of a Spectator; and let I G be perpendicular to the Point G: Then will the Beam, by its Refraction at G, be separated into its several Sorts of Rays, which will paint their respective Colours in that Part of the Drop; of which that

second, because otherwise at the second it could not depend on the first.

9. This Action or Disposition, in its Propagation, intermits and returns at different Intervals in different Sorts of Rays, emerging in equal Angles out of any refracting Surface into the same Medium. Thus in the Experiment of *Art*. 20. and 21. of the last *Annotation*, 'tis plain, the Violet Ray being in a Fit of easy Transmission at its Incidence on the Plate of Air, was again in that Fit at the farthest Surface, in passing through a less Space than that which the Red pass'd through in the Interval of its Fits; for those Spaces were as the Thicknesses of the Glasses, and consequently the Intervals of these Fits were as the Numbers 63, 68, 71, 76, 82½, 88½, 92½, 100, for the Rays respectively from Violet to Red. See *Art*. 32. of the last *Annotation*.

10. Hence when a Ray of Light falls upon the Surface of a Body, if it be in a Fit of easy Reflection, it shall be reflected; if in a Fit of easy Transmission, it shall be transmitted; and thus all thick transparent Substances are found to reflect one Part of the Light which is incident upon them, and to refract the rest.

11. The least Parts of almost all Natural Bodies are in some measure transparent. This is well known to those who are conversant in Experiments with the Common and Solar Microscopes: As also by the Solution of dense and opake Bodies in Menstruums; for then their Particles being so minutely divided become transparent. And therefore, considering the inconceivable Smallness of the Particles of Light, even in comparison of the smallest Parts of Natural Bodies, we may conceive them as always incident on the Surface of transparent Substances.

12. I have observed (briefly) before, that those Superficies of transparent Bodies reflect the greatest Quantity of Light, which have the greatest refracting Power. Thus Glass produces a total Reflection of Light at a less Angle of Incidence on the Air than Water; for in Glass that Angle is but 40° 10′, but in Water it is 48° 35′. Thus also Diamond, whose re-

next

next the Perpendicular IG will be *red*, as being least refracted, and the rest in Order above it. Now it is found by Computation, that the greatest Angle S*e*O, or EOP, (drawing OP parallel to SE) under which the most refrangible Rays can

fractive Power to that of Glass is as 34 to 26 nearly, is found to reflect a much greater Quantity of Light than Glass.

13. Hence 'tis obvious, there can be no Reflection at the Confines of equally refracting Mediums. For let HI be a single Ray of Light passing out of a denser Medium AC into a rarer DE; in this Case there will be a certain Limit or Angle of Incidence HIK, in which the Ray will be reflected into IG, If the Medium AC be supposed to have its Density decreasing, then the Limit or Angle HIK will be continually increasing; or, which is all one, the Ray HI must have a greater Obliquity than HIK that it may be reflected. Therefore when the Density of the Medium AC becomes equal to that of DE, the Angle HIK will become equal to DIK; and so no Ray inclined to the Perpendicular KI can in that Case possibly be reflected.

PlateXLI. Fig. 3.

14. Hence the Reason why uniform pellucid Mediums, as Water, Glass, Crystal, &c. have no sensible Reflection but in their external Superficies, where they are adjacent to other Mediums of different Densities, is because all their continuous Parts have one and the same Degree of Density.

15. Hence also it is, that since in common Substances there are many Spaces, Pores, or Interstices, either empty or replenish'd with Mediums of other Densities, various Reflections must be made in the Confines of these differently refracting Mediums; and thus the Bodies become variously colour'd and opake in different Degrees. As for Example, Water between the tinging Corpuscles wherewith Liquor is impregnated; Air between the aqueous Globules which constitute the Clouds or Mist; and Water, Air, and perhaps other subtil Media between the Parts of solid Bodies, give them their proper Colours and Degrees of Opacity, by a confused and promiscuous Reflection and Refraction of Light.

16. The Parts of Bodies and their Interstices must not be less than of some definite Bigness to render them opake and colour'd: For, as was said before, the opakest Bodies, if their Parts be sufficiently attenuated by Solution, become transparent. Thus the Top of the Water-Bubble being very thin

come

come to the Eye of a Spectator at O, is 40 Deg. 17 Minutes; and that the greatest Angle F O P, under which the least refrangible Rays come to the Eye at O, is 42 Deg. 2 Minutes. And so all the Particles of Water within the Difference Plate L. Fig. 3.

made no sensible Reflection, and therefore exhibited no Colours; but, transmitting the Light, appear'd black. Hence it is that Water, Salt, Glass, Stones, &c. having their Parts and Interstices too small to cause Reflections, become transparent and colourless.

17. The transparent Parts of Bodies, according to their several Sizes, reflect Rays of one Colour, and transmit those of another, on the same Grounds that thin Plates or Bubbles did the same; and this is undoubtedly the Ground and Reason of all their Colour. For if such a thin Plate should be broke into several Fragments, or slit into Threads of the same Thickness, they would all appear of the same Colour; and by consequence, an Heap of those Threads or Fragments would constitute a Mass or Powder of the same Colour the Plate exhibited before it was broken; and the Parts of all Natural Bodies, being like so many Fragments of a Plate, must on the same Grounds exhibit the same Colours.

18. And that they do so will appear by the Affinity of their Properties. The finely-colour'd Feathers of some Birds, as of Peacocks Tails, do in the very same Part of the Feather appear of several Colours in several Positions of the Eye, in the same manner that thin Plates were found to do in *Articles* 16. and 26. of the last *Annotation*; and therefore their Colours arise from the thin transparent Parts of the Feathers, that is, from the Tenuity of the very fine Hairs or *Capillamenta*, which grow out of the Sides of the grosser Parts or lateral Branches of those Feathers.

19. And hence it is, that the Webs of some Spiders being spun very fine have appear'd colour'd; and that the colour'd Fibres of some Silks, by varying the Position of the Eye, do vary their Colours.

20. Another Circumstance in which they agree is, that the Colours of Silks, Cloaths, and other Substances, which Water or Oil can intimately penetrate, become more faint and obscure by being immersed into those Liquors, and recover their Vigour and Vivacity again by being dried, in the same manner as was observed of thin Bodies in *Art*. 19. and 30.

of those two Angles E F will exhibit severally the various Colours of the Prism, and constitute the *interior Bow* in the Cloud.

IF the Beam go not out of the *Drop* at G, but is reflected (a second time) to H, and is there

of the last *Annotation.*

21. A third Circumstance, in which Natural Bodies agree in their colorific Quality with thin Plates, is, that they reflect one Colour and transmit another. Thus Leaf-Gold looks Yellow by reflected Light, and of a blueish Green by the transmitted Light. Also an Infusion of *Lignum Nephriticum* reflects the Blue and Indigo Rays, and therefore by Reflection appears of a deep Mazarine Blue; whereas by refracted Light it appears of a deep Red. And the same Thing is observable in several Sorts of painted Glasses.

22. Again; as thin Plates and Bubbles exhibit different Colours in different Thicknesses, so the Parts of Natural Bodies are observed to undergo a Change of Colour in some Degree from Trituration, and a Comminution of their Parts. Thus some Powders which Painters use, by being elaborately and finely ground, have their Colours a little changed. Thus Mercury by several Chymical Operations has its Parts so alter'd as to look Red in one Case, Yellow in another, and White in a third. Thus Copper in the Mass appears Red, but having its Parts attenuated by Solution in acid Mediums appears intensely Blue. Hence the Production and Changes of Colours by the various Mixture of transparent Liquors. Thus Clouds receive their different compound and beautiful Hues and Tints from the different Sizes of the aqueous Globules of which they consist.

23. The Sizes of the Particles of Bodies, on which their Colours depend, are indicated by those Colours: Thus the least Particles of Light exhibit the *Violet-Colour*, and the least Thickness of the Plate of Air or Water exhibited the same Colour in the several Rings. Again: The largest Particles of Light exhibit a *Red Colour*, and Red is produced by Reflection and Refraction in the thickest Part of the Plate in each Ring; And the intermediate Colours, Blue, Green, Yellow, are produced from Particles of a larger Size in Order.

24. The Magnitude of the Particles of colour'd Bodies may be pretty nearly conjectur'd by the Colours they exhibit: For 'tis pretty certain they exhibit the same Colours with the

refracted in the Direction HS, making the Angle SYA with the incident Ray AN, it will paint on the Part H the several Colours of Light; but in an inverse Order to the former, and more faint, by reason of the Rays lost by the *second*

Plate of equal Thickness, provided they have the same refractive Density; and since their Parts seem for the most part to have much the same Density with Water or Glass, as by many Circumstances is obvious to collect, we need only have Recourse to the Table or Scale (in *Art.* 47, 48, 49. of the last *Annotation)* by which the Thickness of Water or Glass exhibiting the same Colour is shewn.

25. Thus if it be desired to know the Diameter of a Corpuscle, which being of equal Density with Glass shall reflect Green of the third Order; then in the said Table you see under *Glass*, and opposite to *Green* of that Order, the Number $16\frac{1}{4}$, which shews the Corpuscle to be $\frac{16\frac{1}{4}}{1000000}$ Parts of an Inch. But here the Difficulty is to know of what Order the Colour of any Body is: But for this Purpose we may be assisted by viewing the Scheme of the several Orders of Colours; and by laying the parallel Ruler across them severally, you will observe those which are least compounded with others in every Order, and consequently are most vivid and intense.

26. Thus *Scarlets*, and other *Reds*, *Oranges*, and *Yellows*, if they appear pure and intense, you may conclude they are of the Second Order. Good *Greens* may be of the Fourth Order, but the best are of the Third. *Blues* and *Purples* may be of the Second or Third Order, but the best and least compounded are of the Third. *Whiteness*, if most intense and luminous, is that of the First Order; if less strong and bright, it is that arises from the Mixture of the Colours of several Orders.

27. The Reds therefore of Carmine, Cinnabar, Vermilion, of some Roses, Pinks, Peonies, &c. are of the Second Order. The Green of all Vegetables is of the Third Order; Ultramarine is a Blue of the Third Order, Bise a Blue of the Second Order, and the Azure Colour of the Sky seems to be of the First Order. Gold is a Yellow of the Second Order. The Whiteness of Paper, Linen, Froth, Snow, Silver, &c. is of the First Order. Concerning all which see

Reflection.

Reflection. It has been found also, that the least Angle SGO, or GOP, under which the least refrangible Rays can come to the Eye at O, after two Reflections and two Refractions, is 50 Deg.

more in Sir *Isaac Newton*'s *Optics*, p. 230—237.

28. It has been observed, (See *Art.* 41. of the last *Annotation*) that *Whiteness* arises from a promiscuous Reflection of all the Colours together; and this is proved by several Experiments. Thus the Colour of the Sun's Light is White inclining a little to Yellow, as being a Composition of all the different colour'd Rays, among which the Yellow being the brightest is most predominant. Thus also the Rays when separated by a Prism, if received by a broad convex Lens of a large focal Distance, will all be thrown together in a small round Spot in the Focus, and appear of a white Colour. Thus also a Powder compounded of Orpiment, Purple, Bise and Verdigrease, in proper Proportion, appear'd of a perfect Whiteness in the Beams of the Sun.

29. But the most curious Experiment for Proof of this is as follows. Let any circular Area be divided on its Periphe-
Pl. XLI. Fig. 4. ry into such Parts AB, BC, CD, DE, EF, FG, and GA, as are proportional to the Differences of the Lengths of the Musical Strings in an Octave, or the Numbers $\frac{1}{2}$, $\frac{9}{16}$, $\frac{3}{5}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, 1; then striking a Circle *abcdefg* at a small Distance from the Periphery, the several Divisions of this *Annulus* or Ring are to be laid over with the primary Colours proper to each, that is, Red from A to B, Orange from B to C, and the rest in Order as they are wrote in the Figure. Then making all the internal Space very black, let this Area with its painted Ring be whirl'd or spun round in the manner of a Top, and the Ring will appear very white, especially in the Sun-Beams: For in this Case all the Colours are blended together in the View, and must therefore exhibit Whiteness.

30. On the other hand, *Blackness* the Absence of all Colours; for it was observed, that in the Middle or Center of the Rings of Colours, both in the Plates of Air and Water, there was a *black Spot*, which was occasion'd by a Transmission of all the Light in that Part, and consequently by a total Deficiency of Colour.

31. But this happen'd in that Part of the Plate of Air, and Water-Bubble, where it was thinnest; and hence we are taught that the Corpuscles of black Bodies are less than any of those which exhibit Colours. Hence we see the Reason

57 Minutes; and the least Angle HOP, under which the most refrangible Rays can come to the Eye in this Case, is 54 Deg. 7 Minutes. Whence all the Colours of the *exterior Bow* will be form'd

why Fire, and the more subtil Dissolver Putrefaction, by attenuating the Particles of Bodies turn them black. Why a Razor, while setting, turns the Oil upon the Hone black: Why a small Quantity of a black Substance will tinge so great a Quantity of any other so intensely: Why black Substances soonest of all others do become hot in the Sun's Light and burn: Why being soft, and stroked hard with the Hand, they scintillate, or emit Sparks of Light in the dark: Why a black Cloth will, if wet, dry sooner than a white one: Why most Blacks are a little inclined to a blueish Colour: With various other Phænomena of this Kind.

32. From what has been said, the *Newtonian* Method of compounding and decompounding Colours may be easily understood, if we only first premise, that the Colour resulting from a Mixture of any primary Colours is an Effect in which each primary Colour has a Share in Proportion to its Quantity; therefore this compound Colour is analogous to the *Common Center of Gravity* between two Powers acting against each other: For as this Center of Gravity will always be nearest to the greatest Power, so the Hue of the compound Colour will always approach nearest the Complexion of that primary Colour which was largest in the Mixture.

33. Therefore to know what Colour will result from a Mixture of two Parts Yellow, and three Parts Blue; from the Middle of the Yellow Arch at H to the Middle of the Blue at I draw the Line HI, and divide it into five equal Parts, three of which set from the Point H, or two from the Point I, will give the Point K, through which if you draw the Line NL, it will point out the Colour of the Mixture at L, which is Green; but because the Point L is so much nearer the Blue than the Yellow, it will be a blueish Green.

34. Again: If it be required to know what Colour the Mixture shall be of that has two Parts Yellow, three of Blue, and five of Red; then since we have already determined the Point K for the two first Quantities, which are five Parts; also since there are five Parts of Red, if we draw the Line MK, and divide it into two equal Parts in P, and through P draw the Line NO, this, as it falls upon the Orange, but near the Red, shews the Compound will be of an Orange Colour

in

in the Drops from G to H, which is the Breadth of this Bow, *viz.* 3 Deg. 10 Minutes; whereas the Breadth of the other, *viz.* E F, is but 1 Deg. 45 Minutes, and the Distance between the Bows,

inclining to Red. And thus you proceed in other Cases.

35. It must be farther observ'd, that the Colour will be less or more broken or imperfect as the Point of Intersection K or P falls nearer to or farther from the Circumference towards the Center N, where White is represented: That is, the farther the Point K is situated from L towards N, the less pure and intense, or the more broken and mottley, the Green Colour will be.

36. Hence, if it be required to find (on the other hand) what Colours must be taken, and in what Quantity, to exhibit by their Mixture the broken blueish Green at K, let the Line HI be any how drawn through K, and it will shew that if you take such Quantities of Yellow and Blue as are in Proportion to IK and KL, they will when mixed produce the given Green at K. Also the Line LN, passing through the same Point K, shews that a Quantity of pure Green and White, in the Proportion of NK, LK, will in the Mixture produce the same Green Tint at K as required.

37. What has been said relates to Theory, and to the Colours of the Sun's Light; and therefore in Practice we must not expect so great Accuracy on several Accounts; as, (1.) Because the Powders made use of in artificial Mixtures have different Powers of reflecting Light: Thus lighter Materials reflect more, and darker ones less; and therefore their Quantities must be in Proportion. (2.) Different Bodies, being mix'd, operate upon each other; and thereby, either by attenuating the Parts, or by incrassating them, produce Colours quite different from what we might expect from a Mixture of Bodies or Particles which do not affect or act one upon another. (3.) Because all artificial Colours are in themselves more or less compounded, and therefore cannot produce the Effects of pure, unmix'd, and primary Colours. Yet notwithstanding these Exceptions, this Theory, when well consider'd and understood, will be of the greatest Service to Painters.

38. Lastly, I shall apply this Theory to explain and account for several other Phænomena of Colours. Thus in examining *Mineral Waters*, it is usual, in order to discover whether the Salts contain'd in them are of an *Acid, Alcaline*, or

viz.

viz. FG, is 8 Deg. 55 Minutes. And such would be the Measures of the Bows, were the Sun but a *Point*; but since his Body subtends an Angle of half a Degree, it is evident, by so

Neutral Sort, to mix *Syrup of Violets* with them; because then, if there be an *Acid*, it will change the Syrup Red by attenuating its Parts; so that if the Syrup be a Purple of the Third Order, the Acid will change it to a Red of the Second Order, the Particles which reflect that Colour being of the Size next less.

39 Again: If an *Alcali* abound in the Water, the Mixture will turn Green; for the *Alcali* by incrassating the Particles will increase their Size to those of the Green of the Third Order; therefore the Syrup, and consequently the Mixture, will appear of that Colour. But if there be neither an *Acid* nor an *Alcali* in the Water, it will neither turn Green nor Red.

40. Hence also it is, that when even the Fume or subtil Vapour of a strong Acid, as *Aqua fortis*, reaches a Green Cloth, it changes to a Blue, because that in the same Order results from the next less Size of Particles. If the Acid be dropp'd on the Cloth in Substance, it acts more violently in attenuating the Particles, and thereby produces a Yellow of the next preceding Order, whose Particles are less than the aforesaid Blue. And after a like Manner may this Theory be extended, to account for other Phænomena of the same Kind.

41. To conclude: Since any Object becomes visible when it subtends an Angle of *one Minute*, and also because Objects are distinctly view'd in the Focus of a Lens; therefore supposing the Focus of a Lens were $\frac{1}{10}$ of an Inch, (as they have been made thus small) it will be found by Calculation, that an Object in the Focus of such a Lens, subtending an Angle of one Minute, will be equal to 0,0000097 Parts of an Inch in Length. Therefore the Diameter of a Particle less than the Diameter of any colour'd Particle (except those of the First Order) will be visible in the Focus of such a Lens: And therefore the Particles of all colour'd Bodies would become visible by such a Lens, were it not that Particles equally thick appear of the same Colour, and all so very small are transparent; whence, though they are big enough to be visible, yet we may want a Difference of Colour, and some other Means, to render them distinct, and capable of being

much each Bow will be increased, and their Distance diminish'd (CXXIII).

view'd separately from each other. Sir *Isaac Newton* thinks the Discovery of those Corpuscles by the Microscope will be the utmost Improvement of this Science: For it seems impossible to see the more secret and noble Works of Nature within the Corpuscles, by reason of their Transparency.

(CXXIII) 1. Having explain'd the Doctrine of the different *Refrangibility* of the Rays of Light, and the *Theory of Colours* consequent thereupon, it will now be easy to explain and understand the natural Cause of the *Rainbow*, which is wholly owing to the above-mention'd Property of Light. For though it was, by long Observation, known to proceed from the Sun's shining upon the falling Drops of Rain; and even before Sir *Isaac Newton*'s Time it was discover'd to be the Effect of the Sun's Light several times refracted and reflected in the aqueous Globules; first of all by *Antonius de Dominis*, Archbishop of *Spalato*, in a Book publish'd in the Year 1611, and after him by *Descartes*: Yet no one could ever account for the Diversity of Colours, and their inverse Order in the two Bows, or give a direct Method of Calculation, before Sir *Isaac Newton*.

2. To apprehend rightly the different Affections of this remarkable Phænomenon, we must attend to the following Particulars. *First*, That though each Bow be occasion'd by the refracted and reflected Light of the Sun falling on the Drops of Rain, yet neither of them is produced by any Rays falling on any Part of the Drop indifferently, but by those only which fall on the Surface of the Drop BLQG in or about the Point N, as the Ray AN; those which fall nearer to B, or farther towards L, being unconcern'd in this Production.

Plate L. Fig. 2.

3. *Secondly*, The internal Bow is produced by two Refractions and one Reflection. The first Refraction is of the incident Rays extremely near AN, by which they proceed from N to one common Point or Focus at F, from whence they are reflected to G, and are there a second time refracted towards R, and produce the various Colours of the said Bow.

Pl. XLII. Fig. 1.

4. *Thirdly*, There is a Necessity that several Rays should be refracted together to the Point F, that being reflected together from thence to G they may there go out parallel, and so come in Quantity sufficient to excite the Sensation of

HALO'S

HALO's are form'd by Rays of Light coming to the Eye after two Refractions through *Drops*

Colours in a strong and lively Manner. Now those Rays, and those only, which are incident on the Globule about the Point N, can do this, as will appear from what follows: For,

5. *Fourthly*, The Point F makes the Arch QF a *Maximum*; or the Distance QF from the Axis of the Drop SQ is greater than any other Distance from whence any other Rays nearer to the Axis, as SD, SE, or farther from it, as SH, SI, are reflected; because those which are nearer after the first Refraction tend to Points in the Axis produced more remote than that to which the Ray SN tends; and therefore as their Distance from the Axis increases, so likewise will the Distances of their Points of Reflection QP, QO, till the Ray becomes SN; after which the Rays more remote from the Axis, as SH, SI, are refracted towards the Points XY, which are nearer and nearer to the Axis; and this occasions the Points of Reflection on the farthest Side of the Drop to decrease again from F towards Q.

6. *Fifthly*, Hence it will necessarily happen, that some Rays above and below the Ray SN will fall upon the same Point, as O or P, on the farthest Side; and for that Reason they will be so reflected from thence as to go out of the Drop by Refraction parallel to each other. Thus let SE below, and SH above the Ray SN, be refracted both to one Point O; from hence they will be reflected to M and L, and will there emerge parallel, 'tis true, but alone; being divested of their intermediate Rays SN, which going to a different Point F will be reflected in a different Direction to G, and emerge on one Side, and not between those Rays, as when they were incident on the Drop. All which is evident from the Figure.

7. *Sixthly*, As this will be the Case of all the Rays which are not indefinitely near to SN, it is plain, that being deprived of the intermediate Rays, their Density will be so far diminish'd, as to render them ineffectual for exciting the Sensation of Colours; and they are therefore call'd *Inefficacious Rays*, in Contra-distinction to those which enter the Drop near SN; and which, having the same Point F of Reflection, are not scatter'd like the others, but emerge together at G, so as to constitute a Beam GR of the same Density with the incident Beam SN, and therefore capable of exhibiting a vivid Appearance of Colours, and for this Reason are call'd *Efficacious Rays*.

of Rain, or *spherical Hail-stones*; which Light ought to be strongest at the Distance of about

8. These Things premised, we shall now shew the Mathematical Principles on which the Calculations relating to this Phænomenon depend, according to Dr. *Halley*'s most elegant and easy Constructions, a little explain'd and facilitated by Dr. *Morgan*, late Bishop of *Ely*. Let SN, *sn*, be two of the efficacious Rays incident upon a Drop of Rain; these when refracted to the same Point F, and thence reflected to G, *g*, will have the Parts within the Drop on one Side, NF, *n*F, equal to those on the other Side, FG, F*g*, from the Nature of the Circle, and the Angles of Incidence CFN, CF*n*, being equal to the Angles of Reflection CFG, CF*g*. Since the Parts within the Drop are equal and alike situated, they will also be so with it; and therefore as the incident Rays SN, S*n*, are supposed parallel, the emergent Rays GR, *gr*, will be so too.

Pl. XLII. Fig. 2. 9. From C the Center draw the Radii CN, C*n*, CF; then is CNF = CFN the Angle of Refraction, and the small Arch N*n* is the nascent Increment of the Angle of Incidence BCN; and as it measures the Angle at the Center NC*n*, it is double of the Angle at the Circumference NF*n*, which is the nascent Increment of the Angle of Refraction NFC.

Fig. 3. 10. Again: Let the Ray SN enter the lower Part of the Drop, and be twice reflected within the Drop at F and G; then is the Ray NF = the Ray FG, and the Arch NF = to the Arch FG. Draw *fg* parallel to FG, and it will be the reflected Part of some Ray *sn*, whose Obliquity to the Drop is such as obliges it to cross the Ray NF in its Refraction, as it must do if it be a little more oblique than SN, (by *Art.* 6.) Then also will the Part *nf* = *fg*, and the Arch *nf* = *fg*, and the small Arch F*f* = G*g*.

11. Therefore, 2F*f* = (F*f* + G*g* = the Arch FG — the Arch *fg* = the Arch NF — the Arch *nf* =) N*n* — F*f*; consequently N*n* = 3F*f*. That is, the nascent Increment of the Angle of Incidence is equal to *three times* that of the Angle of Refraction. After a like Manner you proceed to shew, that after 3, 4, 5, *&c.* Reflections, the Increment of the Angle of Incidence will be 4, 5, 6, *&c.* times greater than that of the Angle of Refraction.

12. Hence, in order to find the Angle of Incidence of an efficacious Ray, after any given Number of Reflections, we are to find an Angle whose nascent Increment has the same Ratio to the Increment of its corresponding Angle of Re-

26 Degrees from the Sun or Moon, or somewhat less, if the said Hail-stones be a little flatted, as

fraction, generated in the same Time, as the given Number of Reflections (n) increased by Unity has to Unity; that is, in the Ratio of $n+1$ to 1. Now those Increments are as the Tangents of the respective Angles directly; as is thus demonstrated.

13. Let ACD, ABD, be the Angles of Incidence and Refraction proposed; and if we suppose the Line AC to move about the Point A in the Plane of those Angles, the Extremity thereof C will describe the circular Arch Cc; and when AC is arrived to the Situation Ac, the Line BD will be thereby removed into the Situation Bd. Draw cD; then is the Angle ACD = ABC + CAB, and the Angle Acd = ABc + cAB. Wherefore the Excess of Acd above ACD, or the Increment of ACD, is equal to both the Angles CBc and CAc. But since the Angle AcC differs infinitely little from a Right one, a Circle described on the Diameter AC shall pass through the Points D and c; and therefore the Angles CAc, CDc, (insisting on the same Arch Cc of the said Circle) will be equal. Wherefore the Increment of the Angle ACD is equal to CBc + CDc = Dcd. But the nascent Angles Dcd and DBc are as their Sines, that is, as their opposite Sides BD and Dc = DC, because of the Angle CDc infinitely small. But BD : CD :: DE : DA (the Line BE being parallel to AC) :: Tangent of the Angle (EBD =) ACD : Tangent of the Angle ABD. Therefore the Increment Dcd of the Angle ACD is to the Increment CBc of the Angle ABD (generated in the same Time) as the Tangent of the former to the Tangent of the latter directly. Pl. XLII. Fig. 4.

14. Hence, having given the Ratio of the Sine of Incidence I to the Sine of Refraction R, we may find the Angles of Incidence and Refraction of an efficacious Ray, after any given Number (n) of Reflections, thus: In any Right Line AC, let there be taken AC : AD :: I : R; and again, AC : AE :: $n+1$: 1. Upon the Diameter EC describe the Semicircle EBC; and on the Center A with the Radius AD describe the Arch DB, intersecting the Circle in B. Draw AB and BC; then let fall the Perpendicular AF on CB continued out to F. So shall ABF and ACF be the Angles of Incidence and Refraction required. Fig. 5.

15. For drawing BE parallel to AF, the Triangles ACF and ECB are similar. Now the Sine of the Angle ABC or ABF is to the Sine of ACB as AC to AB = AD, that is,

often they are. These *Halo's*, if the Hail be duly figur'd, will be colour'd, and must then ap-

as I to R; therefore if ABF be the Angle of Incidence, ACF will be the Angle of Refraction. Moreover, the nascent Increment of ABF is to that of ACB (generated in the same Time) as CF to BF, (by *Art.* 13.) that is, as CA to AE, (by similar Triangles) that is, as $n+1$ to 1 by Construction. The Ratio therefore of the nascent Increment of the Angle of Incidence ABF, to that of the Angle of Refraction ACB, is that which is required in the Angles of Incidence and Refraction of an efficacious Ray, after a given Number of Reflections, (in *Art.* 12.) Consequently the Angles ABF and ACF are those required. *Q. E. D.*

16. From this Construction we easily deduce Sir *Isaac Newton*'s Rule for finding the Angle of Incidence ABF in *p.* 148, 149. of his *Optics*, thus. We had AC : AB :: I : R, whence $AC = \frac{I}{R} \times AB$. Also CF : BF :: $n+1 : 1$; therefore CF $= \overline{n+1} \times BF$, or (putting $n+1 = m$) $CF = m \times BF$; and because of the Right Angle at F, it is $AC^2 - CF^2 = AB^2 - BF^2$, that is, $\frac{II}{RR} AB^2 - m^2 FB^2 = AB^2 - BF^2$; and therefore $m^2 FB^2 - FB^2 = \frac{II}{RR} AB^2 - AB^2$; and consequently $\frac{FB}{AB} = \sqrt{\frac{II - RR}{m^2 RR - R}}$.

17. Hence, because in the first Bow the Ray emerges after one Reflection, we have $n = 1$, $m = 2$, $m^2 = 4$, $m^2 - 1 = 3$; therefore $\sqrt{3RR} : \sqrt{II - RR}$ :: AB : BF :: Radius : Co-Sine of the Angle of Incidence. In the second Bow, where there are two Reflections, $m^2 - 1 = 8$; whence $\sqrt{8RR} : \sqrt{II - RR}$: AB : BF. In the third Bow, after three Reflections, $m^2 - 1 = 15$; and $\sqrt{15RR} : \sqrt{II - RR}$: AB : BF; and so on for any given Number of Reflections.

18. To find the Values of I and R, it must be remember'd, that the Ratio of the Sines of Incidence and Refraction was shewn to be constant, (in *Annot.* CXVII. 13.) and therefore their Excesses in divers Sorts of Mediums are also in a given Ratio. Thus it was shewn, that in the least refrangible Rays I : R :: 50 : 77, out of Glass into Air; the Excess of R above I is here 27. If the Refraction be made out of Rain-Water

pear

pear *red* within by the least refrangible Rays, and *blue* without by the most refrangible ones.

into Air, then it is I : R :: 3 : 4 very nearly for the least refrangible Rays; the Excess here is $4 - 3 = 1$. Wherefore say, As 1 : 27 :: 3 : 81 :: 4 : 108. Whence it appears, that the Sines I and R out of Water into Air are as 81 to 108, in the least refrangible Rays: And if to the lesser Sine you add the given Differences between those Sines out of Glass into Air for all the other Sorts of Rays, *viz.* $27\frac{1}{8}$, $27\frac{1}{5}$, $27\frac{1}{3}$, $27\frac{1}{2}$, $27\frac{2}{3}$, $27\frac{7}{9}$, 28; we shall have the several Values of R for those Rays, *viz.* $108\frac{1}{8}$, $108\frac{1}{5}$, $108\frac{1}{3}$, $108\frac{1}{2}$, $108\frac{2}{3}$, $108\frac{7}{9}$, 109.

19. But since the Refraction here is not out of Water into Air, but the contrary, we shall have the Values of I and R interchanged; or they will stand for the several Sorts of Rays as below.

For the Red,	I : R :: 108 : 81	Extreme.
For the Orange,	I : R :: $108\frac{1}{8}$: 81	Beginning.
For the Yellow,	I : R :: $108\frac{1}{5}$: 81	Beg.
For the Green,	I : R :: $108\frac{1}{3}$: 81	Beg.
For the Blue,	I : R :: $108\frac{1}{2}$: 81	Beg.
For the Indigo,	I : R :: $108\frac{2}{3}$: 81	Beg.
For the Violet,	I : R :: $108\frac{7}{9}$: 81	Beg.
For Violet,	I : R :: 109 : 81	Extreme.

20. Wherefore in the least refrangible Rays, since $I = 108$, $II = 11664$; also $R = 81$, and $RR = 6561$, and $I^2 - R^2 = 5103$; $\sqrt{3RR} = 140,3$, and $\sqrt{I^2 - R^2} = 71,4$. Therefore say, (by *Art.* 17.)

As	$\sqrt{3RR}$	= 140,3	= 2,147045
Is to	$\sqrt{I^2 - R^2}$	= 71,4	= 1,853913
So is Radius		= 90° 00′	= 10,000000
To the Co-Sine of the Angle of Incidence	BAF	= 30° 37′	= 9,706868

22. Hence the Angle of Incidence ABF is 59° 23′, in the Red or least refrangible Rays. Wherefore in the Drop of Rain whose Axis is SQ, if we make the Arch BN = 59° 23′, we shall have SN the least refrangible Ray. Having given the Angle of Incidence, and the Ratio of I to R, we have also given the Angle of Refraction: For say,

As	I = 108	= 2,033424
Is to	R = 81	= 1,908485
So is the Sine of Incidence	59° 23′	= 9,934798
To the Sine of the Angle of Ref.	40° 12′	= 9,809859

The Reason why there is always a determinate Angle for exhibiting the *Bows*, or *Halo's*, is be-

Pl. XLII. Fig. 6. 22. Therefore, making the Angle $CNF = 40° 12'$, NF will be the refracted Ray; which at F is reflected into FG, and at G emerges in GR. Produce the incident and emergent Rays SN and RG till they intersect each other at X; and as CF bisects the Angle NFG, so when produced it will bisect the Angle SXR. Then $CFN = CXN + FNX$, but $FNX = CNX - CNF$ or CFN; therefore $CFN = CXN + CNX - CFN$; that is, $2CFN - CNX = CXN$. Or $80° 24' - 59° 23' = 21° 01' = CXN$; therefore $2CXN = SXR = 42° 02'$, which is the Measure of the Angle that the incident and emergent Rays, which are the least refrangible, contain with each other.

23. If instead of the Ratio 108 to 81, we take that of 109 to 81, we shall find the Values of $\sqrt{3RR}$ and $\sqrt{I^2 - R^2}$ such as will give the Angle of Incidence BCN, or the Arch $BN = 58° 40'$, and the Angle $SXR = 40° 17'$, which will be the Case for the most refrangible, or extreme Violet Rays.

24. If the Ray be twice reflected, *viz.* at F and G, as in the Production of the exterior Bow, and emerges at H in the Direction HA intersecting the incident Ray SN in Y; then we may find the Angle AYS, which those Rays contain with each other, thus. Produce AH, till it meets GX produced in R; then in the Triangle HGR, the external Angle $HGX = HRG + GHR$. But because of equal Angles of Reflection at F and G, it is $GHR = FGX$; therefore $HGX - FGX = HGF = HRG = 2CGF$ or $2CNF$. And (in *Art.* 22.) we had $SXR = 4CNF - 2CNX$; therefore in the Triangle YXR we have the two internal Angles $R + X = 6CNF - 2CNX =$ the external Angle at Y, *viz.* AYN.

25. In this Case to find the Angles of Incidence and Refraction, we have $\sqrt{8RR} : \sqrt{I^2 - R^2}$ :: Radius : the Co-Sine of the Angle of Incidence; whence the said Angle of Incidence will be found $71° 50' = CNX$. And as 108 : 81 :: Sine of $71° 50'$: Sine of $45° 27' = CNF$ the Angle of Refraction; therefore $45° 27' \times 6 - 2 \times 71° 50' = 129° 02' = AYN$, and therefore its Complement $AYS = 50° 58'$ the Angle required, for the least refrangible Rays.

26. But for the most refrangible Rays, where I : R :: 109 : 81, we have the Angle of Incidence $71° 26'$, and the Angle of Refraction $44° 47'$; and therefore the Angle $AYS = 54° 10'$.

cause

cause there is but one particular Point N in all

After this Manner you proceed to calculate the same Angles after three, four, or more Reflections; but because the Beam in being so often reflected loses so many of its Rays, that the remaining refracted Part is in general too faint to excite the Idea of Colours, we pass it by, and proceed to apply what has been said to account for the *Phænomena* of the Bows, which so strongly strike the Eye; the principal whereof here follow.

127. The *First* is, *That each is variegated with all the Prismatic Colours.* This is a necessary Consequence of the different Refrangibility of the Rays refracted and reflected in Drops of falling Rain. Let *A* be such a Drop, SN a Ray entering it at N, which is refracted to F, from thence reflected to G, where, as it emerges, it is refracted into all the several Sorts of Rays of which it is composed, *viz.* GR the least refrangible or *Red-making Ray*, GO the *Orange*, GY the *Yellow*, GG the *Green*, GB the *Blue*, GI the *Indigo*, and GV the *Violet* or most refrangible Ray.

28. Now we have shewn (*Art.* 22, 23.) that the Angle SFR is to the Angle SFV as $42^{\circ}\ 02'$ to $40^{\circ}\ 17'$; the Difference whereof is the Angle VGR $= 1^{\circ}\ 45'$. Through this Angle all the original Rays are diffused; and though the Angle be small, yet at a great Distance it spreads to a considerable Width; and therefore by coming from the Drop *A* to the Eye of a Spectator at A, they will be sufficiently separated, and fall upon the Eye singly, each Sort of Rays by themselves alone. Pl.XLIII, Fig. 1.

29. Hence, were there only one Drop *A*, the Eye at A would see only one Colour in that Drop, *viz.* the *Red*, by the least refrangible Ray GR; the others, GO, GY, *&c.* being refracted above it, as is evident enough in the Figure. If now we suppose this Drop to descend to the Situation *B*, then would the Orange-making Ray GO fall upon the Eye continuing in A, and then the Drop would exhibit an Orange Colour. If after this it should sink down to *C*, the Yellow-making Ray GY would enter the Eye at A, and excite the Idea of Yellow in the Drop at G. And so continually, if we suppose the Drop to succeed to the several Situations *D*, *E*, *F*, *G*, the other more refrangible Rays GG, GB, GF, GV, will fall upon the Eye successively, and raise the Sensation of their proper Colours, Green, Blue, Indigo, and Violet when the Drop is at *G*.

30. The Truth of this may be easily proved by Experi-

the

the Part of the Drop between B and L, where

ment, by suspending a Glass Globe fill'd with Water in the Sunshine, and viewing it in such a Position that the Rays SN which fall upon it may emerge to the Eye at A, under the several Angles from SFR to SFV; which may be easily effected by letting the Globe descend from *A* to *G* by a String going over a Pulley. And this was the famous Experiment of *Antonius de Dominis* and *Des Cartes*, who by this means confirm'd the Truth of their Doctrine of the Rainbow, which had been demonstrated mathematically. The same Thing may be also shewn, if the Globe be at Rest at *A*, and the Eye be raised from R to V.

31. If now, instead of depressing the Drop from *A* to *G*, we suppose a Drop placed in each Point *A, B, C, D, E, F, G*; then these will severally send an original Ray to the Eye, according to their Situations in respect of it. Thus the Drop in *A* will refract the Red-making Ray GR; the Drop *B* will refract the Orange Go; the Drop *C* the Yellow Gy; and so the other Drops *D, E, F, G*, will by the Rays Gg, Gb, Gi, Gv, excite the several Colours, Green, Blue, Indigo, Violet, all at the same Time; and therefore all that Part of the Rain from *A* to *G* will appear variously colour'd, as is represented in the Scheme.

31. Now let SP be a Line drawn through the Spectator's Eye at A, parallel to the Sun's Rays SN, and conceive the several Rays GR turning about the Line AP as an Axis, and always under the same invariable Angle GRP; 'tis evident, the Extremity of each Ray would in the Cloud or Rain describe a Circle which would be the Base of a Cone whose Axis is AP, and its Vertex A; and for the same Reason that the Drop *A* excites the Sensation of Red, every Drop in the Circle described by the Extremity of the Ray GR will excite the same Sensation; thus will a red circular Arch *AH* be form'd as far as the Rain extends. Next to that the Ray *BA*, by revolving, describes the Arch of an Orange Colour, as *BI*; the Ray *CA* will in like manner trace out the Yellow Circumference, as *CK*; and so of all the rest, as represented in the *Figure*.

32. Hence the *Second Phænomenon, viz.* the *circular Form*, is accounted for; and also the *Third*, which is *the Breadth of the Bow*; for that will be equal to the Angle $ARG = RGV = 1° 45'$, where the Ray, as here, emerges after one Reflection. These Particulars are represented more compleatly in the *Figure* where BGD is the red Circumference

Pl. XLIII. Fig. 2.

the

the Rays AN can enter, so that after a second

form'd by the Rotation of the Ray AG, that can first come to the Eye at A; and CgE is the Violet Arch form'd by the least refrangible Ray gA; after which the Rays are all refracted below the Eye. And thus by the intermediate Rays and Colours the whole interior Bow is produced.

33. The *Fourth Phænomenon* is the *Appearance of Two Bows*. This follows from hence, that after an efficacious Ray of Light SN, entering a Drop of Rain, has been twice reflected on the farthest Side at F and H, it will emerge refracted into all its simple or constituent Rays at G upon the upper Side of the Drop, so as to make with the incident Ray the Angle GYN or SYA $= 54^\circ\ 10'$, if that Ray be the Violet Sort, or most refrangible, (by *Art*. 26.) but if it be of the red or least refrangible Sort, then the said Angle is but $50^\circ\ 58' =$ SyA, (by *Art*. 25.)

34. Therefore all those Drops which are so situated around the Eye, that their most refrangible Rays shall fall upon it, must with those Rays make an Angle with the Line AP passing through the Eye parallel to the Sun's Rays, *viz*. the Angle GAP, equal to the Angle SYA, or GAP $= 54^\circ\ 10'$. These Rays therefore will every where exhibit a Violet Colour in the Arch PGL. For the same Reason those Drops whose least refrangible Rays fall upon the Eye at A, make the Angle gAP $= 50^\circ\ 58'$; and so the Ray Ag, revolving about the Axis AQ, will describe the circular Arch MgK, which will exhibit the deepest Red; and all the Drops between G and g will paint the several other colour'd Peripheries, all which together will compleat the exterior Bow.

35. The *Fifth Phænomenon* is the *greater Breadth of the exterior Bow*. Thus, if from $54^\circ\ 10'$ we subduct $50^\circ\ 58'$, we shall have $3^\circ\ 12' =$ G$g =$ the Width of the outer Bow; which therefore is almost twice as wide as the interior Bow.

36. The *Sixth Phænomenon* is the *Distance between the two Bows*, which is thus determined: From the Angle which the least refrangible Ray in the upper Bow makes with the Axis AP, *viz*. $50^\circ\ 58'$, substract the Angle $42^\circ\ 02'$ which the most refrangible Rays make therewith in the lower Bow, and the Remainder $8^\circ\ 56' =$ gAF is the Arch of Distance between the Bows.

37. The *Seventh Phænomenon* is the *inverse Order of the Colours in the two Bows* This follows from the contrary Parts of the Drop on which the Ray is incident, and from whence it emerges and is refracted. Thus because the Rays

Refraction

SN enter the upper Part of the Drop and emerge from the lower, 'tis evident the Rays refracted in this Case (*viz.* in the interior Bow) will have a Situation quite the reverse of those which enter on the lower Part of the Drop, and are refracted from the upper, as in the exterior Bow, whose Colours are *Violet*, *Indigo*, *Blue*, *Green*, *Yellow*, *Orange*, and *Red*; whilst those of the other are *Red*, *Orange*, *Yellow*, *Green*, *Blue*, *Indigo*, and *Violet*; counting from the upper Parts downwards in both.

38. The *Eighth Phænomenon* is the *Faintness of the exterior Bow in Comparison of the interior one.* This is the Consequence of the Rays being twice reflected within the Drops which form the outer Bow. They who make the Experiment in a dark Chamber may wonder when they observe how large a Part of the Beam (that enters the Globule at N) goes out at F, that there should be enough in the remaining Part FG to exhibit the Colours so strong and vivid in the first Bow as they appear; but then considering how much of this residual Ray is refracted at G, 'tis rather a Wonder how the very small Part reflected to H should there when refracted be in Quantity sufficient to excite any distinct Ideas of Colours at all.

39. The *Ninth Phænomenon* is, *that sometimes more than two Bows appear*; as in a very black Cloud I have myself observed *four*, and a faint Appearance of a *fifth*: But this happens rarely. Now these spurious Bows, as I may call them, cannot be form'd in the Manner as the two principal Bows are, that is, by *Refraction* after a *third*, *fourth*, *fifth*, &c. *Reflection*; for the Beam is by much too weak to exhibit Colours by Refraction, even after the *third* Reflection only, much less would it after a *fourth* or *fifth*. Besides, though after a third and fourth Reflection of the Rays they should be supposed capable of shewing their Colours, yet the Bows made thereby would not appear at the same Time with the other two, nor in the same Part of the Heavens, but in the Rain between us and the Sun, and must be view'd by the Spectator's Face turn'd towards the Sun, and not from it, as in the other Case.

40. To account for the Appearance of these colour'd Rings within the interior primary Bow, we shall here transcribe what the learned Dr. *Pemberton* has wrote upon the Subject. He observes, that Sir *Isaac Newton* takes notice, that in Glass which is polish'd and quicksilver'd there is an irregular Refraction made, whereby some small Quantity of

and G for the *Bows*, there can enough go out to-

Light is scatter'd from the principal reflected Beam. If we allow the same Thing to happen in the Reflexion by which the Rainbow is caused, it seems sufficient to produce the Appearance now mention'd.

41. Let AB represent a Globule of Water, B the Point from whence the Rays of any determinate Species being reflected to C, and afterwards emerging in the Line CD, would proceed to the Eye, and cause the Appearance of that Colour in the Bow which appertains to this Species. Here suppose, that besides what is reflected regularly, some small Part of the Light is irregularly scatter'd every Way; so that from the Point B, besides the Rays that are regularly reflected from B to C, some scatter'd Rays will return in other Lines, as in BE, BF, BG, BH; on each Side of the Line BC. Pl. XLIV. Fig. 1.

42. Now it has been observed, (*Annotat.* CXXI.) that the Rays of Light in their Passage from one Superficies to another, in any refracting Body, undergo alternate *Fits of easy Transmission and Reflection*, succeeding each other at equal Intervals, insomuch that if they reach the farther Superficies in one of those Fits, they shall be transmitted; if in the other, they shall be reflected back. Whence the Rays that proceed from B to C, and emerge in the Line CD, being in a Fit of *easy Transmission*, the scatter'd Rays that fall at a small Distance without these on either Side (suppose the Rays BE and BG) shall fall on the Surface in a Fit of *easy Reflection*, and so will not emerge; but the Rays next to these, *viz.* BF and BH, shall arrive at F and H in a Fit of *easy Transmission*, and so be refracted in the Rays FI and HK.

43. Now these Rays will emerge, so as to contain a less Angle with the incident Beam SN than the Ray CD, which was shewn to make the greatest Angle therewith of all others whatsoever: (See *Art.* 5, 22, 23.) The Colours therefore which they exhibit must appear within those of the primary Bow. And if we suppose other scatter'd Rays without these to emerge (having the intermediate Rays intercepted by Reflection) they will contain Angles still less with the incident Ray SN, and will therefore form colour'd Arches still within the former: And this may be conceived for divers Successions.

44. Now as the scatter'd Rays by various *Reflections* and *Refractions* form Arches variously mix'd together, some of these made by the lighter Colours may be lost in the inferior Part of the primary Bow, and may contribute to the red Tincture which the Purple of that Bow usually has. The

gether

gether at G or H, to form a strong and distinct

darker Colours of those refracted scatter'd Rays form the Arches which reach below the Bow, and are seen distinct; of which the first has a *light Green*, *dark Green*, and *Purple*; the second has a *Green* and *Purple*; the third a *faint Green* and *vanishing Purple*.

45. The Distances between the Bow and these secondary Arches depend on the Size of the Drops; to make them in any degree separate, 'tis requisite the Drops should be exceeding small. It is therefore most likely, that they are form'd in the Vapour of the Cloud, which the Air, being agitated by the Rain, may carry down with the larger Drops; and this may be the Reason why they never appear but under the upper Part of the Bow only, this Vapour not descending very low. As a Confirmation of this, these Arches are seen strongest when the Rain falls from very black Clouds, which cause the fiercest Rains, and therefore produce the greatest Agitation of Air. Thus far Dr. *Pemberton*.

46. But to return: The *Tenth Phænomenon* is, the *Appearance of the Bows in that Part of the Heavens opposite to the Sun*. This necessarily happens from the incident and emergent Ray being both on one Side of the Drop, for 'tis evident, that in order to see the Colours, we must look to that Part against which the Sun shines.

47. The *Eleventh Phænomenon* is, *that they never appear but when and where it rains*. This is because Rain affords a sufficient Plenty of Drops, or aqueous Spherules, proper to reflect and refract the Light fit for this Purpose, which cannot be done without a requisite Size, Figure, and Disposition of the Particles, which the Vapour of the Cloud does not admit, and therefore Clouds alone exhibit no such Appearance.

48. The *Twelfth Phænomenon* is, *the Dimension of the Bows*. This is determined easily, for continuing the Axis
Pl.XLIII. AP to Q the Centre of the Bows, we have the Semidiameter
Fig. 2. of each Bow in the Angle QAg, or QAG; the double of which gives the Angles which the whole Diameters of the Bows subtend, and are therefore the Measure of their Magnitude.

49. The *Thirteenth Phænomenon* is, *the Altitude of the Bow above the Horizon, or Surface of the Earth*. This is equal to the Angle GAT, which may be taken by a Quadrant, or it may be known for any Time by having given the Sun's Altitude, which is equal to the Angle TAQ; which therefore subducted from the constant Angles QAF, or QAY, will always leave the Angle of the apparent Height of the Bow.

Image

Image of the Sun; which Rays, therefore, en-

50. Hence it follows, that when the Sun is in the Horizon, the Lines QA and TA will coincide, and therefore the Points Q and T; whence, in this Case, the Bows will appear compleat Semicircles; as on the other hand, when the Altitude of the Sun is equal to the Angle QAF $= 42^\circ\ 02'$, or to QAY $= 54^\circ\ 10'$, the Summits of the Bows will be depress'd below the Horizon, and therefore within a certain Interval in many Days, in Summer-Time, no Rainbow can appear.

51. We have hitherto consider'd the Bows, and given their Dimensions, such as they would have were the Sun but a Point; but because the Sun subtends an Angle of half a Degree, or 30 Minutes at a Mean, therefore the Breadths of the Bows will be increased, and their Distance decreased by half a Degree, and so the Breadth of the interior Bow will be $2^\circ\ 15'$, and that of the exterior one $3^\circ\ 42'$, and their Distance $8^\circ\ 26'$; also the greatest Semidiameter of the interior Bow $42^\circ\ 17'$, and the least of the exterior Bow $50^\circ\ 43'$.

52. For let SFA be the Angle of any one particular colour'd Ray coming from the Centre of the Sun, and reflected from the Drop to the at Eye A. In the Ray SF take any Point S at Pleasure, and make the Angles FSN, FSM, each equal to $15'$, as also the Angles FAM, and FAN; then will SN be Part of a Ray *n*N coming from the lower Limb of the Sun, SM a Part of a Ray *m*M coming from the upper Limb; and so the whole Angle NSM $=$ *m*S*n* $= 30'$, the Sun's apparent Magnitude. Pl. XLII. Fig. 7.

53. Join SA; and since the Sums of the Angles at the Base SA of the several Triangles ASN, ASF, ASM, are equal among themselves, their vertical Angles at N, F, M, are also equal to each other. Wherefore the Angle SMA will be that which the emergent Ray makes with the incident Rays SM of the same Colour, as before, coming from the highest Point *m* of the Sun, and SNA of that which comes from the lowest Point of the Sun *n*. Therefore, if all the Rays of the Sun were of one Sort, the apparent Breadth of the Bow, measured by the Angle MAN, would be but $30'$ or half a Degree.

54. But since the Rays of the Sun are differently refrangible, conceive the Drop F to be placed any where in the inward or outward Verges of the Bows (above described) and then it is manifest that the Angle FAM must be added to the Inside, and FAN to the Outside of the Angles, which

tering

tering at the Point N, are call'd *Efficacious Rays*;

the Breadths of those Bows subtend at A, to obtain their apparent Breadths; which therefore will be such as are defined in *Article* 51.

55. I design'd here to have added Dr. *Halley*'s Method of discovering the Ratio of the Sine of Incidence to that of Refraction, by having given the Angle which an efficacious Ray, as S N, contains with its emergent Part G A; but as this Angle is determined only by Experiment, and the Calculation brings us to a Cubic Equation, I think it a Matter of too much Intricacy to trouble the Reader with in this Place, especially as it is so easy to determine the refractive Power of any transparent Bodies, by the experimental Methods before deliver'd. (See *Annotat.* CXVII.)

56. I have often taken Notice (with Mr. *Whiston*) of the Silence of Authors concerning the Reason why the *Iris*, or rather a strong and deeply colour'd *Corona* does not appear about the Sun in the falling Drops of Rain, at the Distance every way of about 25 Degrees; because at that Distance from the Axis, the efficacious Rays S N, *sn*, *po*, after Refraction into the Drop are refracted a second Time at F towards the Eye at I. For if S B Q be the Axis of the Drop, we have shewn the Angle B N = 59° 23′ (*Art.* 21.) and the Arch N F = 99° 36′; therefore the Arch F Q = 21°, wherefore in a Glass Globe of Water, held in the Sun's Light in a dark Room, we see a colour'd Circle or *Corona* A D F of about 42 Degrees in Diameter, and the Superficies within it extremely luminous, as containing all the Sun Beams that fall on the fore Part within 60 Degrees all round the Axis.

Pl. XLIV. Fig. 2.

57. But because the Rays are there promiscuously blended together, they produce only a white Light; whereas, on the Circumference F D A, where the efficacious Rays fall, there all the Colours of the Bow appear; and from thence many have wonder'd why we see not a Circle colour'd with stronger Tints than even the primary Bow itself, from this Refraction of the efficacious Rays in all the Drops of Rain between us and the Sun from the Circle F D A. But we are to observe, with Mr. *Whiston*, that the efficacious Rays S N, *sn*, *po*, which are parallel when incident on the Drop, are not so when refracted at their Emergence at F; for being there refracted to one Point, they are not parallel within the Drop, and therefore cannot be so after their Emergence, but will proceed diverging to the Eye at I in the several Directions F I, F *r*, F *q*, and therefore will not be sufficiently dense,

to distinguish them from the rest which are in-

and at the same time too much blended with others to excite any Sensation of Colours.

58. But why it should be said, this variegated Circle ought to appear at the Distance of about 26 Degrees from the Sun, I do not see; for the refracted Ray F I contains an Angle F M G with the Incident Ray S N (produced to G) of 38° Plate
$22'$; for 'tis plain that $NM = MF$, and therefore the Angle XLIV.
$MNF = MFN = CNM - CNF = 59^{\circ}\,23' - 40^{\circ}\,12'$ Fig. 3.
$= 19^{\circ}\,11'$; but the external Angle $IMG = MNF + MFN = 38^{\circ}\,22'$, and consequently this is the Angle of Distance at which such a Bow must appear all around the Sun.

SCHOLIUM.

59. In *Article* 5. it is asserted, that the efficacious Ray S N makes the Arch Q F a *Maximum* by its refracted Part N D. To prove this, let Radius $CN = CB = 1$, the versed Plate
Sine $BA = x$, and $CD = z$, and by the Nature of the Cir- XLIII.
cle $QB : AN :: AN : AB$; therefore $AN = \sqrt{2x}$. Again, Fig. 3.
let $a =$ Ratio of the Incidence and Refraction; then because $I : R :: ND : CD$, we have $ND = \frac{I}{R}CD = az$; likewise from the similar Triangles B N D and F Q D, we have $ND : NB :: QD : QF$; that is, $az : \sqrt{2x} :: z - 1 : \frac{z-1}{az}\sqrt{2x} = QF$.

60. This Value of Q F is to be determined to a *Maximum*; in order to this we neglect the given Part $\frac{\sqrt{2}}{a}$, and take the variable Part $\frac{\overline{z-1} \times \sqrt{x}}{z} = \frac{z\sqrt{x} - \sqrt{x}}{z} = \sqrt{x} - \frac{\sqrt{x}}{z} = x^{\frac{1}{2}} - \frac{x^{\frac{1}{2}}}{z}$; then making its Fluxion $\frac{1}{2}x^{-\frac{1}{2}}\dot{x} - \frac{1}{2}z^{-1}x^{-\frac{1}{2}}\dot{x} + z^{-2}x^{\frac{1}{2}}\dot{z} = 0$; or (multiplying by $2z^{2}x^{\frac{1}{2}}$) $z^{2}\dot{x} - z\dot{x} + 2x\dot{z} = 0$. Whence $2x\dot{z} = z\dot{x} - z^{2}\dot{x}$, and $\dot{z} = \frac{z\dot{x} - z^{2}\dot{x}}{2x}$.

61. We must now find another Value of $\dot{z}$ in order to exterminate it, which we find from the right-angled Triangle N A D, where $ND^{2} = AN^{2} + AD^{2}$; that is, $a^{2}z^{2} = x^{2} + 2z - 2xz + 1$, which in Fluxions is $2a^{2}z\dot{z} = 2z\dot{z} +$

effectual (CXXIV.)

$z\dot{z} - 2\dot{x}z - 2x\dot{z}$; therefore $\dot{z} = \frac{z\dot{x}}{z - a^2 z - x - 1} = \frac{z\dot{x} - z^2\dot{x}}{2x}$; whence $z^2 - a^2 x^2 - xz + a^2 z + 3x - 1 = 0$; now by means of this and the preceding Equation $a^2 z^2 = z^2 + 2z - 2xz + 1$, if we throw out x, we shall get this cubic Equation $z^3 - a^2 z^3 - a^2 z^2 + z^2 + 3z + 3 = 0$; whose Roots will be found $z = -1$, $z = -\sqrt{\frac{3}{a^2-1}}$; $z = \sqrt{\frac{3}{a^2-1}}$; of which the two first being negative, are of no Use; therefore the Arch QF is a *Maximum* when $CD = \sqrt{\frac{3}{a^2-1}}$.

62. By inserting this Value of z into the Equation $a^2 z^2 = z^2 + 2z - 2xz + 1$; we get this Equation $x = \sqrt{\frac{3}{a^2-1}} + 1 + \overline{\tfrac{1}{2} - \tfrac{1}{2}a^2} \times \sqrt{\frac{3}{a^2-1}}$. And since in case of the least refrangible Rays, we have $a = \frac{4}{3}$, therefore $\sqrt{\frac{3}{a^2-1}} = 1{,}964 = CD$; and $AB = 0{,}4908$ the versed Sine of the Angle $BCN = 59^\circ\ 23'$, the same as was found before in *Article* 21. Also if we put $a = \frac{109}{81}$ (See *Art.* 19.) we shall have the Angle $BCN = 58^\circ\ 40'$ for the most refrangible Rays, as in *Article* 23.

63. Also we get the Value of $QF = \frac{z-1}{az}\sqrt{2x} = 0{,}3648$, which is the Chord of $21^\circ\ 02' = QCF$, as was before shewn, *Article* 56. Hence all the Particulars relating to the *principal Bow* are easy to be understood; and this is an egregious Instance of the extreme Usefulness of the *Fluxionary Calculus* in Natural Philosophy. This noble Theorem was first given us by Mr. *Stewart* in his Comment on Sir *If. Newton*'s Quadratures.

64. If BNQ were a Globe of Glass, then $a = \frac{3}{2}$,

CD

CD $= n = 1,5492$, and the Arch BN $= 49° 48'$; also the Arch QF $= 11° 22'$. Some other considerable Uses, which may be made of this Theorem, will be consider'd in the next Lecture of *Optics*.

(CXXIV.) 1. Concerning the Production of Halo's, our illustrious Author has left us to make the best Shift we can in accounting for it; having said nothing of this Phænomenon that can be of any Service to help us in this Disquisition He intimates, indeed, *that Halo's are form'd by the Light which comes through the Drops of Rain by two Refractions* (viz. at N and F) *without any Reflection*; but how this can be is not easy to conceive. We have shew'd that a Rainbow or deeply colour'd Ring might have been expected at the Distance of about 38 Degrees from the Sun, and also why it cannot happen. Pl. XLIV. Fig. 3.

2. For the same Reason we should also not expect an Halo to be form'd by the same refracted Rays, *viz.* on account of their not being refracted parallel to the Eye, and consequently not entering it dense enough to render that Part of the Heavens more luminous than the rest, or to produce the lucid Ring we call by this Name. Again, Sir *Isaac* says, *it ought to appear strongest at the Distance of about* 26 *Degrees from the Sun* (*viz.* when the Angle IMG $= 26°$) *and to decay gradually both ways*. But though our Author did not undoubtedly assert any Thing without very great Reason, yet this does not appear to us.

3. For that the Angle IMG may be 26 Degrees, the Angle of Incidence BCN must be about 46, and then the Angle of Refraction CNF will be near 33 Degrees; but why such an Incidence and Refraction should cause the Rays to be refracted in greater Plenty to the Eye than any other, does not appear to me; nor can I find it by any Experiment. On the contrary, as the Angle IMG increases with the Angle of Incidence, and consequently with the Angle of Refraction, it is evident that with respect to heterogeneal Light, the greater the Angle IMG is; the more will it be refracted and scatter'd; and consequently, the farther the Drops are situate from the Sun, the less dense will be the Light transmitted by Refraction to the Eye, which therefore ought to decrease as the Distance from the Sun increases.

4. As Sir *Isaac Newton* has said but little, so his Expositors, Dr. *Pemberton* and Dr. *s'Gravesande*, have thought fit to be absolutely silent on this Head. Mr. *Huygens* has advanced an Hypothesis by which the *Phænomenon* may be solved, if we grant him all his Petitions. And since none of

our great Philosophers, not even Sir *Isaac* himself, have undertook to disprove it, but on the contrary seem rather to approve of it, as Sir *Isaac* in his *Optics*, and Dr. *Smith* in his *Optics* has adopted the same entirely; I think upon these Accounts, and considering the Character of the great Author, the Reader will be pleased to have the same in a very concise Manner represented to him.

5. His *Postulatum* is, *That there are certain Globules in the Atmosphere consisting of a Coat or Shell of transparent Ice or Water, containing an opake Nucleus or Kernel within*; and that these are made from Particles of Snow, (which is in itself opake) attracting the aqueous Particles in the Vapour or Exhalation by which it is sustain'd, which gathering together form the pellucid Shell of Water, or are frozen into a crystalline Shell of Ice; and this he thinks is proved to be Matter of Fact by the Hail-stones which fall to the Earth, for these (says he) when broken do discover some Snow at the Center.

Plate XLIV. Fig. 4. 6. These Things premised, he addresses himself to the Solution as follows: Let ABCD represent such a Globule, with the opake Nucleus EF in the Middle of it; and let us suppose the Rays coming from G, H, to fall on the Side AD. It is manifest they will be refracted inwards from the Surface AD; from whence it follows, that a great Number of them must strike upon the Kernel EF.

7. Let GA and HD be the Rays which after Refraction touch the Sides of the Kernel EF, and let them be refracted again at B and C, emerging in the Lines BK, CK, crossing each other in the Point K, whose Distance from the Globule is somewhat less than its Semidiameter.

8. Wherefore, if BK and DK be produced towards M and L, it follows, that no Light coming from the Sun through the Globule can proceed to the Eye any where placed within the Angle LKM, or rather in the Cone which that represents, supposing that the Obliquity of the incident Rays HD and GA is such as shall make the Arch QC and QB the greatest possible; (see the last Note, *Art.* 5.) for then all the Rays exterior to HD, GA, will be refracted nearer to Q, and after Emergence cross each other in a Point *k* nearer the Globule than the former, and therefore cannot come at the Eye placed within the said Cone LKM.

Fig. 5. 9. Suppose now the Eye placed at N; and let NR, NQ, be drawn parallel to LK and MK; then 'tis plain, none of the Globules (the same as ABCD) within the Cone RNQ can come to the Eye at N. Thus the Globules at O and P have their refracted Rays *akb* and *ckd* including the Eye in

the

the Cone of Obscurity: But other Globules, which lie without the Cone QNR, as S and T, do not involve the Eye N by their shady Cones *lke* and *fkm*; and therefore some of those Rays, which are more refracted than *ke* or *kf*, will fall upon the Eye, and produce a luminous circular Ring or *Corona*, including a dark Area within, and whose Light outwardly decreases as it is more remote from the Center.

10. Much after a like Manner this great Man undertook to account for the Appearances of *Mock-Suns* and *Mock-Moons*, call'd *Parhelia* and *Paraselenæ*; which I shall not here detain the Reader with, because I cannot help thinking the Whole is but too much like a mere (though ingenious) Hypothesis; having never observed in any Hail-stones any such opake Kernels, so regularly form'd, and surrounded by such regular Shells of pellucid Ice as is here supposed.

LECTURE IX, X.

Of OPTICS.

Of the Science of OPTICS *in general. Of* CATOPTRICS *and* DIOPTRICS. *Of diverging, converging, and parallel* RAYS. *Of the several Kinds of* MIRROURS *and* LENSES. *Of the* FOCUSES *of* Rays; *the* CALCULATIONS *thereof, and* THEOREMS *for every Case. Of* OBJECTS *and their* IMAGES, *with* THEOREMS *relating thereto for every Kind of Glass. The* THEORY *of* VISION *explain'd. The several Parts of the* EYE *described. Of the* DEFECTS *of* VISION, *and how remedied by* SPECTACLES *of several Sorts. Of* READING-GLASSES. *Of* SINGLE-MICROSCOPES *of every Sort by* Reflection *and* Refraction. *Of* DOUBLE-MICROSCOPES *by Reflection and Refraction. Their* Structure *and* Use *explain'd. A* New POCKET-MICROSCOPE *described, furnish'd with a* MICROMETER. *The* Nature, Structure, *and* magnifying Power *of a refracting* TELESCOPE *of every Sort; the Reason of their* Imperfection *explain'd. Of* REFLECTING TELESCOPES, *with their Theory at large explain'd. Of the* CAMERA OBSCURA, *and its various Uses. Of the* SCIOPTRIC BALL *and* SOCKET. *Of the* SOLAR TELESCOPE; *and* SOLAR MICROSCOPES *of several Sorts. Of the*

new-invented HELIOSTATA *of* s'Gravesande, *with its* Theory, *and Manner of Use explain'd.*

WE are now arrived to that Part of Natural Philosophy which treats of *Vision*, and the various Phænomena of visible Objects, by Rays of Light reflected from Mirrours, and transmitted through Lenses, which constitute the Subject of the most delightful Science of OPTICS. (CXXV.)

(CXXV) 1. OPTICS is divided into Two Parts, CATOPTRICS and DIOPTRICS; the former treats of Vision by Light reflected from Mirrours or polish'd Surfaces, and the latter of Vision effected by Light transmitted through Lenses. Of these Lenses the several Sorts in Use are the Plano-Convex A, the Double-Convex B, the Plano-Concave C, the Double-Concave D, the *Meniscus* E, (which is convex on one Side, and concave on the other) and the Hemisphere F. The Line GH, that is perpendicular to and passes through the Middle of each Lens, is call'd the *Axis* of the Lens, and that Middle Point the *Vertex* of the Lens. Plate XLIV. Fig. 6.

2. As Rays of Light fall on these Glasses, they are variously reflected and refracted, as above described in the Lecture. The Theorems which shew the different Effects of all these Glasses in reflecting and refracting the Rays of Light, and forming the Images of Objects, are investigated several Ways; one of which is by *Algebra*. By this means Dr. *Halley* has raised a general Theorem extending to all the particular Cases of every Kind of Optic-Glasses of a spherical Form, and which I have largely applied and exemplified in my *Treatise of Optics*.

3. Another Method of doing this is by *Fluxions*, which is easy and universal, comprehending all the Cases of Mirrours and Lenses of every Form. This I propose to exhibit and illustrate here for Variety, and for the Genuineness and Excellency of this Method above all others, it depending on Principles that are more of a Philosophical than of a Mathematical Nature. It is as follows.

4. Let VBG be the Section of any curved Superficies of a Medium VGHI, V the Vertex, and AI the Axis of the Curve VG. From any Point in the Axis A let a Ray of Fig. 7.

THE principal Things here to be consider'd are, First, *the Rays of Light*; Secondly, *the Glasses by which they are reflected and refracted*; Thirdly, *the Theorems or Laws relating to the Formation of the Images of Objects thereby*; Fourthly, *the Nature of Vision, and Structure of the Eye*; and Fifthly, *the Structure and Use of the principal Optical Instruments*.

THE *Rays of Light* are distinguished into three Sorts, *viz. Parallel*, *Converging*, and *Diverging Rays*; *Parallel Rays* are such as in their Progress keep always an equal Distance from each other, as ABDC; such as are the Sun's Rays, in their natural State, with respect to Sense. *Converging*

Plate L. Fig. 4.

Light AB be incident on the Medium in B, which suppose refracted to a Point F in the Axis. Then, by having given the Distance of the radiating Point AD, and the Sine of Incidence BD, we are to find the focal Distance VF after Refraction.

5. To do this, from the Point B let fall the Perpendicular BD to the Axis; and putting $AV = d$, $AB = z$, $BF = v$, $VD = x$, $BD = y$, and $VF = f$, then will $DF = f - x$, $AD = d + x$, $z = \sqrt{y^2 + d^2 + 2dx + x^2}$, and $v = \sqrt{y^2 + f^2 - 2fx + x^2}$; and therefore in Fluxions we have $\dot{z} = \frac{y\dot{y} + d\dot{x} + x\dot{x}}{\sqrt{y^2 + d^2 + 2dx + x^2}}$, & $\dot{v} = \frac{y\dot{y} - f\dot{x} + x\dot{x}}{\sqrt{y^2 + f^2 - 2fx + x^2}}$.

6. But $\dot{z}$ and $\dot{v}$ being the Fluxions of the incident and refracted Rays, will represent their *Velocities* before and after Refraction, which Velocities we have shewn (*Annot.* CXVII.) are as the Sines of Incidence and Refraction m and n; whence $\dot{z} : \dot{v} :: m : n$. And from the Nature of Refraction (above explain'd) it is manifest that while the incident Ray increases, the refracted Ray decreases; therefore their Fluxions must have contrary Signs, *viz.* $+ \dot{z}$, and $- \dot{v}$. Wherefore $\dot{z} : - \dot{v} :: \frac{y\dot{y} + d\dot{x} + x\dot{x}}{\sqrt{y^2 + d^2 + 2dx + x^2}} : \frac{f\dot{x} - y\dot{y} - x\dot{x}}{\sqrt{y^2 + f^2 - 2fx + x^2}} :: m : n$.

Rays

Rays are ſuch as in their Progreſs approach nearer and nearer to each other, all of them tending towards a certain Point F, where they all unite; as the Rays of the Sun collected by a Glaſs, as C D F. *Diverging Rays* are thoſe which proceed from a Point, as F, and in their Progreſs recede from one another towards the Parts G E.

THE Point F, where the Rays are collected, is call'd the *Focus*, or Burning-Point, becauſe there the Sun's Rays, being united within a very ſmall Compaſs or Circle, are greatly conſtipated and condenſed, by which means their Action or Heat is proportionably increaſed, and therefore Objects poſited in that Point will be greatly heat-

7. Now becauſe in thoſe Mirrours and Lenſes which are of common Uſe in *Optics* we regard only the Focus of thoſe Rays which fall very near the Axis, in which Caſe the Arch B V is very ſmall, and therefore V D $= x = o$ nearly; therefore xx and $x\dot{x}$ may be rejected, without ſenſibly affecting the Value of the Expreſſions; therefore $m : n :: \frac{y\dot{y} + d\dot{x}}{\sqrt{y^2 + d^2}} : \frac{f\dot{x} - y\dot{y}}{\sqrt{y^2 + f^2}}$, and ſo $m \times \frac{f\dot{x} - y\dot{y}}{\sqrt{y^2 + f^2}} = n \times \frac{y\dot{y} + d\dot{x}}{\sqrt{y^2 + d^2}}$.

8. From which Equation we ſhall find $f =$ F V, in any Curve V G from the Equation expreſſing its Nature. Thus if V G be a CIRCLE, its Equation is $yy = 2rx - xx$, (where C B $= r =$ the Radius) the Fluxion of which is $y\dot{y} = r\dot{x} - x\dot{x}$; and ſince $x = o$, we have $yy = o$, $y\dot{y} = r\dot{x}$; and, ſubſtituting theſe Values in the general Equation above, we have $m \times \frac{f\dot{x} - r\dot{x}}{\sqrt{f^2}} = n \times \frac{r\dot{x} + d\dot{x}}{\sqrt{d^2}}$, and, dividing by $\dot{x}$, $m \times \frac{f - r}{f} = n \times \frac{r + d}{d}$; therefore $mdf - mdr = nrf + ndf$, and thence $\frac{mdr}{md - nd - nr} = f =$ V F.

ed,

ed, burnt, or melted.

OF GLASSES there are two Kinds, *viz. Mirrours*, and *Lenses*. A *Mirrour* or *Speculum* is that, which from one polish'd Surface reflects the Rays of Light; and these are either *Convex*, *Concave*, or *Plane*, as will be shewn. A *Lens* is any transparent or diaphanous Body, as *Glass*, *Crystal*, *Water*, &c. through which the Rays of Light do freely pass, and is of a proper Form to collect or disperse them. Of these there are several Species, as a *Plane Lens*, a *Plano-Convex*, *Plano-Concave*, *Double-Convex*, *Double-Concave*, and *Meniscus*.

I SHALL now consider the different Properties and Effects of these Glasses in reflecting and refracting

9. If the Medium be Glass, then $m : n :: 3 : 2$; therefore $\frac{3dr}{d-2r} = f$. And for parallel Rays AB, where d is infinite, we have $\frac{3dr}{d-2r} = \frac{3dr}{d} = 3r = f = VF$. But in Water, where $m : n :: 4 : 3$, we have $f = \frac{4dr}{d-3r}$, and $4r = f = VF$, for parallel Rays AB.

10. This Theorem (in *Art.* 7.) may be also adapted to the ELLIPSIS, the Equation of which Curve is $yy = px - \frac{pxx}{a}$, which in Fluxions is $y\dot{y} = \frac{p\dot{x}}{2} - \frac{px\dot{x}}{a}$; and, putting $x = o$, we have $yy = o$, $y\dot{y} = \frac{p\dot{x}}{2}$, which Values substituted in the general Equation give $\frac{\frac{1}{2}dp}{d-p} = f$; and when d is infinite, or the Rays parallel, then $\frac{p}{4} = VF$, the focal Distance of the Ellipsis VG, a fourth Part of the *Latus Rectum* from the Vertex, for the Sun-Beams. The Expression is also the same for an HYPERBOLA VG, because only $\frac{pxx}{a}$ is af-

fracting the Sun's Light, and forming the Images of Objects: And this all depends *(in Reflection of Light)* on that fundamental Law, *That the Angle of Incidence is equal to the Angle of Reflection.*

Let E H be a concave Mirrour, V its Vertex, and C the Center of its Concavity. Let A be a Ray of the Sun's Light incident on the Point E, and draw E C, which will be perpendicular to the Mirrour in the Point E; make the Angle C E F equal to the Angle A E C, then shall E F be the reflected Ray. Thus also H F will be the reflected Ray of the incident one D H, at an equal Distance on the other Side of the Axis B V. Plate L. Fig. 5.

If now the Points E and H be taken very near

fected with a different Sign, and vanishes in that Equation also.

11. If VG be a Parabola, its Equation is $yy = px$, and in Fluxions $2y\dot{y} = p\dot{x}$; whence, since $x = o$, we have $yy = o$, $y\dot{y} = \frac{p\dot{x}}{2}$, which substituted as before give $\frac{\frac{3}{2}dp}{d-p} = f$; and in case of parallel Rays, or the Sun-Beams, $\frac{p}{4} = VF$, the Focus or Burning-Point of the Parabola.

12. Hence we observe, that in the Circle VG, whose Radius CB is equal to half the *Latus Rectum* of the Ellipsis or Parabola, *viz.* $r = \frac{1}{2}p$, the Focus will be at the same Distance from the Vertex V, or VF will be the same in all; for then it is $\frac{\frac{3}{2}dp}{d-p} = \frac{3dr}{d-2r} = f$ in all the Curves, and consequently the *Circle, Ellipsis,* and *Parabola,* have all the same Degree of Curvature at the Vertex V in this Case.

13. When $d = 2r$, or $d = p$, then the focal Distance $f = \frac{3dr}{o} = \frac{\frac{3}{2}dp}{o} = VF$ becomes infinite; that is, if the Radiant Point A be at the Distance of the Diameter of the Circle, or the Parameter of the Conic Section from the Vertex V of the Medium of Glass, then the Rays will be refracted paral-

the

the Vertex V, we shall have EF, or HF, very nearly equal to FV; but EF = FC; therefore FV = FC = ½ CV. That is, *the Focal Distance* FV *of parallel Rays will be at the Distance of half the Radius* CV *of the Concavity of the Mirrour, from the Vertex* V, *in the Axis* BV.

Plate L. Fig. 6. AFTER the same manner, a *convex Mirrour* is shewn to reflect the Rays AE, DH, into EF, HF, as if they came diverging from a Point F in the Axis CV, which is half the Radius CV distant from the Vertex V. But since the Rays do not actually come at, or from the Focus *f*, it is call'd the *Imaginary or Virtual Focus*.

PARALLEL Rays falling directly on a *plane*

lel to the Axis. And, *vice versâ*, parallel Rays will be refracted from a Substance of Glass by a spherical Surface to the Distance of the Diameter of the Sphere; or from an elliptical or parabolical Surface to the Distance of the *Latus Rectum*, from the Vertex V.

Plate XLIV. Fig. 8. 14. After the same Manner we express the several Cases of a *Spherical*, *Elliptical*, or *Parabolical* reflecting Surface VBG, that is, such a one where the incident Ray AB is reflected from the Point B instead of being refracted; and then since the Angle of Incidence ABL is equal to the Angle of Reflection LBK, the Ray KB will be so reflected from the Point B as if it came from a Point F in the Axis, and therefore that Point F we must consider as the Focus of reflected Rays. In this Case the Velocities of the incident and reflected Rays are the same, viz. $\dot{x} = \dot{v}$, and both affirmative; also $m = n$. Whence $\frac{y\dot{y} + d\dot{x} + x\dot{x}}{\sqrt{y^2 + d^2 + 2dx + x^2}} = \frac{y\dot{y} - f\dot{x} + x\dot{x}}{\sqrt{y^2 + f^2 - 2fx + x^2}}$; or, putting $x = 0$, $yy = 0$, and $y\dot{y} = r\dot{x}$, or $= \frac{1}{2}p\dot{x}$, (as above) then this general Theorem becomes $\frac{dr}{r + 2d} = f =$ VF, in the *Circle*; and $\frac{dp}{p + 4d} = f =$ VF, in the *Ellipsis*, *Hyperbola*, and *Parabola*.

Speculum

Speculum are reflected back upon themselves; if they fall obliquely, they are reflected in the same Angle, and parallel as they fell. Hence there is no such thing, properly speaking, as a *Focus* belonging to a *plane Speculum*, neither *real* nor *virtual*.

The Focus F, or *f*, of parallel Rays, is call'd the *Solar Focus*; because in that the Image of the *Sun* is form'd, and of all Objects very remote. But the Focus of any Object situated near the Mirrour will have its Distance from the Vertex more or less than half the Radius: The Rule in all Cases being as follows:

Multiply the Distance of the Object into the Radius

15. If d, or AV, be infinite, as in parallel Rays, or the Sun-Beams, then $\frac{dr}{r+2d}=\frac{dr}{2d}=\frac{1}{2}r=f=$ VF, in the spherical convex Mirrour VG; but if the said Mirrour be *Elliptical, Hyperbolical*, or *Parabolical*, then $\frac{pd}{p+4d}=\frac{1}{4}p=f=$ VF. But because the Rays BK do not actually proceed from the Point F, that Point is in this Kind of Mirrours call'd the *Virtual Focus*.

16. If the Radius BC $=r$ of the convex Mirrour be infinite, the spherical Surface VBG will become a Plane, *viz.* a plane Speculum or Looking-Glass, as VBG in the following Figure; and the Theorem $\frac{dr}{r+2d}=\frac{dr}{r}=d=f=$ VF, that is, AN is equal to VF, or the incident Ray AB is so reflected at B into BK as if it came from a Point F, just as far behind the Glass as the Radiant A is before it. Plate XLIV. Fig. 9.

17. Furthermore, if $r=$ BC be supposed greater than Infinite, or from affirmative to become negative, the Center C will then lie on the contrary Side, the Speculum VBG will become concave, and in the Theorem above r must have a negative Sign, which then will be $\frac{-dr}{2d-r}=f=$ VF, which shews that in concave Mirrours, when d is less than Fig. 10.

of

of the Mirrour, and divide that Product by the Sum of the Radius and twice the Distance of the Object; the Quotient will be the Focal Distance of a Convex Mirrour.

AGAIN; for a *Concave Mirrour, the same Product of the Radius into the Distance of the Object, divided by the Difference of Radius and twice the Distance of the Object, will give the Focal Distance* V F *or* V *f*. And here we are to observe, that as twice the Distance of the Object is lesser or greater than the Radius, so the Focus will be positive or negative, that is, behind the Glass or before it.

THE Image of every Object is form'd in the

$\frac{1}{2}r$, that is, when AV is less than $\frac{1}{2}$CV, the Focus *f* will be affirmative, or on the same Side as before; or the Ray AB will be so reflected at B into BK as if it came from a Point F behind the Speculum.

18. When $d = \frac{1}{2}r$, or AV $= \frac{1}{2}$CV, then is the Focus F at an infinite Distance, the Theorem then being $\frac{-dr}{0} = f$; so that in this Case all the Rays AB will be reflected parallel to the Axis, as B*K*. But when *d* is greater than $\frac{1}{2}r$, then the Focus *f* will be negative, or it will be $\frac{-dr}{2d-r} = -f$. Wherefore in this Case the Focus F will be on the same Side with the Radiant A.

19. Lastly, when $d = r$, then also $f = r$; that is, if the Radiant A be placed in the Center C, the Focus F will be there too; or, in other Words, Rays proceeding from the Center will be reflected back upon themselves.

20. On the contrary, (in all these Cases) *converging Rays* KB are reflected to a Point in the Axis less distant than $\frac{1}{2}$CV, or half the Radius. *Parallel Rays* KB are reflected to that Point F of the Axis where FV $= \frac{1}{2}$CV. This will therefore be the *Burning Point of the Sun's Rays*, and is the *Solar Focus* above mention'd. *Diverging Rays* have their Focus at a Distance from the Vertex V, greater than half the Ra-

Focus

Focus proper to its Distance: And since the Writers on *Optics* demonstrate, that *the Angles under which the Object* O B *and its Image* I M *are seen from the Center or Vertex of the Mirrour* C *are always equal*; it follows, that the Image I M will be always in Proportion to the Object O B, as the Focal Distance V F to the Object's Distance G V. Plate L. Fig. 7.

THE Position of the Object will be always erect at a *positive Focus*, or behind the *Speculum*; diminished by a convex, and magnified by a concave one. Hence, since a convex has but one, *viz.* an *affirmative Focus*; so it can never magnify any Object, howsoever posited before it.

THE Position of the Image in a *negative Focus*,

dius C V.

21. If V B G be an Ellipsis, Hyperbola, or Parabola, the Theorem is found in the same Manner to be $\frac{-dp}{4d-p}=f$, in concave Speculums of this Sort; and all that has been said with respect to d and $\frac{1}{2}r$ in the spherical Speculums, is true of d and $\frac{1}{4}p$ in these. Thus when $d=\frac{1}{4}p$, the Rays will be reflected parallel to the Axis; and on the other hand, parallel Rays will be reflected to a Point in the Axis whose Distance from the Vertex V is $\frac{1}{4}p$. Thus the Sun's Rays are collected at the Distance of *one Fourth Part of the Parameter* (in each Section) from the Vertex; and as this is the *Burning Point*, we see the Propriety of its being call'd the *Focus* of those Curves.

22. As within the Curve of an Ellipsis V G v H there are two of those Focus's, 'tis observable, that if the Radiant A be in one Focus, the Rays will be reflected to the other at F, wherever the Point B be taken in the Perimeter of the Ellipse. For in this Case $Vv=a$, $Av=x$, $AP=y=\frac{1}{2}p$, (for $PP=p$) $AV=d=a-x$, and $FV=Av=f=-x$; therefore writing $a-x$, and $-x$ for d and $-f$, in the Equation above, we shall have $\frac{px-pa}{4a-4x-p}=-x$, and so $4ax-4xx=pa$, or $ax-xx=\frac{1}{4}pa$, which is the known Pl. XLV. Fig. 1.

or

or that before the Glass, will be ever inverted; and if nearer the Vertex than the Center C, it will be less; if farther from it, it will be greater than the Object; but in the Center, it will be equal to the Object, and seem to touch it.

THE Image form'd by a *plane Speculum* is erect; large as the Life; at the same apparent Distance behind the Glass, as the Object is before it; and on the same Side of the Glass with the Object. These Properties render this Sort of Mirrour of most common Use, *viz.* as a LOOKING-GLASS.

IF the Rays fall directly, or nearly so, on a plane Mirrour, and the Object be opake, there will be but *one single Image form'd*, or at least be

Property of the Ellipsis.

23. And the same thing holds with respect to the Foci of two opposite Hyperbola's V B and $v\,b$; for if the Radiant be
Pl. XLV. Fig. 2. in the Focus A of one, any Ray A B' will be so reflected into B K, as if it came from the Focus F of the opposite Hyperbola $v\,b$, as is evident in the Figure. In the Parabola V B G,
Fig. 3. if the Radiant be placed in the Focus A, the reflected Rays B K, tending to the other Focus at an infinite Distance, will be all parallel to the Axis V C, agreeable to what is said above, *Article* 21.

24. If we resolve the Equation $\frac{dr}{r+2d}=f$, into an Analogy, we shall discover that the Axis of the Mirrour is divived harmonically in the Points V, F, C, and A; or that it is A V : A C :: V F : F C. For supposing it to be so, we have $d : d \pm r :: f : r \pm f$, which gives us the above Theorems $\frac{dr}{2d+r}=f$, in the convex Speculum; and $\frac{-dr}{2d-r}=f$, in the Concave. This curious Property of Speculums was first discover'd by the late Mr. *Ditton*.

25. We now proceed to apply this Method to *Dioptric Problems*, that is, to find the Focus of Rays refracted thro' any Sort of Lenses. To this End we must recollect, that in *Article* 8. we had $mdf - mdr = nrf + ndf$; whence deduce

visible;

visible; and that by the second Surface of the *Speculum*, and not by the first, through which the Rays do most of them pass.

But if the Object be luminous, and the Rays fall very obliquely on the *Speculum*, there will be more than one Image form'd, to an Eye placed in a proper Position to view them. The first Image being form'd by the first Surface will not be so bright as the second, which is form'd by the second Surface. The third, fourth, &c. Images are produced by several Reflections of the Rays between the two Surfaces of the *Speculum*; and since some Light is lost by each Reflection, the Images from the second will appear still more

this other Equation $\frac{m}{n}=\frac{f+r}{d-r}\times\frac{f}{d}=\frac{AC}{CF}\times\frac{VF}{AV}$, which in Words is thus express'd: *The Ratio of the Sine of Incidence to the Sine of Refraction is compounded of the Ratio of the Distances of the Foci* A *and* F *from the Centre* C, *and of the Ratio of their Distances from the Vertex* V.

26. If then we consider Bb (in the double convex Lens V Dv) as a converging Ray refracted from Glass into Air; we shall find the Distance vf, at which the refracted Ray bf shall intersect the Axis of the Lens, by the Rule in *Article* 25. Only here we must consider, that the Point A will be negative, or on the same Side with the Focus f, *viz.* at *A*. And as the Refraction is out of Glass into Air, we must use the Ratio $\frac{n}{m}$ instead of $\frac{m}{n}$; then $\frac{n}{m}=\frac{Ac}{fc}\times\frac{fv}{Av}$. Plate XLV. Fig. 4.

27. Let the Thickness of the Lens be V$v=t$, and vf$=$f; also let the Radius of the second Surface be $cb=$r; then $\frac{n}{m}=\frac{f+r-t}{f+r}\times\frac{f}{f-t}$; whence $f=\frac{nfr-nft-ntr}{mf-mt+mr-nf}$ $=v$f the focal Distance required. But if the Thickness t be inconsiderable, as it commonly is, it may be neglected, and then f$=\frac{nfr}{mf+mr-nf}$; whence $f=\frac{mfr}{nr+nf-mf}=$

faint and obſcure, to the eighth, ninth, or tenth, which can ſcarcely be diſcerned at all.

We proceed now to *Lenſes*; and here, ſince all Viſion by them is effected by the Refraction of Rays through their Subſtance, it will be too intricate an Affair to ſhew the particular Manner how Rays are collected by them to their ſeveral Focus's: It muſt ſuffice only to ſay, *That parallel Rays are refracted through a plano-convex Lens to*

$\frac{mdr}{md - nd - nr}$, which Equation reduced gives $\mathrm{f} = \frac{ndr\mathrm{r}}{mrd - nrd + md\mathrm{r} - nd\mathrm{r} - nr\mathrm{r}}$; and putting $\frac{n}{m-n} = 9$, we have $\mathrm{f} = \frac{9dr\mathrm{r}}{rd + d\mathrm{r} - 9r\mathrm{r}}$. But in Glaſs, $9 = 2$; and if we ſuppoſe the Lens equally convex, or $r = \mathrm{r}$, we have $\mathrm{f} = \frac{dr}{d-r} = v\mathrm{f}$, the focal Diſtance of the Ray A B after paſſing through the Lens, as required.

Pl. XLV. Fig. 5. 28. If d be infinite, then $r = f$; therefore parallel Rays, or the Sun-Beams, will be collected in a Point f, whoſe Diſtances from the Lens is equal to the Radius of Convexity.

Fig. 6. 29. If one of the Radii r, r, be infinite, the Lens will be a *Plano-Convex*, and $\mathrm{f} = \frac{2dr}{d - 2r}$; and for parallel Rays where d is infinite, $\mathrm{f} = 2r$.

Fig. 7. 30. If both the Radii be infinite, the Speculum then is no other than a *plain Glaſs* terminated by two parallel Sides; and the Focus f will be at an infinite Diſtance for parallel Rays, or they will be parallel after Refraction as they were before.

Fig. 8. 31. If one Radius r be infinite, and the other r negative, then will the Lens be a *Plano-Concave*; then will the Theorem be $\frac{-2dr}{d + 2r} = \mathrm{f}$, which is therefore negative, or the Rays proceed diverging after Refraction. When d is infinite, the Theorem is $\frac{-2dr}{d} = -2r = \mathrm{f}$, or parallel Rays diverge from a Point F, at the Diſtance of twice the Radius of Concavity.

a Point

a Point or Focus, which is the Diameter of the Sphere of its Convexity distant from it:

THAT *the same Rays are collected by a double and equally convex Lens in a Point which is the Center of the Sphere of its Convexity:*

THAT *parallel Rays are refracted through a plano-concave Lens in such a manner, as though they came from a Point distant from it by the Diameter of its Concavity:*

32. If both of the Radii be negative, the Lens becomes a *Double Concave*; and if d be infinite, and the Radii equal, viz. $\mathrm{r} = r$, the Theorem then is $\frac{dr}{-d-r} = \frac{dr}{-d} = -r = -f$; so that parallel Rays, or the Sun-Beams, are so refracted through a double and equally concave Lens, as if they proceeded from a Point f at the Distance of the Radius of Concavity from the Vertex of the Lens. Pl. XLV. Fig. 9.

33. If one of the Radii, as r, be affirmative, and the other r negative, the Lens becomes a Meniscus, and the Theorem then is $\frac{-2dr\mathrm{r}}{dr-d\mathrm{r}} = \frac{-2r\mathrm{r}}{r-\mathrm{r}} = \mathrm{f}$; which shews that when $r = \mathrm{r}$, and d is infinite, the Focus f is at an infinite Distance, or the Rays are parallel after Refraction as before; as in the case of a *Watch Glass*. If r be greater than r, or the Concavity less than the Convexity, the Focus f will be affirmative, or parallel Rays will be converged to a real Focus; but if r be less than r, the Focus f will be negative, or parallel Rays will proceed diverging after Refraction. Fig. 10.

34. We now proceed to determine the Position, Magnitude, Form, &c. of the Images of Objects form'd by Mirrours and Lenses, having first premised, that the Images of an Object always appears in the Place from whence the Rays diverge after Reflection or Refraction; or, in other Words, the Image appears in that Place, which we have hitherto call'd the Focus of the Rays. This Sir *Isaac Newton* has deliver'd as an Axiom, as being very evident, because the Species, or several Points of the Image of an Object, are brought to the Eye by the reflected or refracted Rays.

35. Let A V G be a reflecting Speculum, C its Centre, V B its Axis, F the solar Focus; and let O B be an Object at the Distance V B; thro' the Centre C draw OA, which as Fig. 11, 12

AND *that the same Rays are refracted through a double and equally concave Lens, in such manner as though they proceeded from a Point which is the Center of the Concavity.*

AND in case of a double and equally convex Lens, we have this general Rule for finding the *Focus* of Rays universally, be the Distance of the Object and Radius of Convexity what it will, *viz.*

it is perpendicular to the Speculum will be reflected back upon itself, and therefore the proper Focus of the Point O will be in the Line AO, and that of the Point B in the Line or Axis BV. Those focal Points are easily found, thus: Draw OV and VD making equal Angles with the Axis VB; also draw BA, and AE, making equal Angles with the Axis OA; then shall those two refracted Rays VD and AE intersect the Perpendiculars OA and BV in the Points M and I, which will therefore be the focal Points where the Representation of the extreme Points O and B will be made; and consequently all the Points between O and B will be represented between M and I, and therefore the Line IM will be the true Representation or Image of the Object OB.

Plate XLV. Fig. 11. 36. Hence also 'tis easy to observe, that the Position of the Object OB is inverted in the Image IM, and consequently the same Parts of the Object and Image are on contrary Sides of the Axis in a *concave Mirrour*, where the Rays have a real Focus, or form a real Image: But in a *convex* Fig. 12. *Mirrour*, where the Rays have no real but an imaginary Focus, or form not a real but an apparent Image, no such Inversion can happen, but the Object and Image both appear in an erect Position, as is easy to understand from the Figure.

37. Again; the Object and Image are commutable, or may be taken the one for the other in the Schemes. Thus if OB be the Object, then IM will be its Image; but supposing IM the Object, then will OB be its Image.

38. Hence also it appears, that if IM represent an Object placed before a convex Mirrour nearer to the Vertex V than the Solar Focus F, the Rays will be so reflected as to form an apparent Image OB behind the Speculum; and this Case will be every way the same with that of the convex Speculum reversed.

Multiply

Multiply the Distance of the Object by the Radius of Convexity, and divide that Product by the Difference of the said Distance and Radius; the Quotient will be the Distance of the Focus requir'd.

HENCE, if the Distance of the Object be greater than the Radius, the Focus will be *affirmative*, or behind the Lens; the Image will be inverted, and diminish'd in Proportion of its Distance to the Distance of the Object.

39. It is farther obvious, that the Object OB and Image IM subtend equal Angles, both at the Vertex V and Center C of the Mirrour, whether concave or convex; for at the Vertex the Object OB subtends the Angle $OVB = BVD$ or IVM, which the Image subtends, (by *Art.* 35.) And at the Center C, the Angles OCB and ICM, under which the Object and Image appear, are equal, as is evident by Inspection, they being vertical to each other.

40. Therefore the Triangles OVB and IVM, also the Triangles OCB and ICM, are similar, as having all their Angles respectively equal; therefore we have $OB : IM :: VB : VI$; also $OB : IM :: BC : IC$. That is, the Lengths of the Object and Image are proportional to the Distances from the Vertex or Center of the Speculum.

41. Hence in Symbols (putting $O =$ Object, and $I =$ Image) we have $O : I :: d : f$; whence $\frac{Id}{O} = f = \frac{dr}{2d - r}$; therefore $O : I :: 2d - r : r$. Wherefore, by having given the Radius of the Speculum, you may place the Object at such a Distance, that it shall bear any given Proportion to its Image, as that of m to n; for then, since $m : n :: 2d - r : r$, we have $mr = 2dn - rn$, and $mr + rn = 2dn$; consequently, $d = r \times \frac{m + n}{2n}$ for a concave Speculum, and $d = r \times \frac{m - n}{2n}$ for a convex one.

42. From hence it is manifest, no Object can be magnified by a convex Speculum; for, because in that Case n is greater than m, $r \times \frac{m - n}{2n}$ would be a negative Quantity, and so d would have a negative Value, which is impossible. And when

AGAIN; if the Distance of the Object be less than the Radius, the Focus will be *negative*, or on the same Side of the Lens as the Object; and the Image will be magnified, and in an erect Position.

IF the Distance be equal to the Radius, the Focus will be at an infinite Distance; that is, the Rays, after Refraction, will proceed parallel, and will therefore enlighten Bodies at a vast Distance.

$m = n$, then $d = o$; or the Object and Image are then only equal, when they coincide at the Vertex of the concave Mirrour.

43. In a concave Mirrour, while m is greater than n, it is plain the Distance d of the Object is greater than the Radius r of the Mirrour. But when $m = n$, then $d = r$; or the Object and Image are equal in the Center of the Mirrour. When m is less than n, or the Object is magnified, then d is less than r. Now this may be done two different Ways in a concave Speculum; for n may be affirmative, or the Image real and form'd before the Glass, then $d = r \times \frac{m+n}{2n}$; or n may be negative, or the Image only apparent and represented behind the Mirrour, then $d = r \times \frac{n-m}{2n}$; in which Case, 'tis plain, the Object cannot be diminish'd. But lastly, if n be infinite in respect of m, then $rn = 2dn$, or $r = 2d$, that is, $d = \frac{1}{2}r$. Or when the Object is placed in the Solar Focus, the Image is form'd at an infinite Distance, and infinitely large.

44. Such are the Theorems for *Specula*; those for *Lenses* are raised after a like Manner. For let GVA be a double and equally convex Lens; C its Center, or CV the Radius of Convexity $= r$; OB an Object, EV its Distance (in the Axis of the Lens) $= d$, IM the Image, and FV $= f$, the focal Distance at which it is form'd. Then as the Point E in the Object is form'd in the Point F in the Axis of the direct double Pencil of Rays EGFA, so the Point O will be form'd at M in the Axis of the Pencil OGMA; and since these two Axes cross each other in the Middle of the Lens at V, therefore the Points O and M, and (for the same Reason) B and I,

Pl. XLV. Fig. 13.

stance. Hence the Contrivance of the *Dark Lantborn* for this Purpose.

LASTLY: If the Distance of the Object be equal to twice the Radius, then will the Distance of the Focus and Image be equal to the Distance of the Object; and consequently the Image will be equal in Magnitude to the Object, but inverted. Hence the Use of these Lenses to Painters, and Draught-Men in general, who have often Oc-

will be on contrary Sides of the Axis EF, and consequently the Image in respect of the Object is inverted.

45. Because the Angles OVB and IVM are equal, as being vertical, the Object and Image have the same apparent Magnitude if view'd from the Vertex of the Lens V; and are in Proportion to each other as their Distances from the Lens, that is, OB : IM :: VE : VF.

46. Hence, if (as before) we make OB : IM :: $m : n$ :: $d : f$, we have $\frac{nd}{m} = f = \frac{dr}{d-r}$; whence $m : n :: d-r : r$; and so $mr = dn - rn$, or $mr + rn = dn$; wherefore $d = r \times \frac{m+n}{n}$. If $m = n$, then $2r = d$; and if n be infinite in respect to m, $r = d$. And if n be negative, or on the same Side of the Lens with the Object, then $d = r \times \frac{n-m}{n}$, which shews the Object in that Case is always magnified.

47. If the Lens be a single or double Concave, the Rays cannot be converged to a Focus, (as is manifest from *Art.* 32.) and consequently no real Image can be form'd, but only an imaginary one; and because it is in this Case $d = r \times \frac{-m-n}{n}$, 'tis plain when $m = n$, then $d = r \times \frac{-o}{n} = o$, that is, the Image can only be equal to the Object when they coincide at the Lens.

48. The Form of the Image IFM is not a right or strait Line, but a Curve; for let VE $= d$, VF $= f$, and VO $=$ d, VM $=$ f; then since $\frac{dr}{d-r} = f$, and $\frac{\mathrm{d}r}{\mathrm{d}-r} = \mathrm{f}$, we have $f : \mathrm{f} :: \frac{dr}{d-r} : \frac{\mathrm{d}r}{\mathrm{d}-r}$; but if IFM were a Right Line,

casion for the Images of Objects as large as the Life, to delineate or draw from.

As to *Plano-concaves*, they, having no real Focus, form no Images of Objects; so that we shall pass them to proceed to the Structure of the Eye, the Manner of performing Vision therein, the several Defects thereof, and how remedied by Glasses; which will be illustrated by the Dissecti-

it would be $f : \mathrm{f} :: d : \mathrm{d}$. Neither is the Image of a circular Form, unless the Object be so; because in that Case $f = \mathrm{f}$, which cannot be but when $d = \mathrm{d}$, or $VE = VO$; so that if the Object be the Arch of a Circle, the Image will be the Arch of a Circle concentric with the Object, or else of a Conic Section, as before observed of Images form'd by Mirrours, *Art.* 36.

49. If the Object be a Surface, the Image will be a Surface similar thereto; and since Surfaces are in duplicate Proportion of their like Sides, (*Annot.* II. *Art.* 3.) therefore $m : n :: OB^2 : IM^2$, in this Case. And if the Object be a Solid, the Image will be a similar Solid, and they will be in the triplicate Proportion of their homologous Sides; whence $m : n :: OB^3 : IM^3$.

50. Though *Speculums* and *Lenses* are of most general Use in *Optics*, yet it will be necessary to consider the Property of a *Globe* or *Sphere*, as also of an *Hemisphere*, with respect to their Power of converging the Rays of Light to a Focus. If therefore in the Theorem of *Art.* 27. we put $t = 2r =$ Diameter of the Globe, and because $\mathrm{r} = r$, we shall have $\frac{dr + 4rr}{2d - r} = f$, the Focus of diverging Rays; and when d is infinite, the Theorem is $\frac{dr}{2d} = \frac{r}{2} = f = V\mathrm{f}$. Therefore a Globe of Glass will converge the Rays of the Sun to a Focus at the Distance of half the Radius.

Plate XLV. Fig. 14.

51. But in case the Globe be Water, then in the aforesaid Theorem we have $m = 4$, $n = 3$, and the rest as before; then by Reduction it will become $\frac{dr - 2rr}{d - r} = f$, for diverging Rays; and for parallel Rays, where d is infinite, we have $\frac{dr}{d} = r = f$, just twice as large as in Glass.

on

on of a *natural Eye*, and exemplified by an *artificial one*.

THE *Eye* is the *noble Organ of Sight or Vision*: It consists of various Coats and Humours, of which there are Three remarkable, *viz.* (1.) The *Aqueous or Watry Humour*, which lies immediately under the *Cornea*, and makes the Eye globular before. (2.) The *Vitreous Humour*, which is by much the greatest Quantity, filling the Cavity of

52. In an *Hemisphere* of Glass, when the convex Side is turn'd towards the Radiant, having r infinite, and $t = r$, the Theorem will become $\frac{4dr + 4rr}{3d - 6r} = f$, the focal Distance of diverging Rays; but for parallel Rays it becomes $\frac{4dr}{3d} = \frac{4}{3}r = f$. Plate XLV. Fig. 15.

53. If the plane Side of the Hemisphere be turn'd towards the Radiant, the Theorem for diverging Rays will be $\frac{6dr + 4rr}{3d - 4r} = f$; and for parallel Rays, $\frac{6dr}{3d} = \frac{6}{3}r = 2r = f$; which is $\frac{2}{3}r$ greater than before.

54. In an Hemisphere of Water, the convex Part being towards the Radiant, we have $\frac{9dr - 9rr}{4d - 12r} = f$; and for parallel Rays it is $\frac{9dr}{4d} = \frac{9}{4}r = f$. But if the Radiant be opposed to the plane Side, then $3r = f$, greater by $\frac{3}{4}r$ than before.

55. We have hitherto consider'd the Property of *spherical Bodies* only, with respect to their Power of refracting a Ray of Light; let us now consider the Nature of Refraction in Bodies whose Figures are derived from the Curves of the *Conic Sections*. In order to this, let DBKC be an Ellipsis, DK its transverse Axis, H, I, its two Foci, and AB a Ray of Light parallel to the Axis be incident on the Point B. Let BE be a Tangent in the said Point, and LG drawn perpendicular to the Tangent through the Point B; join HB and IB; make Fig. 16.

the

the Eye, and giving it the Form of a Globe or Sphere. (3). The *Crystalline Humour*, situated between the other two, near the Fore-part of the Eye, and is the immediate Instrument of Sight; for being of a lenticular Form, it converges the Rays, which pass through the Pupil, to a Focus on the Bottom of the Eye, where the Images of external Objects are by that means form'd and represented (CXXVI).

AB = IB, and from the Points A and I let fall the Perpendiculars AL, IG, on the Line LG; produce IB to O, and draw HO parallel to LG.

56. Then in the similar Right-angled Triangles ALB, ING, we have AL : IG :: AB : NI :: IB : NI, because AB = IB. But IB : NI :: IO : IH, because of the similar Triangles BNI and OHI. Again, the Angle HBG = GBI from the Nature of the Curve; whence GBI = HOB (= HBG) = BHO; therefore the Triangle HBO is Isosceles, or BH = BO. But IB + BH = DK, *per Conics*; therefore IB + BO = IO = DK. Consequently, AL : IG (:: IB : NI :: IO : IH) :: DK : IH.

57. Since LG is perpendicular to the Tangent or Curve in the Point B, 'tis evident that AL is the Sine of Incidence, and IG the Sine of Refraction to the Radius AB = BI. If therefore a Solid be generated by the Revolution of an Ellipsis about its Axis, which Ellipsis has its transverse Axis DK to the Distance between the Foci in the Ratio of the Sine of Incidence to that of Refraction; then parallel Rays AB, falling on every Point B of its Surface, will be refracted to the remote Focus I.

58. After the same Manner we proceed for the *Hyperbolic Conoid*; but as Lenses made of these Forms are extremely difficult to work, and are likely never to be of Use, (since the great Defect of these Glasses is owing to quite a different Cause, as we shall shew in the next *Annotation*) I shall say no more of them here, but refer the inquisitive Reader to the *Dioptrics* of M. *Des Cartes*, who treats largely of this Subject.

(CXXVI) 1. In order to exhibit a just Idea of the *true Theory of Vision*, I shall here give a more exact and particular

OVER

OVER all the Bottom of the Eye is spread a very fine and curious Membrane, call'd the *Retina*, which is an Expansion of the *Optic Nerve*; upon which the Images of Objects being painted and impress'd, they are by that means convey'd to the *Common Sensory* in the Brain, where the Mind views and contemplates their Ideas; but this in a Manner too mysterious and abstruse for us to understand.

Description of the EYE and of its several Parts, with an Account or Calculation of the various Refractions of the Rays of Light through the several Humours, for forming the Images of Objects on the *Retina* at the Bottom of the Eye.

2. To this End I have here represented a Section of the *Human Eye* in its true or natural Magnitude; which consists of two Segments of two different Spheres, *viz.* one larger, as BNB, and a lesser BIB. The larger Segment consists of three Tunics or Coats, of which the outmost is of a hard, thick, white, opake Substance, call'd the *Sclerotica*, as BNB. Within this is another thin, soft, and blackish Tunic, call'd the *Choroïdes*; which serves as it were for a Lining to the other, or rather as a delicate *Stratum* for the third Tunic call'd the *Retina*, which is a curious fine Expansion of the Optic Nerve YZ over all the larger Segment of the Eye, every Way to BB. Pl. XLV. Fig. 17.

3. The lesser Segment consists of one Coat or Tunic, call'd the *Cornea*, as resembling a Piece of transparent Horn; this is more convex than the other, and is denoted by BIB. Within this Coat, at a small Distance, is placed a circular Diaphragm, as B*o*, B*o*, call'd the *Uvea*, or *Iris*, because of the different Colours it has in different Eyes. In this is a round Hole in the Middle call'd the Pupil, as *oo*, which in some Creatures is of a different Figure, *viz.* oblong, as in Cows, Cats, &c.

4. As the *Cornea* by its Transparency admits the Light to enter the Eye, so the Pupil is destined to regulate the Quantity of the Rays that ought to enter the interior Part of the Eye for rendering Vision distinct, and the Images of Objects properly illumined. To this Purpose it is composed of two Sets of muscular Fibres, *viz.* one of a circular Form, which, by corrugating, contract or diminish the Pupil; and the other

THE *Crystalline Humour* is of such a Convexity, that in a sound State of the Eye its Focus falls precisely on the *Retina*, and there paints the Objects; and therefore Vision is not distinct, unless by Rays which are parallel, or nearly so; for those only will have their Focus at the Bottom of the Eye: Now Rays proceeding from any Point more than 6 Inches distant from the Eye, will,

is an *Annulus* of radial Fibres, tending every where from the Circumference BB of the *Uvea* to the Center of the Pupil, which, by contracting, dilate and enlarge the Pupil of the Eye.

5. Immediately within the *Uvea* is another *Annulus* of radial Fibres, which on the extreme Part is every where connected with the *Cornea*, where it joins the *Sclerotica* at BB; and on the other Circumference it is connected with the anterior Part of the *Capsula* including the Crystalline Humour; and is call'd the *Ligamentum Ciliare*, and sometimes the *Processus Ciliares*, and is denoted by B*a*, B*a*.

6. The Bulk or Body of the Eye is made up of three Substances, commonly call'd *Humours*, *viz.* the *Aqueous*, the *Crystalline*, and the *Vitreous*. The *Aqueous Humour* is properly so call'd, being every Way like Water, in respect of its Consistence, Limpidity, specific Gravity, and refractive Power. It is contain'd between the *Cornea* and the *Ligamentum Ciliare*, as BIB*aa*B. This Humour gives the protuberant Figure to the *Cornea*, which makes the first Refraction of the Rays of Light.

7. The second Humour (improperly so call'd) is the *Crystalline*, having its Name from resembling Crystal in Clearness and Transparency. It is denoted by GKHL, and is in Form of a thick Lens unequally convex, whose anterior Surface GKH is the Segment of a larger Sphere, and its posterior Surface GLH the Segment of a lesser. This Humour is of a solid Consistence, and very little exceeds the specific Gravity of Water, *viz.* in the Proportion of 11 to 10 nearly, as I have often found by Experiment. It is contain'd within a most delicate Tunic or *Capsula*, call'd *Arachnoïdes*, every where pellucid as the Crystalline itself. This natural Lens conduces most to the Refraction and Convergency of the Rays of Light.

when

when they enter the Pupil, be very nearly coincident with parallel Rays; and therefore to a sound Eye distinct Vision cannot be effected at less than 6 or 8 Inches Distance, as is evident to any who tries the Experiment.

SINCE then there is a certain and determinate Degree of Convexity in the *Cornea* and *Crystalline Humour*, for forming the Images of Objects

8. The third Humour is the *Vitreous*, (being clear as *Glass*) and is largest of all in Quantity, filling the whole Orb of the Eye BMB, and giving it a globular Shape. This Humour is exactly like the White of an Egg, and but a little exceeds the specific Gravity and refractive Power of Water.

9. We proceed now to give the Dimensions of the Eye and its several Parts, (in order for Calculation) as they have been determined by actual Measurement in a great Number of human Eyes with the greatest Care and Exactness. These Measures are express'd in Tenths of an Inch, as follows.

	Tenths.
The Diameter of the Eye from Outside to Outside, taken at a Mean from six adult Eyes, — — —	IN = 9,4
The Radius of Convexity of the *Cornea*,	BIB = 3,3294
The Radius of Convexity of the anterior Surface of the Crystalline, from twenty-six Eyes, — — — —	GKH = 3,3081
The Radius of Convexity of the hinder Surface, from the same Eyes, at a Mean, — — — —	GLH = 2,5056
The Thickness of the Crystalline, from the same Eyes, — — —	KL = 1,8525
The Thickness of the *Cornea* and Aqueous Humour together, — —	IK = 1,0358

10. Moreover, it is found by Experiment, that the Ratio of Refraction at the *Cornea* I is as 4 to 3, being the same with that of Air into Water; the Ratio of Refraction at K as 13 to 12, and at L as 12 to 13. These Things premised, let AX be the Axis of the Eye, and ED a Ray parallel thereto, and incident on the *Cornea* very near it at D; we are to determine the Foci of the several Refractions of this Ray at the several Surfaces I, K, and L.

on

on the *Retina*; if it happens that the Convexity of those Parts should be more or less than just, the Focus of Rays will fall short of, or beyond the *Retina*, and in either Case will cause indistinct Vision. The first is the Case of short-sighted or *purblind* People; the latter of the *Aged*.

A *purblind Person*, having the Convexity of the Eye and Crystalline Humour too great, will have

11. The first Focus is determin'd by the Theorem (in *Annot.* CXXV. *Art.* 27.) $\frac{mdr}{md-nd-nr}$; for supposing all behind the *Cornea* BIB were the Aqueous Humour continued; then since in this Case $m=4$, $n=3$; $r=3,3294$, and d is infinite, we have $f=4r=13,3176=IQ$, the focal Distance from I by the first Refraction.

12. The Ray tending by this means from D to Q falls converging on the anterior Surface of the Crystalline Humour at S. We must now find the Focus of the converging Ray DS refracted through a Medium every where the same with the Crystalline Humour. This we do by the same Theorem; for at K we have $m:n::13:12$, and $r=3,308$, and $IQ-IK=12,2818=KQ=d$, the Distance of the Radiant Q from the Point K; but d is in this Case negative, or $-d$, and the Theorem is $\frac{-mdr}{-md+dn-nr}=f=10,06=$ KP, the new focal Distance from K, by the second Refraction.

13. The Ray converging from S to P is intercepted by the hinder Surface of the Crystalline at T, and meeting there with a Medium of different Density, and a concave Surface, is again refracted by it; and here we have $m:n::12:13$, (by *Art.* 10.) also the Radius r is negative, as well as d; and here it is $-r=2,5056$, and $-d=KP-KL=LP=8,31$; therefore the Theorem is $\frac{mdr}{-md+nd+nr}=\frac{12dr}{d+13r}=f=LM=6,112$, the last focal Distance required.

14. The Point M therefore is that in which parallel Rays RD are collected within the Eye, and where the Images of

the Rays united in a Point before they reach the Bottom of the Eye, and consequently the Images of Objects will be form'd, not upon the *Retina*, (as they should be) but above it in the Glassy Humour, and therefore will appear indistinct or confused.

THIS Defect of the Eye is remedied two Ways, *viz.* (1.) By diminishing the Distance between

remote Objects are form'd. The Distance of this Point from the *Cornea* is $IM = IK + KL + LM = 1{,}036 + 1{,}852 + 6{,}112 = 9$. Then $IN - IM = 9{,}4 - 9 = 0{,}4 = NM$. Now the Thickness of the *Sclerotica* is by the Micrometer found to be very nearly $0{,}25$; then $0{,}4 - 0{,}25 = 0{,}15$; which is much about equal to the Thickness of the *Choroïdes* and *Retina* together. *Hence we see the Forms and refractive Power of these several Humours are such as nicely converge parallel Rays to a Focus upon the Retina in the Bottom of the Eye.*

15. From hence it follows, that since parallel Rays only have their Focus upon the Retina, they alone can paint an Image there distinctly, or produce a distinct Vision of an Object. If therefore the Object be so near, that the Rays from any particular Point come diverging to the Pupil, they will necessarily require a greater focal Distance than I M, and therefore, as the Rays are not united upon the Retina, that Point cannot be there distinctly represented, but will appear confused.

16. Thus if A B, A B, are two parallel Rays falling upon the Pupil of the Eye, then any other two Rays, as C B, C B, though really diverging, yet as the Point C, whence they proceed, is remote from the Eye, they will at the Entrance of the Eye be so nearly coincident with the parallel Rays, as to have nearly the same focal Point on the Retina; whence the Point C will there be distinctly represented by *c*. But if any other Point E be view'd very near the Eye, so that the Angles E B A which they contain with the parallel Rays be very considerable, they will after Refraction tend towards a Point *f* in the Axis of the Eye produced, and upon the Retina will represent only a circular indistinct Area like that at *e*, whose Breadth is equal to *ab*, the Distance of the Rays upon the Retina. The same Point at D will not be quite so much Plate XLVI. Fig. 1.

the

the Object and the Eye; for by lessening the Distance of the Object, the Distance of the Focus and Image will be increased, till it falls on the *Retina*, and appears distinct. (2.) By applying a concave Glass to the Eye; for such a Glass makes the Rays pass more diverging to the Eye, in which Case the Distance of the Focus will be also enlarged, and thrown upon the *Retina*, where distinct Vision will ensue.

dilated and indistinct; the Rays D B, D B, having a less Degree of Divergence.

17. It is found by Experience, that the nearest Limit of distinct Vision is about six Inches from the Eye; for if a Book be held nearer to the Eye than that, the Letters, and Lines will immediately become confused and indistinct. Now this Cause of indistinct Vision may be in some measure remedied by lessening the Pupils, which we naturally do in looking at near Objects, by contracting the annular Fibres of the *Uvea*; and artificially, by looking thro' a small Hole made with a Pin in a Card, *&c.* for then a small Print may be read much nearer than otherwise; the Reason is plain, for the less the Diameter of the Aperture or Pupil B B, the less will the Rays diverge in coming from D or E, or the more nearly will they coincide with parallel Rays.

18. Besides the Contraction of the Pupil, Nature has furnish'd the Eye with a Faculty of adapting the Conformation of the several Parts to the respective Positions of Objects as they are nigh or more remote; for this Purpose, the *Cornea* is of an elastic yielding Substance, and the Crystalline is inclosed with a little Water in its *Capsula*, that by the Contraction and Relaxation of the Ciliary Ligament, the Convexity of both the Surfaces of the *Capsula* may be a little alter'd, and perhaps the Position of the Crystalline, by which means the Distance from the Retina may be fitted and adjusted to nigh Objects, so as to have their Images very distinctly form'd upon the Retina.

19. I have mention'd *nigh* Objects only, (by which I mean such as are near the Limit of distinct Vision, as between six and a hundred Inches Distance) because Objects more remote require scarce any Change of the Conformation of the Eye, the focal Distance in them varying so very little. Thus sup-

HENCE

HENCE the Use of *Concave Spectacles*: And the *Myops* or purblind Person, who uses them, has the three following Peculiarities, *viz.* (1.) To him Objects appear nearer than they really are, or do appear to a sound Eye. (2.) The Objects appear less bright, or more obscure, to them than to other People, because a less Quantity of Rays of Light enter the Pupil. (3.) Their Eyes grow

pose all the Refractions of the Eye were equivalent to that of a double and equally convex Lens, whose Radius $r = 1$ Inch; if then the Object were 10 Inches distant, or $d = 10$, we should have the focal Distance $f = \frac{dr}{d-r} = \frac{10}{9} = 0{,}11111$; and if another Object be distant 100 Inches, then $d = 100$, and $f = \frac{dr}{d-r} = \frac{100}{99} = 0{,}10101$. The Difference between these two focal Distances is but 0,0101, *viz.* the hundredth Part of an Inch, which the Eye can easily provide for. If we go beyond this, suppose to an Object 1000 Inches distant, we have $f = \frac{dr}{d-r} = 0{,}1001001$, which is only a thousandth Part of an Inch less than the former, and is therefore inconsiderable.

20. We have seen the natural Limit of distinct Vision for *nigh Objects*; we shall now consider what the Limit on the other hand may be for *remote Objects*; for Objects may appear indistinct and confused by being removed too far from the Eye, as well as when they are too near it. And in this Case we find Objects will appear distinct so long as their Parts are separate and distinct in the Image form'd on the Retina. Those Parts will be separate so long as the Axes of the Pencils of Rays which paint them are so at their Incidence on the Retina; that is, so long as the Angle they contain is not less than *one Tenth of a Degree*; for it is found by Experience that Objects and their Parts become indistinct when the Angle they subtend at the Pupil of the Eye is less than that Quantity.

21. Thus suppose OB be a Circle $\frac{1}{10}$ of an Inch Diameter, it will appear distinct with its central Spot till you recede to the Distance of 6 Feet from it, and then it becomes confused; and if it be $\frac{1}{4}$ of an Inch, it will begin to be confused better Plate XLVI. Fig. 2.

better with Age; for whereas the Fault is too great a Convexity of the Eye, the *Aqueous Humour*, and also the *Crystalline*, wasting with Age will grow flatter, and therefore more fit to view distant Objects.

THE other Defect of the Eyes arises from a quite contrary Cause, *viz.* the *Cornea* and *Crystalline Humour* being too flat, as is generally the

at 12 Feet Distance, and so on; in which Cases the Angle subtended at the Eye, *viz.* OAB, is about $\frac{1}{10}$ of a Degree, or 6 Minutes. And thus all Objects, as they are bigger, appear distinct at a greater Distance; a small Print will become confused at a less Distance than a larger; and in a Map of *England* the Names of Places in small Letters become first indistinct, where those in Capitals are very plain and legible; at a bigger Distance these become confused, while the several Counties appear well defined to a much greater Distance. These also at last become so indistinct as not to be known one from another, when at the same Time the whole Island preserves its Form very distinctly to a very great Distance; which may be so far increased, that it also at last will appear but a confused and unmeaning Spot.

22. We have seen the Causes of indistinct Vision in the *Objects*, and shall now enquire what may produce the same in the *Eye* itself. And first it is to be observed, that there is a proper Degree of Convexity in the *Cornea* KPL, and *Crystalline* ST, for converging parallel Rays to a Focus on the Bottom of the Eye in a sound State; hence every distant Object OB will have its Image IM accurately depicted on the *Retina*, and by that means produce distinct Vision. (Pl.XLVI. Fig. 3.)

23. But if the *Cornea* KPL, or Crystalline ST, or both, should chance to be a little more convex than just, it will cause the Pencil of Rays *o*C*o*, which comes to the Pupil *oo*, from any Point C in the Object OB, to unite in a Focus before they arrive at the Retina in the Bottom of the Eye; the Image IM of the Object OB will be form'd in the Body of the Vitreous Humour, and will therefore be very confused and indistinct on the Retina at *im*. A Person having such an Eye is call'd a *Myops*, in Allusion to the *Eye of a Mouse*, by reason of its great Convexity. (Fig. 4.)

24. To remedy this Defect of the Eye, a concave Lens

Case

Case of an old Eye. This Defect is remedied by *Convex Lenses*, such as are the common *Spectacles*, and *Reading Glasses*. For since the Rays, in these Eyes, go beyond the Bottom of the Eye, before they come to a Focus, or form the Image; a convex Glass will make the Rays fall more converging to the Pupil, and on the Humours, by which means the focal Distance will be shorten'd, and

EF is applied before it; for by this means the Rays C*a*, C*b*, which fall diverging on the Lens, will, after Refraction through it, be made to proceed still more diverging, *viz.* in the Direction *ar*, *br*, (instead of *ao*, *bo*, as before) as if they came from the Point *C* instead of C. All which is plain from the Nature of a concave Lens above described.

25. Hence it follows, that since the Rays are made to fall with greater Divergence upon the Eye, they will require a greater focal Distance to be united in the Axis, and consequently the Focus may be made to fall very nicely on the Retina, by using a Lens EF of a proper Degree of Concavity; and therefore distinct Vision will be effected in the same Manner as in an Eye of a just Conformation, by painting the Image on the Retina.

26. Since the Point *C* is nearer to the Eye than the Point C, the *apparent* Place of Objects seen through a concave Lens is nearer than the *true* Place; or the Object will appear at *OB*, instead of OB. And also since converging Rays O*a*, B*b*, proceed less converging after Refraction than before, the Object appears under a less Angle, and therefore the apparent Magnitude of Objects seen by a concave Lens is less than the true.

27. The Object is less luminous or bright seen through such a Lens than without it; because the Rays being render'd more divergent, a less Quantity enters the Pupil of the Eye than otherwise would do. But the Picture is always more or less bright or enlighten'd, according as it is made by a greater or less Quantity of Rays.

28. Lastly, it appears from what has been said, that when a concave Lens EF cannot be applied, we may still effect distinct Vision by lessening the Distance between the Object and the Eye; for it is plain, if OB be situated at *OB*, the Image at IM will recede to *im* upon the Retina, and be di-

adjusted to the *Retina*; where distinct Vision of Objects will then be effected.

By convex Spectacles Objects appear *more bright*, because they collect a greater Quantity of Rays on the Pupil. And they appear at a greater Distance than they are; for the nearer the Rays approach to parallel ones, the more distant the Point will be to which they tend.

stinct, in the same Manner as when made so by the Lens EF.

Pl.XLVI. Fig. 5. 29. On the other hand, when the *Cornea* or Crystalline is too flat, (as often happens by Age) an Object OB, placed at the same Distance from the Eye PC as before, will have the Rays C*o*, C*o*, after Refraction in the Eye proceed to a Focus beyond the Bottom of the Eye, in which if a Hole were made (in an Eye taken out of the Head) the Rays would actually go on, and form the Image *im*; which Image must therefore be very confused and indistinct on the Retina.

30. To remedy this Defect, a convex Lens GH is applied, which causes the diverging Rays C*a*, C*b*, to fall less diverging upon the Eye, or as if they came from a Point more remote, as *C*; by which means the focal Distance is shorten'd, and the Image duly form'd on the Retina at IM, by which distinct Vision is produced.

31. Hence the apparent Place of the Object is at *C*, more distant than the true Place at C; and its apparent Magnitude *OB* is greater than the true, because the converging Rays O*a*, B*b*, are by this Lens after Refraction made to unite sooner than before, and so to contain an Angle OP*B* greater than the true OPB. The Object appears through a convex Lens brighter than without, because by this means a greater Quantity of Rays enter the Pupil; for the Rays *ao*, *bo*, are by the Lens made to enter in the Directions *ar*, *br*, which are nearer together, and leave Room for more to enter the Pupil all around between *o* and *r*.

32. As the Image of the Object painted on the Retina is greater or less, so will the apparent Magnitude of the Object

Fig. 6. be likewise; or, in other Words, the Angle IPM subtended by the Image is always equal to the Angle OPB subtended by the Object at the Eye, and therefore the Image IM will be always proportional to the Object OB. Hence it follows, that the Angle OPB under which an Object appears is the Measure of its apparent Magnitude.

I HAVE

I HAVE already observed, that if the Object be placed nigher to the convex Glass than its Focus, it will appear erect and magnified; which makes them of such general Use as *Reading Glasses*.

IF an Object be placed in the Focus of a convex Lens, the Rays which proceed from it, after they have pass'd through the Glass, will proceed parallel; and therefore an Eye placed any where

33. Therefore Objects of different Magnitudes, as OB, AC, DE, which subtend the same Angle at the Eye, have the same apparent Magnitude, or form an equal Image in the Bottom of the Eye. Hence it is that Objects at a great Distance have their Magnitude diminish'd proportionally: Thus the Object DE removed to *DE* appears under a less Angle *DPE*, and makes a less Image on the Retina, as is shewn by the dotted Lines.

34. The Angles of apparent Magnitude OAB, OCB, when very small, are as their Sines, and therefore as the Sides OC and OA, or BC and BA; that is, the apparent Magnitude of the Object OB, at the Distances BC and BA, is inversely as those Distances; or its Magnitude at C is to that at A as AB to CB.

35. The more directly any Object is situated before the Eye, the more distinctly it will appear; because those Rays only which fall upon the Eye near its Axis can be convened to a Point in the Bottom of the Eye on the Retina, and therefore that Part of the Image only which is form'd by the direct Pencil of Rays can be clear and distinct; and we are said to *see* an Object by such a Pencil of Rays, but only to *look at* it by the others which are oblique.

36. Suppose A, B, C, represent three Pieces of Paper stuck up against the Wainscot of a Room at the Height of the Eye; if then a Person places himself so before them, and shutting his Right Eye views them with his Left, it is very remarkable that the Paper B, whose Pencil of Rays falls upon the Insertion D of the Optic Nerve DE, will immediately vanish or disappear, while the two extreme Papers C and A are visible; and by altering the Position of the Eye, and its Distance, any of the Papers may be made to vanish, by causing the Pencil of Rays to fall on the Point D. Plate XLVI. Fig. 8.

37. Why the Rays of Light should not excite the Sensa-

in the Axis will have the most distinct View of the Object possible; and if it be a Lens of a small focal Distance, then will the Object appear as much larger as it is nearer, than when you view it with the naked Eye. And hence their Use as *Single Microscopes*: To give an Instance of which, suppose the focal Distance of a Lens were one Tenth of an Inch, then will the Diameter or

tion of Vision in that Point D where the Fibres of the Nerves begin to separate and expand every way to form the Retina, I cannot tell. But 'tis highly worth our Notice, that the Nerve DE is for that Reason placed *on one Side of the Eye*, where only the oblique Rays come, the Loss of which is not considerable, and no way affects or hinders the Perfection of Sight. Whereas had it enter'd in the Middle of the Bottom of the Eye, it had render'd useless all the direct Rays, by which the most perfect and distinct Vision is effected; and we could have had only a confused and imperfect Perception of Objects by oblique collateral Rays. How glaring an Instance is this of Contrivance and Design in the Construction of this admirable Organ!

38. I shall conclude this Head with observing, that the Nature of a *Reading-Glass* is the same with that of common *Spectacles*; only in the latter Case we use a Lens to each Eye, but in the former one Lens is made large enough for both. Also in the Use of them we have different Ends to answer; for by Spectacles we only propose to render Objects distinct at a given Distance, but the Reading Glass is applied to magnify the Object, or to render the reading of a small Print very easy, which otherwise would be apt to strain the Eye too much. Therefore the Size of a Lens for Spectacles is not required larger than the Eye; but that of a Reading-Glass ought to be big enough to take in as large a Part of the Object, at least, as is equal to the Distance between both the Eyes.

Plate XLVI. Fig. 9. 39. In the Reading-Glass ECD the Object or Print AB is always nearer to the Glass than its Focus F; because in this Case it is necessary the Image or magnified Print GH should be erect, and on the same Side of the Glass with the Object; that is, the Distance d is negative in the Equation $\frac{dr}{d-r} = f$.

Length

Length of an Object appear 60 times larger than to the naked Eye at 6 Inches Distance: Also the Superficies of an Object will be 3600 times larger; and the whole Magnitude or Bulk will be 216000 times larger than to the naked Eye it will appear at the abovesaid Distance (CXXVII.)

Hence the Pencil of Rays AED, proceeding from any Point A, will after Refraction through the Lens be divergent, but less so than before, and therefore will seem to come from a Point G. Thus also the Point B will be referr'd to H, and the Print at GH will be magnified in Proportion of GC to AC. All which is evident from the latter Part of the last *Annotation*.

(CXXVII) 1. I shall here give a succinct Account of every Sort of Microscope, with respect to their Nature and Theory. MICROSCOPES are distinguishable into two Kinds, *viz.* *Dioptric* by Refraction, and *Catoptric* by Reflection; and each of these is either *Single*, as consisting of one Glass only, or *Compounded* of two or more.

2. A SINGLE MICROSCOPE, of the Dioptric or Refracting Sort, is either a *Lens* or a *Spherule*. Thus if any Object *ab* be placed in the Focus *c* of a small Lens ACB, the Rays proceeding from thence will after Refraction go parallel to the Eye at I, and produce distinct Vision; and the Object will be magnified in the Proportion of 6 Inches to the focal Distance C*c*, according to the Example above. Plate XLVII. Fig. 1.

3. Again; if an Object *ab* be applied to the Focus *c* of a Spherule AB, it will produce distinct Vision thereof by means of parallel Rays, (by *Annot*. CXXVI. *Art*. 15.) and it will appear under an Angle equal to DCE, and be magnified in Proportion of 6 Inches to the focal Distance C*c* from the Center. And here it is remarkable, that if the focal Distance of the Lens and Spherule be the same, the Object will be three times farther distant from the Lens than from the Spherule, because CD the Semidiameter of the Sphere is $\frac{2}{3}$ of C*c*, the Distance of the Lens AB; consequently an Object is view'd by a Lens to a much greater Advantage than by a Sphere, in regard to the Light, &c. Fig. 2. Fig. 3.

4. A Single Microscope of the Catoptric Kind is a small concave Mirrour, as ADB, having the Object *ab* placed before it, nearer to the Vertex than its Focus F. In that Case

Plate XLVIII. Fig. 1, 2.

COMPOUND Microscopes, especially the common Sort, are constructed with three Glasses, *viz.* the Object-Lens *de*, and two Eye-Glasses DE and GH. The Object *abc* being placed at a little more than the Focal Distance from the Lens *de*, will have its Image form'd at a

Plate XLVII. Fig. 4.

the Image *IM* will be form'd behind the Speculum, very large, erect, and distinct, as has been already shewn. Such a Speculum is of admirable Use to view the Eye; for being turn'd to the Light, the *Uvea* or *Iris*, the *Pupil*, the *Cornea*, and all the visible Part of the *Tunica Albuginea*, with the fine Ramifications of the Blood-Vessels, and Part of the *Glandula Lachrymalis*, are all by this means greatly magnified, and render'd curious Subjects of our Sight.

Fig. 5.

5. Also if the Object *ab* be placed any where between the Center C and Focus F of the said small Speculum AB, then will its Image *IM* be form'd at a great Distance from the Glass, and may be made to bear any assign'd Proportion to the Object, by placing the Object nearer to or farther from the Focus F: But for common Objects the Room ought to be dark, or the Object extremely lucid, as a Candle, &c. But more of this when I come to speak of the *Solar Microscope*.

Fig. 6.

6. The next Sort of Single Microscope is a Cata-dioptric one, which performs its Effect by Reflection and Refraction; the Theory of which being curious, I shall give the Reader as follows. DBLH is a Globule of Water; and it was shewn that an Object A*a*, placed in its Focus A, would be seen distinct and magnified by refracted Rays ABDE. (See *Art.* 3.) Now 'tis evident we may consider the Ray BD either as the refracted Ray of AB, or the reflected Ray of FB, the Angle CBF being equal to the Angle CBD; and since in each Case the Ray BD is at D refracted into DE parallel to the Axis GK, it follows, that distinct Vision will be produced of an Object F*f* placed in the Focus by Reflection F within the Drop, as the Object A*a* in its Focus A by Refraction without it.

7. In order to determine the focal Distance IF by Reflection from the Concave BLi, for converging Rays D*b*, we have HK = 4HC, or 4IC, (by *Annotat.* CXXV.) whence IK = 2IC, that is, $d = 2r$, in the Theorem $\frac{dr}{2d-r} =$

greater

greater Distance on the other Side, and proportionably large, as at MN; which large Image is contracted into one ABC somewhat less, by the lower Eye-Glass DE; and this Image is view'd by the Eye through the upper Eye-Glass GH; where it also distinctly views the MICROMETER

f, (in the same *Annotation.*) Also because the reflecting Surface is here concave, and the Rays converging, the Theorem will become $\frac{dr}{-2d-r} = f = \frac{2rr}{-5r} = \frac{2}{5}r = IF$; whence $CF = \frac{3}{5}r$. Also it has been shewn that $AI = IC$, or $CA = 2r$; and therefore $CF : CA :: \frac{3}{5} : 2 :: 3 : 10 :: 1 : 3\frac{1}{3}$.

8. And since the same Object will appear as much larger at F than at A, as the Angle FCf is greater than ACa, or the Distance CA greater than CF, it follows, that an Object in a Globule of Water seen by Reflection is magnified $3\frac{1}{3}$ times more than it would be in the Focus A by Refraction. Suppose then $CA = \frac{1}{5}$ of an Inch, then will $CF = \frac{3}{50}$ of an Inch; and therefore, since $6 : \frac{3}{50} :: 100 : 1$, it appears that the Diameter of an Object at the Focus F is seen 100 times larger than at the Distance of 6 Inches from the Eye.

9. In a Glass Globule, $HK = 3r$, $IK = 1{,}5r = d$; and the Theorem $\frac{dr}{-2d-r} = \frac{1{,}5rr}{-4r} = \frac{3}{8}r = IF$; whence $CF = \frac{5}{8}r$. And because $IA = \frac{1}{2}r$, we have $CA = \frac{3}{2}r$; consequently, $CA : CF :: \frac{3}{2} : \frac{5}{8} :: 2\frac{1}{5} : 1$; that is, an Object is magnified $2\frac{1}{5}$ times more at the Focus F within, than at the Focus A without a Glass Globe: And hence it appears, that equal Globes of Glass and Water magnify by Reflection in Proportion of $2\frac{1}{5}$ to $3\frac{1}{3}$. Also because $CF = \frac{3}{5}$ in Water, and $CA = \frac{3}{2}$ when the same Globe is Glass, it appears that Objects are magnified in the Water Globule more than when seen through the Glass Globule in the Proportion of $\frac{3}{2}$ to $\frac{3}{5}$, or $2\frac{1}{2}$ to 1.

10. A DOUBLE MICROSCOPE is composed of two convex Lenses, *viz.* an *Object* and an *Ocular* Lens. The Object Lens is df, placed a little farther distant from the Object ab, than its focal-Distance ef; because then its Image AB will be form'd at the required Distance eC; and as ec : eC :: ab : AB. If this Image AB be view'd by a Lens DF placed at its focal Distance from it, it will appear distinct, because Plate XLVII. Fig. 7.

opq,

THE TELESCOPE is of two Sorts, *viz.* *Dioptric*, or *Refracting*; or *Cata-Dioptric*, by *Reflection* and *Refraction* conjointly. A *refracting Telescope* consists of an Object-Glass xz, by which the Image fd of an Object O B, at a distance, is

Plate L. Fig. 8.

the Radius Hc describe the Arch ca, and draw aH; this will be the Axis of all the Rays which go from the Point a to the Lens G K; consequently, the Ray aK will after Refraction be parallel to the Axis, *i. e.* the Ray KO is parallel to aH; therefore the Image of the Object being in the Focus c of the Lens G K, will be seen under the Angle KOH, which is equal to the Angle aHc; but it is seen from the Lens df under the Angle aec. But the Angle aHc : aec :: ec : cH. Wherefore the second Part of the Ratio for magnifying is that of ec to cH, or $\frac{ec}{cH}$.

18. Lastly; let C be the Focus of the Lens DF, and with the Radius EC describe the Arch Ce; then will eE be the Axis of the Pencil of Rays proceeding from the Point e to the Lens DF, of which eF being one, it will be refracted into FI parallel to eE; and so the Angle FIE = CEe. But FIE is the Angle under which the Image is view'd through the Lens DF, which is to the Angle COA as CO to CE. Therefore the third and last Part of the Ratio for magnifying is $\frac{CO}{CE}$.

19. If now we compound the several Parts of the Ratio now found into one, it will make the Ratio of $\frac{CO}{CE} \times \frac{ec}{cH} \times \frac{6}{ec}$ to 1. For Example, let ec = ½ an Inch, cH = 3½, ec = 2, CE = 1½; then HE being 2, and HO = 11,66, we have CO = 10,66; whence the above Ratio in Numbers will be $\frac{10,66}{1,5} \times \frac{2}{3,5} \times \frac{6}{0,5} = 40,87$. Therefore the Diameter of any Object is magnified near 41 times by such a Compound Microscope.

20. If this Calculation be enquired into, we shall find that the Glass GK diminishes the magnifying Power, which is greater by the Eye-Glass DF alone, and more distinct. Thus in Fig. II. of the large Plate XLVIII, if the lower Glass

form'd

form'd in the Focus *e* of the said Glass, and in an *inverted Position*. This Image may be view'd by a single Lens *a b*, placed at its Focal Distance, as is usually done for viewing the heavenly Bodies, because in them we regard not the Position: But

DE were taken away, the Rays would go on and be united in a Focus at the Points M, P, N, and there form an Image of the Length MN; but by replacing the Glass DE we shall have the large Image MN contracted into a lesser *mn*. Now this larger Image MN may be consider'd as form'd by the Lens DE at a negative Focus from an Object *mn*, whose Distance FB is less than the focal Distance of the said Lens: All which is easy to understand from the foregoing Theory of *Dioptrics*.

21. Now *ac* : MN :: *bf* : *f*P; and drawing MF and NF, we have MN : *mn* :: FP : FB, because the Object and its Image do in every Case subtend the same Angle from the Vertex of the Lens, as was shewn before. Since FP is given, so also is FB, from the common Theorem $\frac{dr}{d-r}=f$, for a double and equally convex Lens; or $\frac{2dr}{d-2r}=f$, for a plano-convex one. For since the Focus f is negative, or $\frac{dr}{d-r}=-f=$ FP, therefore $dr=-df+rf$, and so $dr+df=rf$; therefore $\frac{rf}{r+f}=d=$ FB; and $\frac{2fr}{2r+f}=d$, in a Plano-Convex.

22. It is evident from the Scheme, that no more of the large Image MN, or of the contracted one *mn*, can be view'd through the Eye-Glass HG, than what is contain'd between the perpendicular Lines HC and GA; and that therefore a much greater Part of the Object can be seen in the Image *mn*, than in the Image MN, which is wholly owing to its being contracted by the large Lens DE; and this is all the Reason of its Use.

23. The next Sort of Microscope I shall take notice of is a *Catadioptric* one, *i. e.* such an one as performs its Effects by *Reflection and Refraction jointly*; for it is constructed with a small Object-Speculum fed, whose Focus is at F; and it has been shewn, that if a small Object ab be placed a little far- Plate XLVII. Fig. 9.

for

for viewing Objects near us, whose Image we would have erect, we must for that Purpose add a second Lens $p\,q$, at double its Focal Distance from the other, that the Rays which come from $a\,b$ may cross each other in the Focus O, in order

ther from the Speculum than the Focus f, there will be form'd a large Image thereof AB; which Image will be inverted, and in Proportion to the Object as the Distance Ce to the Distance ee, as when an Object-Lens was used.

24. Part of this Image is view'd by an Eye-Glass FD, which is or ought to be a *Meniscus*, as here represented; because the Image being form'd by Reflection, it will be more perfect, and admit of a deeper Charge in the Eye-Glass DF; and those of the *Meniscus* Form are best for this Purpose, because the Errors of the Rays, and consequently the Confusion caused thereby, in the Refraction made at the convex Surface, are in a great measure rectified by the contrary Refraction at the concave Surface, as is easy to understand from what has been said of refracted Light, *Annot.* CXVII.

Plate XLVII. Fig. 10. 25. Another Sort of *Catoptric* or *Reflecting* Microscope is constructed with two Speculums, abcd and ABCD, with a central Hole in each. The large Speculum is concave, the other convex, and both of equal Sphericity. They have their Focus at one Inch Distance, and placed at the Distance of $1\frac{1}{2}$ Inch from each other, that so an Object OPQ, being placed a little before the small Speculum, might be nearer to the large one than its Center E.

26. This being the Case, the Rays PA, PD, which flow from the Point P to the Speculum AD, will be reflected towards a Focus p, where an Image opq would be form'd, if the Rays were not intercepted by the convex Speculum ab; and the Point p being nearer than its Focus f, the Rays Aa, Dd, which tend towards it, will be reflected to a Focus P, where the last Image OPQ will be form'd, to be view'd by the Eye-Glass G, transmitting parallel Rays to the Eye at I.

27. The Power of magnifying in this Microscope is thus estimated. (1.) The Object OP seen from the Vertex V of the Speculum AD is to the same seen at the Distance of 6 Inches from the naked Eye as 6 to VP, or as $\frac{6}{VP}$. (2.) The first Image opq, (to be consider'd now as a *virtual Object*) seen from the Vertex V of the Mirrour AD, is to the same

to

to erect the Image $g\,n$, which it will form in its own Focus m, because the Rays come parallel from the first Lens $a\,b$. Lastly, a third Lens $i\,c$ is added, to view that secondary Image $g\,n$. These three Lenses, or Eye-Glasses, are usually of the

seen from the Vertex v of the Mirrour ad as vp to Vp, or as $\frac{Vp}{vp}$. (3.) Lastly, the Image OPQ, seen from the Vertex v of the Speculum ad, is to the same seen through the Eye-Glass G, as GP to Pv, or as $\frac{Pv}{GP}$. Where the whole magnifying Power is as $\frac{6}{VP} \times \frac{Vp}{vp} \times \frac{Pv}{GP}$ to 1. This Contrivance we owe to Dr. *Smith* of *Cambridge*.

28. But a better Form and easier Method of constructing a Catoptric Microscope, with *two reflecting Mirrours*, is that which follows. ABCDEF is a Case or Tube, in one End of which is placed a concave Speculum GH, with a Hole IK in the Middle; the Center of this Speculum is at c, and its Focus at O, so that VO = Oc. At the open End of the Tube is placed a small convex Speculum def, on a Foot ef, by which it is moveable nearer to or farther from the larger Speculum GH, as Occasion requires. Plate XLIX. Fig. 10.

29. If now an Object ab be posited in the Centre c of the large Speculum, the Image thereof $a\,b$ will be form'd in the same Place, as has been shewn already; and this Consideration is all the Reason of this Form of a Microscope; for, if now we look upon the Image $a\,b$ as an Object nearer to the convex Speculum df than its Focus f, 'tis plain a larger Image AB will be form'd thereby at the Focus C; or that Rays cG, cH, proceeding from any Point c in the Object ab, will be reflected back upon themselves, as being perpendicular to the Speculum; but the refracted Rays meeting with, or impinging on, the convex Surface of the Speculum df, will (as they tend to a Point c nearer than the Focus f) be reflected to a Focus C, which is found by the Theorem $\frac{dr}{r+2d} = f$ (*Annot.* CXXV.)

30. For in this Case $f = ec$, and $d = eC$; and since $dr = rf + 2df$, we have $\frac{rf}{r-2f} = d$. Thus if we put the

same

same Size and Focal Length; and the Power of magnifying is always *as the Focal Length of the Object-Glass* e w *divided by the Focal Length of the Eye-Glass* l m *or* h e. For instance: Suppose *ew* = 10 Feet or 120 Inches, and *he* or *lm* =

Radius of the small Speculum $r = 2$ Inches, then $ef = 1$; and let $ec = f = 0{,}8$; then $\frac{rf}{r-2f} = \frac{1{,}6}{2-1{,}6} = \frac{1{,}6}{0{,}4} = 4$ Inches $= eC$; and $ab : AB : 0{,}8 : 4 :: 1 : 5$, or the Image *AB* will be 5 times longer than the Object a b. This Image *AB* is view'd by the *Meniscus* Eye-Glass L M, whence 'tis easy to observe that this Form of a Microscope is the same with that in *Article* 23, 24. only there is but one Reflection, and here is two; and there a small Concave was used, but here a Convex; because by this means the Instrument is shorter by twice the focal Distance e *f* nearly, which is very considerable, as being a $\frac{1}{3}$ Part of the whole.

31. I shall shew in the next *Annotation* how both these Microscopes may be had very conveniently in the reflecting Telescope, and conclude this with an Account of the Nature and Use of the Micrometer for measuring the smallest Parts of natural Bodies; and here I shall not take Notice of the several uncertain conjectural Methods described by others, but only such as I use in my own Microscopes, which is strictly Mechanical, and gives the Measurement absolutely.

32. The MICROMETER consists of a graduated circular Plate X, of a Screw *qo*, and its Index *qr*. The Threads of the Screw are such, that 50 make the Length of one Inch exactly. When it is to be used, the Point *o* is set to the Side of the Part to be measured, and then the Index is turn'd about with the Finger, till the Eye perceives the Point has just passed over the Diameter of the Part; then the Number of Turns, and Parts of a Turn, shewn by the graduated Circle, will give the Dimensions in Parts of an Inch, as I shall shew by the following Example.

33. Suppose it required to measure the Diameter of an human Hair, and I observe the Index is turn'd just once round while the Point *o* passes over it. Then 'tis plain the Diameter of the Hair in the Image is $\frac{1}{50}$ of an Inch. Now if the Microscope magnifies 6 times, or makes the Image 6 times larger in Diameter than the Object, then is the Diameter of the Hair itself but $\frac{1}{6}$ of $\frac{1}{50}$, that is but $\frac{1}{300}$ Part of an Inch.

3 Inches;

3 Inches; then will the Length of the Object appear to the Eye through such a Telescope 40 times larger than to the naked Eye; and its Surface will be magnified 1600 times, and its Bulk or Solidity 64000 times. (CXXVIII).

34. Also it is to be observed, that as there are ten large Divisions, and twenty small ones, on the Micrometer-Plate, so each of those small Divisions are the $\frac{1}{20}$ of $\frac{1}{50}$, or the $\frac{1}{1000}$ Part of an Inch. Therefore, if in measuring any Part of an Object, you observe how many of these smaller Divisions are pass'd over by the Index, you will have so many 1000dth Parts of an Inch for the Measure required. All which is so plain, that nothing can be said to illustrate the Matter.

35. In *Plate* XLVIII. I have given a Print of the Form of my NEW POCKET-MICROSCOPE furnish'd with the MICROMETER above described. This Microscope is of the most simple Structure, most easy and expeditious for Use, and comes at the least Price of any hitherto invented of the compound Sort. But for a particular Account of its Theory, and also of another, in a large Form, mounted on a BALL and SOCKET, for universal Use; as also a large Account of all Kinds of *Microscopic Objects*, and the Manner of applying them; I refer the Reader to my Treatise on that Subject, entituled MICROGRAPHIA NOVA.

(CXXVIII.) 1. The Nature and Structure of a common refracting Telescope is above described, and is so evident from the Figure, that I shall say nothing farther relating to its Composition, but shall proceed to shew the Imperfection of this Telescope, and that it arises from the different Refrangibility of common Light, and not from the spherical Figure of the Glass, as the *Opticians* before Sir *Isaac Newton*'s Time imagined, and therefore proposed to bring them to greater Perfection by introducing the Method of grinding and polishing Glasses of the Figure of one or other of the *Conic Sections*.

2. But this great Philosopher soon shew'd them their Mistake, by proving that the Error arising from the Figure of the Glass was many hundred times less than that which proceeded from the unequal Refrangibility of the Rays, and was so small as to be altogether inconsiderable; and this he did by an elegant Method in his *Lectiones Opticæ*, which I shall here translate from that admirable Book.

IF, instead of a *convex Eye-Glass* we should use a *concave* one of the same Focal Length, it would represent the Object erect, equally magnified, and more distinct and bright; but the Disadvan-

Pl. XLIX. Fig. 1. 3. Let NBM be a spherical Surface, C the Center, CB the Semidiameter or Axis parallel to the incident Rays, AN an incident Ray, and NK the same refracted, cutting the Axis CB produced, in the Point K; and let F be the Solar or principal Focus, where the Rays meet the Axis which are infinitely near to the Axis. The Error KF is now to be determined.

4. Let fall the Perpendiculars CE upon NK, and NG upon CK, and call $CB = a$, $GB = x$, and $CK = z$; and from the Nature of the Circle we have $NG^2 = 2ax - xx$, to which add $GK^2 = \overline{(z - a + x)}^2 z^2 + 2xz - 2az + x^2 - 2ax + a^2$, and the Sum will be $NK^2 = z^2 + 2xz - 2az + aa$.

5. Now since $NG : CE :: I : R$, *viz.* as the Sine of Incidence to the Sine of Refraction; and because of similar Triangles CEK and NGK, it is $NG : CE :: NK : CK :: I : R$; therefore $I^2 : R^2 :: (NK^2 : CK^2)\ z^2 + 2xz - 2az + a^2 : z^2$; and $I^2 z^2 = z^2 + 2xz - 2az + a^2 \times R^2$, and by Reduction $zz = \dfrac{2azR^2 - 2xzR^2 - a^2R^2}{R^2 - I^2}$;

and (putting $\dfrac{R^2a - R^2x}{R^2 - I^2} = s$, we have $z^2 = 2zs - \dfrac{R^2a^2}{R^2 - I^2}$, and $z^2 - 2sz = -\dfrac{R^2a^2}{R^2 - I^2}$, and compleating the Square, $z^2 - 2sz + ss = ss - \dfrac{R^2a^2}{R^2 - I^2}$; and extracting the Root, $z = s + \sqrt{ss - \dfrac{R^2a^2}{R^2 - I^2}}$; whence by Substitution we have) $z = \dfrac{R^2a - R^2x + R\sqrt{I^2a^2 - 2R^2ax + R^2x^2}}{R^2 - I^2}$.

6. And, reducing the radical Part to an infinite Series, we have $z = \dfrac{Ra}{R - I} - \dfrac{R^2x}{IR - I^2} - \dfrac{R^3x^2}{2I^3a} - \dfrac{R^5x^3}{2I^5a^2}$ &c. Now when $x = 0$, $z = \dfrac{Ra}{R - I} = CF$; whence $CF - CK =$

tage

tage of this Glaſs is, that it admits of but a ſmall *Area*, or *Field of View*, and therefore not to be uſed when we would ſee much of an Object, or take in a great Scope; but it is uſed to great Ad-

$KF = \frac{R^2 x}{IR - I^2} + \frac{R^3 x^2}{2 I^3 a}$, &c. which is the Value of the Error required.

7. Hence when BG or x is exceeding ſmall, $\frac{R^2 x}{IR - I^2} =$ KF nearly, becauſe in that Caſe the other Terms, where the aſcending Powers of x are found, become extremely ſmall, and nothing in regard to the firſt Term where x is ſingle.

8. Again; putting $NG = y$, we have $\frac{R^2 y^2}{2IRa - 2I^2 a} =$ KF nearly; for $NG^2 = BG \times \overline{BC + CG} = BG \times 2BC$ nearly, (from the *Elements*) that is, $y^2 = 2ax$ nearly, or $\frac{y^2}{2a} = x$. If then for x in the Equation of the laſt *Article* we ſubſtitute its Value $\frac{y^2}{2a}$, it gives the Equation above in this.

9. Hence alſo it follows, that the Error KF is always as the *Sagitta* or Verſed Sine GB, or the Square of the Semi-chord NG.

10. If the Ray ANK be given in Poſition, and an be any other parallel Ray nearer to the Axis, and on the other Side; of which let nk be the refracted Part cutting the Axis in k, and the refracted Ray NK in Q, and from Q draw Qo perpendicular to the Axis: Then will the Line Ko become greateſt of all, or a *Maximum*, when the Ray an is about half the Diſtance of the Ray AN from the Axis.

11. For drawing ng perpendicular to the Axis, and put $ng = v$, $Ko = s$, $GK = f$, and $KF = b$; and ſince, by *Art.* 9, we have $NG^2 : ng^2 :: KF : kF$, or $y^2 : v^2 :: b : \frac{v^2 b}{y^2} =$ kF, therefore $KF - kF = Kk = b - \frac{bv^2}{y^2} = \frac{byy - bvv}{yy}$.

12. Moreover, $GK : GN :: Ko : Qo$; wherefore $Qo = \frac{ys}{f}$. Alſo $gn : GK$ ($= gk$ nearly) $:: Qo : ok = \frac{ys}{v}$; there-

vantage in viewing the *Planets* and their *Satellites*, *Saturn's Ring*, *Jupiter's Belts*, &c. This is call'd the *Galilean Telescope*, from *Galileo*, the Inventor, and is the first Sort of Telescope ever made.

fore $ko + Ko = \frac{ys}{v} + s = \frac{ys + vs}{v} = Kk = \frac{byy - bvv}{yy}$; and dividing by $v + y$, and reducing the Equation, we have $s = \frac{bvy - bvv}{yy}$.

13. Now to determine s a *Maximum*, we must make its Fluxion $= o$, that is, $\dot{s} = \frac{b\dot{v}y - 2bv\dot{v}}{yy} = o$; whence we get $b\dot{v}y - 2b\dot{v}v = o$, that is, $by = 2bv$, or $2v = y$, or $2ng = NG$, when s or Ko is greatest of all.

14. Therefore Ko, when greatest, is equal to about $\frac{1}{4}$ of KF; for if in the Equation expressing the Value of s (in *Article* 12.) you write $2v$ for y, there will arise $\frac{1}{4}b = s$.

15. Also because $CF - CB = BF = GK$ nearly, therefore $GK = \frac{Ra}{R - I} - a = \frac{Ia}{R - I}$. Whence since GK $\left(= \frac{Ia}{R - I}\right) : GN \,(= y) :: Ko \left(= \frac{1}{4}KF = \frac{R^2y^2}{8IRa - 8I^2a}\right) : Qo = \frac{R^2y^3}{8I^2a^2}$.

16. If the Arch BM be taken equal to the Arch BN, and $Bm = Bn$, and Rays incident on M and m are refracted intersecting each other in the Point P, then 'tis evident $PQ = 2Qo = \frac{R^2y^3}{4I^2a^2}$; and it is also plain, that all the Rays which fall on the Curve between N and M are so refracted as to pass through the Space PQ, and that the said circular Space PQ is the least possible in which all the Rays can be congregated; and therefore that this Space is the Focus or Place of the Image of an Object, which sends parallel Rays upon the whole Surface of the Lens NBM.

17. For no Rays can be refracted without this Space, because since Qo is in a given Ratio to Ko, it will be at the same time a *Maximum* with it; and therefore the Point Q is the most remote from the Axis, in which any of those re-

THE *Cata-dioptric* or *Reflecting Telescope* is the most noble and useful of all others; the *Mechanism* whereof is as follows: ABEH is the large Tube or Body of the Instrument, in which BE is a large Plate L. Fig. 9.

fracted towards F can possibly intersect the external Ray NK. Neither can they be refracted into a less Space, because the Rays MK, NK, cut the external Rays *nk* and *mk* in the Points P and Q, by which the Space PQ is terminated.

18. If the Aperture of the Circle (or Lens) NBM be increased or diminished, the lateral Error PQ will be as y^3, or as the Cube of the Breadth of the Aperture NM. Also, if the Aperture of the Lens remain the same, the said Error PQ will be reciprocally as aa, or as CB^2, and therefore as BF^2, since CB and BF are in a given Ratio. But if neither the Magnitude of the Circle nor of the Aperture be constant, the Error PQ will be as $\frac{y^3}{a^2}$, or as $\frac{NM^3}{BF^2}$, as is evident from its Value $\frac{R^2y^3}{4I^2a^2}$, wherein the Part $\frac{R^2}{4I^2}$ is constant, and therefore omitted. Thus far Sir *Isaac*.

19. In all that has been said in the preceding *Articles*, we are to understand Sir *Isaac*'s Design is to shew what the Quantity of the Error is, and in what Proportion it varies, that arises from the circular Figure of the Glass only in refracting the same Ray as it is nearer to or farther from the Axis. And therefore we are to understand that the Rays here meant are homogeneal, or all of the same Sort, and which admit of no Error from a different Refrangibility.

20. Hence we are able to compare the Errors arising from the different Refrangibility of the Rays, and from the spherical Figure of the Glass, (supposing it a *Plano-Convex*, as it commonly is) in a Telescope of any given Length. For Example: In a refracting Telescope of 100 Feet Length, that is, where $BF = 2BC = 2a = D =$ Diameter of the Sphere $= 1200$ Inches, $y = NG = 2$ Inches, and let $I : R :: 20 : 31$ out of Glass into Air. Then will the Expression for the lateral Error from the Figure of the Glass be $\frac{R^2y^3}{4I^2a^2}$ $= \frac{31 \times 31 \times 8}{4 \times 20 \times 20 \times 600 \times 600} = \frac{961}{72000000}$ Parts of an Inch, the Diameter of the circular Space PQ.

reflecting Mirrour, with a Hole in the Middle CD. This Mirrour receives the Rays $a\,c$, $b\,d$, coming from the Object at a distance, and reflects them converging to its Focus e, where they cross

21. But the Diameter of the little Circle through which the Rays are scatter'd by unequal Refrangibility is about the 55th Part of the Breadth of the Aperture of the Object-Glass, (as we have already shewn) that is, in the present Case, a 55th Part of 4 Inches, or $\frac{4}{55}$. Wherefore the Error arising from the spherical Figure of the Glass is to that arising from the different Refrangibility of the Rays as $\frac{961}{72000000}$ to $\frac{4}{55}$, that is, as 1 to 5449; and therefore being in comparison so very small, deserves not to be consider'd in the Theory of Telescopes.

Pl.XLIX. Fig. 2. 22. Let us now see, according to Sir *Isaac*'s Method, what the Value of this lateral Error PQ is in Rays reflected from a spherical Surface, where every Part is denoted by the same Letters as before; only now the refracted Ray NK is the reflected Ray: And here also $NG^2 = 2ax - xx$, and $GK^2 = (\overline{a - z - x}^2 =)\ a^2 - 2ax - 2az + x^2 + 2zx + zz$, as before, (*Article* 4.) therefore $NG^2 + GK^2 = NK^2 = a^2 - 2az + 2xz + zz = zz$; because $NK = CK$, from the Law of *Reflection*. Whence $a^2 = 2az - 2xz$, and therefore $z = \frac{a^2}{2a - 2x} = CK$; but $CF = \frac{1}{2}a$, therefore $CK - CF = FK = \frac{a^2}{2a - 2x} - \frac{1}{2}a = \frac{ax}{\overline{a - x} \times 2}$.

23. Hence, when x is indefinitely small, $FK = \frac{ax}{2a} = \frac{1}{2}x = \frac{1}{2}GB$ nearly; and because $yy = 2ax$ nearly, (see *Article* 8.) therefore $\frac{y^2}{4a} = \frac{1}{2}x = FK$; and hence it appears, that the Error KF is always as x or the versed Sine GB, or as y^2, or Square of the Sine or Semi-Aperture NG.

24. Again; every thing in *Art.* 10, 11, 12, 13, and 14, each

each other, and form the inverted Image I M. xy is a small concave Mirrour, whose Focus is at f, at a small Distance from the Image. By this means the Rays coming from the Image are re-

is the same here as there; and so $Ko = \frac{1}{4}KF = \frac{y^2}{16a}$. And because G K is nearly equal $BF = \frac{1}{2}a$, therefore $GK : GN :: Ko : Qo$; that is, $\frac{1}{2}a : y :: \frac{y^2}{16a} : \frac{y^3}{8aa} = Qo$; consequently $2Qo = PQ = \frac{y^3}{4aa}$. *Q.E.I.*

25. Hence if we put $a = BG =$ Radius of the reflecting Sphere N B M, we shall have P Q in the refracting Surface or Lens, to P Q in the reflecting Surface or Mirrour, as $\frac{R^2y^3}{4I^2aa}$ to $\frac{y^3}{4aa}$, or (if $a = a$) as $\frac{R^2}{I^2}$ to 1, that is, as 2,4 to 1; so that *the Error by Refraction is near twice and a half greater than that by Reflection*, when the Radius of the Sphere is the same in both.

26. If the Medium be given, or the Ratio of I to R, and also the Aperture $NM = 2y$; then the Error by Reflection is to that by Refraction as $\frac{1}{aa}$ to $\frac{1}{aa}$. Hence, since if the focal Distance of a reflecting Telescope and a refracting one be equal, we have $a = 4a$, therefore $\frac{1}{aa}$ to $\frac{1}{aa}$ as $\frac{1}{16}$ to 1, it appears that the Error P Q in the Refractor is to that of the Reflector as 16 to 1.

27. Again; it appears, that in the Reflector, as well as the Refractor, the Error is *(cæteris paribus)* proportional to y^3, or the Cube of the Aperture of the Object-Metal N B M.

28. Lastly, we observe in the refracting Telescope, if the Radius $CB = a$, and Semi-Aperture $NG = y$, be given, the Error P Q will be as $\frac{R^2}{I^2}$. Hence, if the Lens be Glass, we have $\frac{R^2}{I^2} = \frac{31 \times 31}{20 \times 20} = 2{,}4$; and if the Lens be Water, we have $\frac{R^2}{I^2} = \frac{4 \times 4}{3 \times 3} = \frac{16}{9} = 1{,}7$. Therefore the Error

flected back through the central Hole CD of the large Mirrour, where they fall on the plano-convex Lens W X, and are by it converged to a Focus, and there form a second Image R S, very

by Refraction in a Glass-Lens is to that in a Water-Lens (*cæteris paribus*) as 2,4 to 1,77, or as 4 to 3 nearly.

29. Before Sir *Isaac Newton*, all Opticians imagined the Indistinctness or Imperfection of Telescopes was owing wholly to the Figure of the Glass or Lens; which put them upon introducing the Figures of the *Conic Sections*, because, being acquainted with the Ratios of Incidence and Refraction, they could find by Geometry that an Aberration of Rays from the principal Focus F would be occasion'd by the Curvature of the Glass, and that was always less of course as the Curvature was less; and that therefore if NBD, EBF, OBP, and Plate XLIX. Fig. 3. QBR represent the curved Surface of a *Circle*, an *Ellipsis*, a *Parabola*, and an *Hyperbola*, whose common Focus is C, 'tis plain, if a parallel Ray AN be incident on each of these Curves in the Points N, *a*, *b*, *c*, the Aberration or Error caused in the Ray by Refraction in each will be as the Curvature is less, or as the Radius of Curvature in the Points N, *a*, *b*, *c*, increases; and it has been shewn to be as the Square of that Radius inversely. (See *Art.* 18 and 26.) Consequently, since the Aperture and principal Focus is the same in all those Lenses, the Errors of the Rays will be lessen'd in each of them respectively.

30. But if the Imperfection of the refracting Telescope had been owing only to the spherical Figure of the Glass, Sir *Isaac Newton* proposed a Remedy without Recourse to the *Conic Sections*, which was by composing the Object-Glass of two Meniscus-Glasses, with Water between them. Thus let Fig. 4. ADFC represent the Object-Glass composed of two Glasses ABED and BEFC, alike convex on the Outsides AGD and CHF, and alike concave on the Insides BME, BNE, with Water in the Cavity BMEN.

31. Now let the Sines of Incidence and Refraction out of Glass into Air be as I to R, and out of Water into Air as K to R; then out of Glass into Water they will be as I to K, (*Annot.* CXVII.) And let the Diameter of the Sphere to which the convex Sides are ground be D, and the Diameter of the Sphere to which the concave Sides are ground be to D as the Cube Root of $\overline{K-I}\times K$ to the Cube Root of $\overline{K-I}$ $\times R$. Then the Refractions on the concave Sides of the large

large and erect, which is view'd by a *Meniscus Eye-Glass* YZ by the Eye at P, through a very small Hole in the End of the Eye-Piece YCDZ.

In the first Lens WX were taken away, the

Glasses will be very much corrected by the Errors of Refractions on the convex Sides, so far as they arise from the Sphericalness of the Figure.

32. But since those compound Lenses of Glass and Water are with Trouble and Difficulty made, Opticians have applied themselves to invent the best Figure of Lenses for this Purpose, that is, such that the Refraction at the second Surface might correct the Errors of Refraction at the first Surface (arising from the Figure of the Glass only) as much as possible: And the famous *Hugens* has given us a Theorem by which he proves the following Particulars.

33. *First*, That when parallel Rays fall upon the plane Side of a plano-convex Lens, the (longitudinal) Aberration of the extreme Ray is $\frac{9}{2}$ of the Thickness, and is less than the like Aberration caused by any Meniscus-Glass whose concave Side is exposed to the incident Rays.

34. *Secondly*, When the said Glasses have their convex Sides turn'd to the incident Rays, the Aberration of the extreme Ray in the Plano-Convex is $\frac{7}{6}$ of its Thickness, and is less than the like Aberration of any Meniscus in this Position.

35. *Thirdly*, That a double-convex Glass, whose Radius of the first Surface, on which the Rays fall, is to that of the second Surface as 2 to 5; is just as good as the Plano-Convex in its best Position, the Error being in both $\frac{7}{6}$ of their common Thickness.

36. *Fourthly*, When the Radii of a Double-Convex are equal, the Aberration is $\frac{5}{3}$ of the Thickness; and therefore such a Lens is not so good as a Plano-Convex of the same Thickness in its best Position.

37. *Fifthly*, But if the Radius of the first Surface be to that of the second as 1 to 6, it is then the best Glass of all, its Aberration then being the least possible, *viz.* $\frac{15}{14}$ of its Thickness. But if this best Glass be turn'd with its other Side to the Rays, the Aberration will be $\frac{145}{42}$, and therefore becomes much worse than before.

38. *Sixthly*, When a Plano-Concave has its plane Side turn'd towards parallel Rays, the Aberration of the extreme Rays is also $\frac{9}{2}$ of the Thickness; and when inverted it is only

Image

Image would be form'd somewhat larger at *Mg*; but the *Area*, or *Scope*, would be less, and therefore the View not so pleasant. At T V is placed a circular Piece of Brass, with a Hole of a proper

3. In a Double-Concave likewise, whose Radii of the first and second Surfaces are as 1 to 6, the Aberration is the least possible, *viz.* $\frac{15}{14}$, as above in the like Convex.

39. Hence the Glasses of common Spectacles ought to have the Figure of the Convex in *Art.* 37, and those Hand-Glasses which short-sighted People use ought to be such Concaves as are last mention'd.

40. In all the above mention'd Glasses the same Aperture, Thickness, and focal Distance is supposed, and that they differ in nothing but the Figure arising from the various Magnitude and Position of their Radii respectively. But after all, since, as we have shewn, the Aberration caused by the Figure bears so small a Proportion to that by the different Refrangibility of Rays, the Perfection of refracting Telescopes becomes desperate, and can only admit of Improvement by increasing their Length.

41. From hence long Telescopes became of common Use; and so great were the Improvements of this Sort, that for viewing the celestial Bodies the Tube of the Telescope was thrown aside, and a Method invented by *Hugenius* of managing them with much greater Ease, and of a greater Length. For he contrived to fix the Object-Glass upon the Top of a long upright Pole, and directed its Axis towards any Object by means of a Silk-Line coming down from the Glass to the Eye-Glass below. In this manner were Telescopes made to the Length of 123 Feet.

42. These were call'd *Aerial Telescopes*, as being used without a Tube, in a dark Night; for the Use of a Tube is not only to direct the Glasses, but also to make the Part dark where the Images of Objects are form'd; for in Telescopes, as well as in the *Camera Obscura*, we ought to have no other Light come to the Eye than what proceeds from the Pictures made of the Objects abroad.

43. In order to understand in what Proportion Telescopes are to be lengthen'd, so that they shall magnify in any proposed Degree with the same Distinctness and Brightness of the Object, we are to consider that the Indistinctness of Vision consists in this, *that the sensible Image of a lucid Point in the Object is not a Point in the Image, but a circular Area*; and

Size

Size to circumscribe the Image, and cut off all superfluous or extraneous Rays, that so the Object may appear as distinct as possible.

As the Image is form'd by Reflection, the

that two contiguous Points in the Object make two of those Areas in the Image, whose Centers are contiguous; and therefore as those two Areas are mix'd almost entirely with each other, the Representation of the said two Points in the Object is not distinct but confused.

44. And since this is the Case with respect to every other Point in the Object, 'tis evident there will be a Mixture of so many Points of an Object in every Point of the confused Picture, as there are Points in the Circle of Aberration; since the Center of any one Circle of Aberration will be cover'd by all other Circles of Aberration, whose Centers fall within the Perimeter of the first mention'd Circle; or, in other Words, there will be such a Number of Points in the Object mix'd in any one Point in the confused Image, as is proportional to the Area of the Circle of Aberration.

45. Hence, since this confused Representation of several Points in one is impress'd on the Retina by the Eye-Glass, and from thence convey'd to the Common Sensory, it appears *that the Indistinctness of an Object is as the Area of a Circle of Aberration in the Focus of a Telescope*, or as the Square of its Diameter.

46. To illustrate this Matter, let A be a given Point, BC an Object-Glass of a Telescope, BCA a Pencil of Rays coming from the Point upon the Glass; each Ray, AB, AC, will be so refracted through the Lens, as that the most refrangible Part of each will meet and intersect each other in the Point F in the Axis, the mean refrangible Part will go to *c*, and the least refrangible Part will meet and intersect the most refrangible on each Side in the Points D and E; therefore DE will be the Diameter of the confused Image or Circle of Aberrations *a*D*b*E, and *c* its Center. Plate XLIX. Fig. 5.

47. Let HI be the Eye-Glass, and G its Center; then will the Angle DGE be that under which the Circle of Aberrations is seen at the Eye-Glass, and consequently at the Eye, (as we have shewn already). But this Angle is as the Subtense DE directly, and as the Perpendicular G*c* inversely, that is, DGE is as $\frac{DE}{Gc}$; for it increases as DE increases while G*c* remains the same, and as G*c* decreases while DE is con-

Rays

Rays of every Sort will be united nearly in one Point, and will therefore admit of an Eye-Glass YZ of a deep Charge, or small Focal Distance; and so the Power of magnifying will be propor-

stant; wherefore, since DE is always as the Angle DGE, we have $DE : \frac{DE}{Gc}$, and so $DE^2 : \frac{DE^2}{Gc^2}$. But DE^2 is as the Area of the Circle of Aberration, and therefore as the Indistinctness of Vision; consequently the apparent Indistinctness of a given Object will be as $\frac{DE^2}{Gc^2}$.

48. Therefore the Distinctness of Vision will be as $\frac{Gc^2}{DE^2}$; or, because $DE = \frac{1}{55} CB$ the Diameter of the Aperture of the Object-Glass, therefore DE^2 will be as CB^2; and so the Distinctness of a given Object will always be as $\frac{Gc^2}{CB^2}$, that is, *as the Square of the focal Distance of the Eye-Glass directly, and as the Square of the Diameter or Area of the Aperture inversely.*

49. If then in any one refracting Telescope the Distinctness of an Object be represented by $\frac{Gc^2}{BC^2}$, and in any other Telescope of the same Sort by $\frac{\mathit{Gc}^2}{\mathit{BC}^2}$; then if $\frac{Gc^2}{BC^2} = \frac{\mathit{Gc}^2}{\mathit{BC}^2}$, we have $BC^2 \times \mathit{Gc}^2 = \mathit{BC}^2 \times Gc^2$, or $BC \times \mathit{Gc} = \mathit{BC} \times Gc$; and therefore $BC : \mathit{BC} :: Gc : \mathit{Gc}$; that is, *two refracting Telescopes shew an Object equally distinct, when the Diameters of the Apertures of the Object-Glasses are as the focal Distances of the Eye-Glasses.*

50. In reflecting Telescopes the Diameter of the Circle of Aberrations was $PQ = \frac{y^3}{4aa} = \frac{y^3}{D^2}$, (supposing $D = 2a =$ Diameter of the Sphere; see *Article* 24.) whence $PQ^2 = \frac{y^6}{D^4}$. Let F = focal Distance of the Eye-Glass, then the Indistinctness of Vision will be as $\frac{PQ^2}{F^2}$ (*Article* 47.) $= \frac{y^6}{D^4 \times F^2}$.

tionally

tionally greater; for it will be in a Proportion compounded of $\frac{Qe}{eG}$ and $\frac{Gk}{kt}$, if only one Eye-Glass Y z be used. Thus, in Numbers, suppose $Qe = 12$ Inches, $eG = 3,5$; $Gk = 18$, and $kt = 1$; then will $\frac{12}{3,5} \times \frac{18}{1} = \frac{216}{3,5} = 61,71$.

51. Therefore if the same Parts in another Telescope of this Sort be represented by $\frac{pQ^2}{F^2} = \frac{y^6}{D^4 \times F^2}$, and since the Distinctness in each will be as $\frac{D^4 F^2}{y^6}$, and $\frac{D^4 F^2}{y^6}$; then if we suppose the Object seen equally distinct in both, we shall have $D^4 F^2 \times y^6 = D^4 F^2 \times y^6$, or $D^2 F y^3 = D^2 F y^3$; Hence $F : F :: \frac{y^3}{D^2} : \frac{y^3}{D^2}$; that is, *Reflecting Telescopes shew an Object equally distinct, when the focal Distances of the Eye-Glasses are as the Cubes of the Diameters of the large Specula or Object Metals, divided by the Square of the Diameter of the Spheres to which they are ground, or by the Square of the focal Distance of the Metals.*

52. In any Telescope or Double Microscope, the Brightness of a given Image will be as the Quantity of Light by which it is shewn; that is, as the Area of the Aperture of the Object-Glass, or as the Square of the Diameter.

53. Also, if the Area of the Aperture of an Object-Glass be given, the Brightness of the Image will be inversely as its Area, or Square of its Diameter or Breadth: For the less the Area of the Picture is, the greater will be its Brightness by the same Quantity of Light.

54. Therefore when neither the Apertures of the Glasses nor the Amplifications of the Picture are given, or the same, the Brightness is as the Square of the Diameter of the Apertures directly, and the Square of the linear Dimensions of the Pictures inversely.

55. Hence in all Sorts of Telescopes a given Object appears equally bright, when the Diameters of the Apertures are as the linear Dimensions of the Pictures: But the Picture is larger as the focal Distance of the Object-Glasses is so, and also as the focal Distance of the Eye-Glass is less; therefore

nearly;

nearly; whence by such a Telescope the Length of an Object will be magnified 50 times, the Surface 2500 times, and the Solidity 125000 times; yet the Telescope not above 20 Inches long; an Effect equal to that of a refracting Telescope 16 Feet in Length.

As to the *Camera Obscura*, and *Magic Lant-*

the linear Dimensions of Pictures are as the focal Distances of the Object-Glasses directly, and as the focal Distances of the Eye-Glasses inversely. Let these be represented by F and *F*, and f, *f*, in any two Telescopes; let D, *d*, be the Diameters of the Apertures, and L, *l*, the linear Dimensions of the Pictures; then we have $D : d :: L : l :: \frac{F}{f} : \frac{F}{f}$, when Objects appear equally bright in both.

56. Hence, since the Brightness of a Picture or Image is as $\frac{D^2}{L^2}$ (*Art.* 54.) $= \frac{D^2 f^2}{F^2}$, (because $L = \frac{F}{f}$ by the last) therefore if D or f be each increased in any Ratio, the Distinctness will remain the same as before, (by *Art.* 49) and the linear Dimensions of the Image will be diminish'd in the same Ratio, (since L is inversely as f) but the Brightness of the Image will be increased in the quadruplicate Ratio of what it had before. For,

57. Suppose F or the focal Length of the Telescope given, then the Brightness of the Picture will be in this Case as $D^2 f^2$; and if D and f be increased each in the Ratio of 1 to m, then will the Brightness be in this Case as $m^2 D^2 f^2 m^2 = D^2 f^2 m^4$; so that the former Brightness is to this as $D^2 f^2$ to $D^2 f^2 m^4$, that is, as 1 to m^4; which Ratio is quadruplicate of the Ratio 1 to m.

58. Because we had $D : \frac{F}{f}$, or $Df : F$, when Objects appear equally bright, (by *Art.* 55.) and when they are shewn equally distinct we had $D : f$ (by *Art.* 49); therefore in refracting Telescopes of various Lengths, that Objects may appear equally bright and equally distinct, it is requisite that $D^2 : F$, and $f^2 : F$, or that $D : f : \sqrt{F}$; that is, *the Diameter of the Aperture and also the focal Length of the Eye-Glass should each be as the Square Root of the focal Distance or Length of the Telescope.*

born,

born, they both perform their Effects by a single Lens; the former being only the Object-Glass of a long Telescope applied in a *Scioptric Ball* to the Hole of a Window-Shutter, in a darken'd Room; which gives a lively Picture of all the Objects

59. In this Case likewise *the linear Dimensions of the Picture or Image are in the same subduplicate Ratio of the Length of the Telescope*; because, as was shewn, (*Art.* 55.) the linear Dimensions are directly as the Diameter of the Aperture, which is here shewn to be as the Square Root of the Length of the Telescope.

60. In reflecting Telescopes, when the Distinctness is given, we have $F : \frac{y^3}{D^2}$, and therefore $y^3 : D^2 F$. (See *Article* 51.) Also when the Brightness is given we have $y : \frac{D}{F}$, (*Art.* 55.) therefore $F : \frac{D}{y}$. Hence, when the Distinctness and Brightness are both given, we have $y^3 : (D^2 F) : \frac{D^3}{y}$, or $y^4 : D^3$, or $y : D^{\frac{3}{4}}$.

61. The linear Dimensions of the Picture $\frac{D}{F}$ were as y; that is, in this Case, $\frac{D}{F} : D^{\frac{3}{4}}$, and therefore $D : F D^{\frac{3}{4}}$; whence $F : \frac{D^{\frac{3}{4}}}{D} : D^{\frac{1}{4}}$. *Hence in reflecting Telescopes of different Lengths a given Object will appear equally distinct and bright, when the Diameters of the Object-Metals are as the Biquadrate Roots of the Cubes of the Diameters of the Spheres or focal Lengths of the Specula*; or, *when the focal Distances of the Eye-Glasses are as the Biquadrate Root of the focal Distance of the Specula.*

62. According to the Theorems in *Art.* 48, 49, Mr. *Hugens* calculated a Table of the linear Aperture of the Object-Glass, the focal Distance of the Eye-Glass, and the linear Amplification or magnifying Power of the Telescope, from one which he found by Experience was constructed in the best Manner. I have reduced his *Rhinland* Measures to *English* Feet, Inches, and Decimal Parts, as follows.

which

which lie before it, in true Perspective, but in an *inverted Position*, on a white Sheet or Plane held

Focal Distance of the Object-Glass.	*Linear Aperture of the Object-Glass.*	*Focal Distance of the Eye-Glass.*	*Magnifying Power.*
Feet.	Inch. Dec.	Inch. Dec.	
1	0,545	0,605	20
2	0,76	0,84	27,6
3	0,94	1,04	33,5
4	1,08	1,18	39,5
5	1,21	1,33	44
6	1,32	1,45	49
7	1,43	1,58	53
8	1,53	1,69	55
9	1,62	1,78	59
10	1,71	1,88	62
15	2,10	2,30	76
20	2,43	2,68	88
30	3,00	3,28	108
40	3,43	3,76	125
50	3,84	4,20	140
60	4,20	4,60	152
70	4,55	5,00	164
80	4,83	5,35	176
90	5,15	5,65	187
100	5,40	5,95	197
120	5,90	6,52	216

63. Since it has been shewn that the Errors arising from the different Refrangibility of Rays, and of consequence the Indistinctness of Vision by refracting Telescopes is so very great, a Question may be put, How it comes to pass Objects appear through such Telescopes so distinct as they do? To which it may be answer'd, 'tis because the erratic Rays are not uniformly scatter'd over all the Area of the Circle of Aberration, but collected infinitely more densely in the Center than in any other Part of that circular Space, growing rarer and rarer towards the Circumference, where, in comparison, they are infinitely rare, and affect not the Sense any where but in the Center, and very near it, on that account.

64. 'Tis farther to be observed, that the most luminous of

at

at the Focal Diſtance of the ſaid Glaſs: And on the other hand, the *Magic Lanthorn* is only a large

all the Priſmatic Colours are the Yellow and the Orange; theſe affect the Senſes more ſtrongly than all the reſt put together; and next to theſe in Strength are the Red and Green. The Blue compared with theſe is a faint and dark Colour, and the Indigo and Violet are much darker and fainter; ſo that theſe, compared with the ſtronger Colours, are little to be regarded.

65. The Images of Objects are therefore to be placed not in the Focus of the mean refrangible Rays, which are in the Confine of Green and Blue, but in the Focus of thoſe Rays which are in the Middle of the Orange and Yellow, there where the Colour is moſt luminous and fulgent; that is, the brighteſt Yellow, that Yellow which inclines more to Orange than to Green.

66. Now it has been ſhewn (*Annot.* CXVIII. 9.) that the Diameter of the Circle in which both thoſe Colours will be contain'd is but the 260th Part of the Diameter of the Aperture of the Object-Glaſs; and farther, about $\frac{2}{5}$ of the brighter Halves of the Red and Green (on each Side) will fall within this Circle, and the remaining $\frac{3}{5}$ without it, which will be ſpread over twice the Space nearly, and therefore become much rarer. Of the other Half of the Red and Green, about one Quarter will fall within this Circle, and $\frac{3}{4}$ without, and be ſpread through four or five times the Space, and therefore become much rarer. Alſo this extreme Red and Green is much rarer and darker than the other Parts of the ſame Colours; and the Blue and Violet being much darker Colours than theſe, and more rarified, may be quite neglected.

67. Hence the ſenſible confuſed Image of a lucid Point is ſcarce broader than a Circle whoſe Diameter is the 260th Part of that of the Aperture of the Glaſs, if we except the dark miſty Light round about, which we ſcarce regard. And therefore in a Teleſcope whoſe Aperture is 4 Inches, and Length 100 Feet, it exceeds not $2\frac{3}{4}''$, or $3''$; and in a Teleſcope whoſe Aperture is 2 Inches, and Length 20 or 30 Feet, it may be about $5''$ or $6''$, and ſcarce above. And this anſwers well to Experience; for it is obſervable that in Teleſcopes of 20 or 30 Feet long, the Diameters of the Fixed Stars appear to be about $5''$ or $6''$, or at moſt not more than $8''$ or $10''$.

68. Now if we ſuppoſe the ſenſible Image of a lucid Point to be even a 250th Part of the Diameter of the Aperture of

convex Lens, with a short Focal Distance, which by being placed at a proper Distance from small

the Glass, yet will this be much greater than if it were only from the spherical Figure of the Glass, *viz.* (in an 100 Foot Telescope) in the Ratio of $\frac{4}{250}$ to $\frac{961}{72000000}$, or of 1200 to 1. (See *Art.* 20, 21.) Therefore the Image of a lucid Point would still be a Point, but for the various Refrangibility of the Rays; and this alone is the invincible Obstacle to perfect Vision by any refracting Instruments.

Pl.XLIX. Fig. 6. 69. The magnifying Power of a refracting Telescope is thus estimated. Let AB be the Object-Glass, and CD the Eye-Glass; and let HFI and GFM be two Rays coming from the extreme Parts of a distant Object, and crossing each other in the Center F of the Glass AB. Then is the Angle GFM = IFM that under which the Object appears to the naked Eye; but IEM = CKD is that under which the Image appears as magnified by the Eye-Glass CD. But the Angle IEM is to the Angle IFM as LF to LE, or *as the focal Distance of the Object-Glass to the focal Distance of the Eye-Glass*; and in that Proportion is the Object magnified, as was observed before in *Art.* 55.

Fig. 7. 70. The magnifying Power of a reflecting Telescope is thus computed. The parallel Rays KB and LE are reflected by the large Object-Metal AF to its Focus *a*, where the Image IM is form'd; which Image is defined by two other Rays NQ, PQ, coming from the extreme Parts of the Object at a remote Distance, and meeting in the Center of the large Speculum at Q; for it has been shewn that the Object and its Image both appear under the same Angle from the Vertex of the Mirrour. (*Annot.* CXXV.)

71. Now if *f* be the Focus of the small Mirrour GH, supposing the Image were form'd in the said Focus *f*, (that is, that both the Foci *a* and *f* were coincident) then the Rays proceeding from the Image IM will proceed parallel after Reflection, and produce distinct Vision of the Image, which will then subtend an Angle IOM at the Center O of the Speculum GH; which is to the Angle IQM, under which the Object appears to the naked Eye, as *a*Q to *a*O or *f*O. So that the magnifying Power would in this Case be as $\frac{aQ}{fO}$.

72. But to increase this magnifying Power, the Image IM

transparent-

transparent-colour'd Pictures or Figures, forms a large and surprizing Image thereof at a great Di-

is not placed in the Focus of the small Speculum, but at a small Distance beyond it; by which means the Rays coming from the Image to the Speculum GH will be reflected converging to a distant Focus R, where a secondary large Image *IM* is form'd from the first Image IM; which Image *IM* is seen under the same Angle *IOM* with the former from the Center of the Speculum GH, but from the Center of the Eye-Glass TV it is seen under the large Angle *ISM*. But the Angle *ISM* is to the Angle *IOM* as OR to SR; wherefore the second Ratio or Part of the magnifying Power is that of $\frac{OR}{SR}$.

73. Consequently, the whole magnifying Power of the Telescope is $\frac{aQ}{aO} \times \frac{OR}{SR}$ (because in this Case fO becomes aO). Or, in other Words, the Angle NQP, under which the Object appears to the naked Eye, is to the Angle *ISM*, under which the large magnified secondary Image *IM* appears to the Eye, through the Eye Glass, as $\frac{aQ \times OR}{aO \times SR}$. Such is the Theory of the Telescope first contrived by Dr. *J. Gregorie*, and therefore call'd the *Gregorian Telescope*; but it received its last Improvement from the late Mr. *Hadley*, and is now in common Use.

74. A small Alteration was made in the Structure of this Telescope by Mr. *Cassegrain*, viz. in using a convex Speculum *GH*, instead of the concave one GH. Now if they are equally spherical, that is, if they are Segments of the same Sphere, then will f be also the virtual Focus of the Convex *GH*; and if all other Things remain the same, the first Image IM will be virtually the same as before, and the last Image *IM* will be really the same; so that the magnifying Power of this Form of the Telescope is $\frac{aQ \times OR}{aO \times SR}$, which is equal to that of *Gregorie*'s Form.

75. And to shew this is a curious Proposition, I shall give Pl. XLIX.
the following easy Demonstration thereof. Let HD be a Fig. 8.
concave Speculum, and EC a convex one, both described with the same Radius CD, on the common Axis BCD: The Point N, bisecting the Radius CD, will be the Solar Focus to each Speculum. Let F be a radiant Point, from whence

ſtance; in order to which, it is neceſſary to illuminate them very ſtrongly with the Light of the

a Ray FH is incident upon the concave Mirrour in H, or to which the Ray KE incident upon the convex Mirrour tends; both thoſe Rays will be reflected to the ſame Point B in the Axis, and in the ſame Line EB. Laſtly, let GF be an Object; the Image thereof *ab* form'd by the Concave is equal to the Image AB made by the Convex. This is evident from the Theorems $\frac{dr}{2d+r}=f$, and $\frac{dr}{r-2d}=f$, thoſe Specula reſpectively.

76. For as $d=$ FC, CB $=f$ in the Convex; ſo in the Concave let FD $=D$, and DB $=F$; and then we have in the former $d:f::2d+r:r$, and in the latter $D:F::r-2D:r$. But $D=d+r$, therefore $2D=2d+2r$; whence $r-2D=2d+r$, conſequently $d:f::D:F$; that is, CF : CB :: DF : DB. But the Object and Image are to each other in the ſame Ratio in either Glaſs; and therefore ſince the Object is the ſame in both, the Image will be ſo likewiſe, or AB $=ab$.

Plate XLIX. Fig. 9. 77. Sir *Iſaac Newton* order'd this Teleſcope to be made in a different Form or Manner, as follows. ABCD was a large octogonal Tube or Caſe; EF a large poliſh'd Speculum, whoſe Focus is at *o*; GH a plane Speculum truly concenter'd, and fix'd at half a Right Angle with the Axis of the large one. Then parallel Rays *a*E, *b*F, incident on the large Speculum EF, inſtead of being reflected to the Focus *o*, were intercepted by the ſmall plane Speculum GH, and by it reflected towards a Hole *cd* in the Side of the Tube, croſſing each other in the Point O, which is now the true focal Point; and from thence they proceed to an Eye-Glaſs *ef* placed in that Hole, whoſe focal Diſtance is very ſmall, and therefore the Power of magnifying may be very great in this Form of the Teleſcope; becauſe the Image IM is made by one Reflection, (for that of the plane Speculum only alters the Courſe of the Rays, and adds nothing to the Confuſion of the Image) and will for that Reaſon bear being view'd by a Glaſs of a very deep Charge, in compariſon of an Image form'd by differently refrangible Rays.

78. This Teleſcope is a very good one, as to its Effect or Performance, but is not ſo commodious for common Uſe as thoſe of the *Gregorian* Form, and is therefore now pretty much laid aſide. They who would ſee a larger Account here-

Candle

Candle thrown on them by another *very large and very convex Lens* (CXXIX).

of may consult Sir *Isaac*'s *Optics*, and several *Philosophical Transactions*, where he describes it at large, and the Reasons which induced him to make choice of this Structure rather than that of Dr. *Gregorie*: Or see a compendious Account of the whole in the last Edition of Dr. *Gregorie*'s *Elements of Optics*.

(CXXIX) 1. The CAMERA OBSCURA, or *Darken'd Room*, is made after two different Methods; one is the *Obscura Camera* or Darken'd Chamber at large, and properly so call'd; that is, any large Room or Chamber made as dark as possible, so as to exclude all Light but that which is to pass through the Hole and Lens in the Ball fix'd in the Window of the said Room.

2. The other is in small, and made in various Ways, as that of a Box, a Book whose Sides fold out, &c. for the Conveniency of carrying it from Place to Place, for taking an Optic View in Picture of any proposed Place or Part of the Country, Town, &c. and hence it is call'd the *Portable Camera Obscura*.

3. The following Particulars are to be attended to in this Philosophical Contrivance. *First*, That the Lens be extremely good, or free from any Veins, Blebs, &c. which may distort and blemish the Picture.

4. *Secondly*, That the Lens be always placed directly against the Object whose Picture you would have perfectly form'd to contemplate; for if the Glass has any other Position to the Object, the Image will be very imperfect, indistinct, and confused.

5. *Thirdly*, Care ought to be taken, that the Ball be sufficiently large, and the Frame in which it is placed not too thick, that so there may be sufficient Room for turning the Ball every way, to take in as many Objects as possible, and to render the Use thereof most compleat.

6. *Fourthly*, The Lens ought to be of a just Magnitude or Aperture; for if it be too small, the Image will be obscure, and the minute Parts not visible at a distance for want of requisite Light. On the other hand, if the Aperture be too large, the Image will be confused, and become indistinct by too much Light.

7. Therefore, *Fifthly*, if by Experience I find that an

THE *Solar Microscope* is of the same Kind with the *Magic Lanthorn*; only here the Objects are very small, and strongly enlighten'd by the Sun

Aperture of 2 Inches Diameter is best for a Lens of 6 Feet focal Distance, I know (from what has been said in the last *Annotation*) that the Diameter of any other Lens of a different focal Distance ought to be in the subduplicate Ratio of 6 to the said focal Distance, that the Object, or its Image rather, may be equally bright and distinct in both.

8. *Sixthly*, We ought not to attempt to exhibit a Picture of Objects in a dark Room, unless the Sun shines upon or strongly illuminates the Objects; for mere Daylight is not sufficient for this Purpose, the greatest Beauty in this Phænomenon being the exquisite Appearance and Contrast of Lights and Shadows, none of which can appear but from an Object placed in the Sun-Beams; without which every thing looks dark and dull, and makes a disagreeable Figure.

9. Therefore, *Seventhly*, the Window, or that Side of the Room where the Scioptric Ball is used, ought to look towards that Quarter directly upon which the Sun shines, that so the illumined Sides of Objects may present themselves to the Lens, and appear more glorious in the Picture.

10. *Eighthly*, Hence it is easy to infer, that the best Time of the Day for this Experiment is about Noon, because the Sun-Beams are then strongest, and of course the Picture most luminous and distinct: Also that a North Window is the best; though for viewing the Shadows in greatest Perfection, an East or West Window will answer the End best.

11. *Ninthly*, As the Image is form'd only by the reflected Rays of the Sun, so due Care should be taken that none of the Sun's direct Rays fall on the Lens in the Window; for if they do, they will, by mixing with the former, greatly disturb the Picture, and render it very confused and unpleasant to view.

12. *Tenthly*, As white Bodies reflect the incident Rays most copiously, and black ones absorb them most; so to make the Picture most perfect it ought to be received upon a very white Surface, as Paper, a painted Cloth, Wall, *&c.* border'd round with Black, that so the collateral Rays which come from on each Side the Object may be stifled, and not suffer'd to disturb the Picture by Reflection.

13. These are the necessary Precautions for the due ordering of the various Circumstances of this Experiment. I shall

through

through a concave Lens; they are also magnified by a small Lens, of a very short Focal Distance, that the Images may be thrown large and distinctly

now enumerate the several *principal Phænomena* of the Dark Chamber. The *First* of which is, that an exact and every-way similar Image is form'd of an external Object; for Pencils of Rays coming from all Points of the Object will represent those Points in such a Manner and Position as will be very proportional and correspondent to their respective Positions and Distances in the Object, so that the Whole in the Image shall bear an exact Similitude or Likeness of the Object in every Respect.

14. The *Second Phænomenon* is, that the Image will bear the same Proportion to the Object, whether a Line, Superficies, or Solid, as their Distances from the Glass respectively: This is evident from what has been said relating to the Effect of a convex Lens. Hence the larger the focal Distance of the Glass, the more ample will be the Picture of the same Object, but the less will be the Space or Compass of the Plan or Perspective View.

15. The *Third Phænomenon* is, that the Image or Picture of the Object is *inverted*; and this is not the Effect of the Glass, but the crossing of the Rays in the Hole through which they pass into the Room; for if a very small Hole were made in the Window-Shutter of a darken'd Room, the Objects without would be all seen inverted, those which come from the upper Part of the Object going to the lower Part of the Image, and *vice versâ*. All that the Glass does is to render the Image distinct, by converging the Rays of every Pencil to their proper Focus in the Picture, the Position of each Point being the same as before.

16. The *Fourth Phænomenon* is the Motion or Rest of the several Parts of the Picture, according as those of the Object are in either State. The Reason of this is very obvious; and this it is that gives Life and Spirit to the Painting and Portraits of Nature, and is the only Particular inimitable by Art. And indeed a more critical Idea may be form'd of any Movement in the Picture of a darken'd Room, than from observing the Motion of the Object itself: For Instance, a Man walking in a Picture appears to have an undulating Motion, or to rise up and down every Step he takes; whereas nothing of this Kind is observed in the Man himself, as view'd by the bare Eye.

on the opposite Wall of a darken'd Room: Which, if well perform'd, is one of the most exquisitely curious and most delightfully surprizing

17. The *Fifth Phænomenon* is the *Colouring of the Optic Picture*; every Piece of Imagery has its proper Tints and Colours, and those always heighten'd and render'd more intense than in the Object; so that in this respect it is an Improvement of Nature itself, whereas the Art of the greatest Master can only pretend to a distant Resemblance and faint Imitation. The Reason why the Image is coloured is because the several Points of the Object reflecting several Sorts of colour'd Rays to the Glass, those Rays will give a Representation of those several Points respectively, and in their own Colour, and therefore in those of the Object; but those Colours will be heighten'd, because they are crowded into a less Space.

11. The *Sixth Phænomenon* is the *Claro Oscuro*, as the *Italians* call it; that is, the Intensity of *Light and Shadow* in the Picture: And this, as well as the Colouring, is greatly heighten'd above what it is in the Object, by reason of the lesser Area of the Picture. Here every Light and every Shade is express'd in its proper Degree, from the most brillant in the one, to the most jetty Black of the other, inclusive of a wonderful Variety in the several Parts, arising from the different Situations of the several Parts of the Object, and the different Angles of Reflection. A just Imitation of Nature in the Distribution of Light and Shadows is perhaps the most difficult Part of the Art of Painting, and on which its greatest Perfection depends.

19. The *Seventh Phænomenon* is the *Optical Perspective*, or Projection of the Image, which is not *in Plano*, or on a Plane, as in common Perspective, but on a Surface described by the Revolution of a *Conic Section* about its Axis, as is evident from what was observed in *Annot*. CXXV. Therefore, though in general a plane Surface is made use of, and may do very well in large Representations, yet in smaller ones, as those of the *Portable Camera's*, it is necessary, to have the Image or Picture compleat, or every where well defined, that it be received upon the Surface of an *Elliptic Figure*, and such as is suited to the middle Distance of the Objects. But this is a Nicety which few will think worth regarding, who do not aim at a very great Accuracy indeed in what they do.

20. I shall finish this Subject with an Observation that may

Effects

Effects that can be produced by any Optical Instrument whatsoever (CXXX).

be useful to Persons concerned in Drawing, and that is, *That if an Object be placed just twice the focal Distance from the Glass without, the Image will be form'd at the same Distance from the Glass within the Room, and consequently will be equal in Magnitude to the Object itself.* The Truth of this is demonstrated in *Annot.* CXXV.

21. Although every thing that has been said of the *Camera Obscura* is plain enough in itself to be understood, yet as a Representation thereof may facilitate the Idea, I have here given a Diagram for that Purpose; where ABCD is the Prospect of a *House, Trees*, &c. EF a darken'd Room, or *Camera Obscura*; on one Side is the Picture GH of the said View inverted, form'd by a convex Lens in the Ball fix'd before a Hole in the other Side IK at V. All which is so easy that nothing more remains to be said to explain it. Plate LI. Fig. 1.

(CXXX) 1. The SOLAR TELESCOPE and SOLAR MICROSCOPE, as they ought to make a Part of the Amusement of every Virtuoso and Gentleman, so they deserve a Particular Account, and the several Ways in which they are used merit a particular Description, which I shall illustrate by a Draught of each.

2. The SOLAR TELESCOPE is applied to Use in the following Manner. AB represents a Part of the Window-Shutter of a darken'd Room, CD the Frame, which (by means of a Screw) contains the Scioptric Ball EF, placed in a Hole of the said Shutter adapted to its Size. This Ball is perforated with a Hole *abcd* through the Middle; on the Side *bc* is screw'd into the said Hole a Piece of Wood, and in that is screw'd the End of a common refracting Telescope GHIK, with its Object-Glass GH, and one Eye-Glass at IK; and the Tube is drawn out to such a Length, as that the Focus of each Glass may fall near the same Point. Fig. 2.

3. This being done, the Telescope and Ball are moved about in such manner as to receive the Sun-Beams perpendicularly on the Lens GH, through the cylindric Hole of the Ball; by this Glass they will be collected all in one circular Spot *m*, which is the Image of the Sun. The Lens IK is to be moved nearer to or farther from the said Image *m*, as the Distance at which the secondary Image of the Sun is to be form'd requires, which is done by sliding the Tube IKLM backwards and forwards in the Tube LMNO. Then

Then of the first Image of the Sun *m* will be form'd a second Image PQ, very large, luminous, and distinct.

4. In this Manner the Sun's Face is view'd at any time, without Offence to weak Eyes; and whatever Changes happen therein may be duly observed. The *Spots* (which make so rare an Appearance to the naked Eye, or through a small Telescope in the common Way) are here all of them conspicuous, and easy to be observed under all their Circumstances of Beginning to appear, Increase, Division of one into many, the Uniting of many into one, the Magnitude, Decrease, Abolition, Disappearance behind the Sun's Disk, *&c.*

5. By the *Solar Telescope* we also view an Eclipse of the Sun to the best Advantage, as having it in our Power by this means to represent the Sun's Face or Disk as large as we please, and consequently the Eclipse proportionably conspicuous. Also the Circle of the Sun's Disk may be so divided by Lines and Circles drawn thereon, that the Quantity of the Eclipse estimated in Digits may this way be most exactly determined: Also the Moments of the Beginning, Middle, and End thereof, for finding the Longitude of the Place: With several other Things relating thereto.

6. The Transits of *Mercury* and *Venus* over the Face of the Sun are exhibited most delightfully by this Instrument. They will here appear truly round, well defined, and very black; their comparative Diameters to that of the Sun may this way be observed, the Direction of their Motion, the Times of the Ingress and Egress, with other Particulars for determining the Parallax and Distance of the Sun more nicely than has hitherto been done.

7. By the *Solar Telescope* you see the Clouds most beautifully pass before the Face of the Sun, exhibiting a curious Spectacle according to their various Degrees of Rarity and Density. But the beautiful Colours of the Clouds surrounding the Sun, and refracting his Rays, are best seen in the Picture made by the *Camera-Glass*. The fine Azure of the Sky, the intensely strong and various Dyes of the Margins of Clouds, the *Halo's* and *Corona's*, are this way inimitably express'd. And since the Prismatic Colours of Clouds, so variously compounded here, make so noble and delightful a Phænomenon, I have often wonder'd to see no more Regard had thereto by Painters, whose Clouds (though near the Sun) are seldom or never seen tinged or variegated with those natural Tints and Colours.

8. I cannot here omit to mention a very *unusual Phænomenon* that I observed about ten Years ago in my darken'd Room. The Window look'd towards the West, and the Spire

Spire of *Chichester* Cathedral was directly before it, at the Distance of about 50 or 60 Yards. I used very often to divert myself in observing the pleasant Manner in which the Sun pass'd behind the Spire, and was eclipsed by it for some time; for the Image of the Spire and Sun were very large, being made by a Lens of 12 Feet focal Distance. And once as I observed the Occultation of the Sun behind the Spire, just as the Disk disappear'd, I saw several small, bright, round Bodies or Balls running towards the Sun from the dark Part of the Room, even to the Distance of 20 Inches. I observed their Motion was a little irregular, but rectilinear, and seem'd accelerated as they approach'd the Sun. These luminous Globules appear'd also on the other Side of the Spire, and preceded the Sun, running out into the dark Room, sometimes more, sometimes less together, in the same manner as they follow'd the Sun at its Occultation. They appear'd to be in general about $\frac{1}{20}$ of an Inch in Diameter, and therefore must be very large luminous Globes in some Part of the Heavens, whose Light was extinguish'd by that of the Sun, so that they appear'd not in open Daylight; but whether of the Meteor-Kind, or what Sort of Bodies they might be, I could not conjecture.

9. The SOLAR MICROSCOPE (said to be the Invention of a *German*, from whom at least it had its Name) is a most curious Improvement in *Optics*, and deserves to be greatly valued; as it is the best Method which Nature will admit of, or Art can furnish, for magnifying and exhibiting very small transparent Objects to the View of Spectators.

10. To this End it has been contrived very commodiously in several different Forms, two of which I shall here illustrate by Diagrams. The first is as follows: AB is a Section of the Window-Shutter of a dark Room, CD of the Frame containing a Scioptric Ball EF; in the Fore-part whereof is screw'd the Tube GIKH, at one End of which is a Lens GH, which by converging the Sun-Beams into a narrow Compass does strongly enlighten the small Object *ab* placed upon a Slip of Glass or otherwise in the Part of the Tube NQ, where a Slit is made on each Side for that Purpose. Within this Tube there slides another L*mn*M, which contains a small magnifying Lens *mn*. Plate LI. Fig. 3.

11. By moving the exterior Tube IGHK one way and the other, the Glass GH will be brought to receive the Rays of the Sun directly, and will therefore most intensely illuminate the Object *ab*. The other Tube LM being slid backwards and forwards will adjust the Distance of the small Lens *mn*, so that the Image of the Object *ab* shall be made very distinct

distinct on the opposite Side of the Room at OP; and the Magnitude of the Image will be to that of the Object as its Distance from the Lens *mn* is to the Distance of the Object from it, as has been shewn in *Annot.* CXXV.

12. Thus for Example: Suppose the focal Distance of the Lens *mn* to be 1 Inch $= r$, and let the Distance at which it is placed from the Object be $1,1 = d$; then if the Lens be double, and equally convex, (as usual) the Distance of the Image will be $\frac{dr}{d-r} = f = 110$; therefore the Image will be 110 times larger than the Object in its linear Dimensions, and $110 \times 110 = 12100$ times larger in Surface, and in Solidity it will be $110 \times 110 \times 110 = 1331000$ times larger than the Object.

13. If the Lens, instead of 1 Inch, were but $\frac{1}{2}$ an Inch focal Distance, then would the Diameter of the Image be twice as large, or 220 times larger than the Object; and the Superficies 4 times larger, *viz.* $4 \times 12100 = 48400$; and the Solidity 8 times larger, *viz.* $8 \times 1331000 = 10648000$, that is, above 10 Millions of times larger than the Object.

14. Once more; for very small Objects we may use a Lens $\frac{1}{4}$ of an Inch focal Distance, and then the Image at the same Distance as before will be in Diameter $4 \times 110 = 440$ times larger than the Object; in Superficies, $16 \times 12100 = 193600$ times larger; and in Solidity, $64 \times 1331000 = 85184000$ times larger; that is, any solid small Object will at the Distance of 9 Feet 2 Inches, by means of a Lens $\frac{1}{4}$ Inch focal Distance, be magnified above 85 Millions of times.

15. Or more directly thus: Let the focal Distance of the Double-Convex *mn* be $\frac{1}{4} = r$, and let the Distance at which the Image is form'd be 12 Feet or 144 Inches $= f$; then $\frac{rf}{f-r} = d = 0,2504$, which therefore may be taken for $\frac{1}{4}$ of an Inch; consequently the Distance of the Image is 576 times the Distance of the Object from the Lens, and so much larger will it be in Diameter, and in Surface it will be $576 \times 576 = 331776$ times larger, and in Solidity it will be $576 \times 576 \times 576 = 191102976$ times larger: Or, a small Blood-Globule, or other solid Particle, will be magnified above 191 Millions of times; an Effect prodigious, and incredible to those who are not conversant with Glasses, or understand not the Rules of *Optics*.

16. If the linear Dimensions of the Image be nicely taken by a By-stander with a graduated Scale of equal Parts, the Dimension of the Object will be known of course from the Distances of the Image and Object from the Lens; and in exceeding

ceeding small Objects, such as the Pores of Cork, the Particles of Blood, *Animalcula in Semine*, &c. there is no other Way of measuring them so well: And thus the *Solar Microscope* becomes a *Micrometer* in the last Degree of possible Mensuration.

17. The Form of this Instrument, as it has been hitherto described, is that which I have contrived for my own Use, and for theirs who regard more the general Convenience than the Grandeur of an *Apparatus*. However, that those of a different Taste may be gratified, the common Form is to be very much commended for their Use; of which it will be sufficient to give a bare Description, illustrated by a Print.

18. This Instrument consists of several Parts, *viz.* A, a square Frame of Mahogany to be fix'd to the Shutter of a Window by means of the Screws 1, 1. To this Frame is applied a circular Collar B of the same Wood, with a Groove on its Periphery on the Outside, denoted by 2, 3. This Collar is connected by a Cat-Gut to the Pulley 4 on the upper Part, which is turn'd round by the Pin 5 within. On one Part of the Collar, on the Outside, is fasten'd by Hinges a Looking-Glass G in a proper Frame, to which is fix'd the jointed Wire 6, 7; by which means, and the Screw H 8, it may be made to stand in an Angle more or less inclined to the Frame. In the Middle of the Collar is fix'd a Tube of Brass C, near two Inches in Diameter; the End of which, on the Outside, has a convex Lens 5 to collect the Sun-Beams thrown on it by the Glass G, and converging them towards a Focus in the other Part, where D is a Tube sliding in and out, to adjust the Object to a due Distance from the Focus. To the End G of another Tube F is screw'd one of *Wilson*'s *Single Pocket Microscopes*, containing the Object to be magnified in a Slider; and by the Tube F, sliding on the small End E of the other Tube D, it is brought to a due focal Distance. Plate LII. Fig. 1.

19. The great Artifice and Conveniency of this Solar Microscope is, that by means of the Glass G the oblique Rays of the Sun are made to go strait along the dark Room parallel to the Floor, instead of falling upon it. Thus let AB denote a Section of the Looking-Glass, and SC the Rays of the Sun impinging upon it at C, by which they are reflected to the Lens D, and from thence converged towards E to illuminate the Object to be magnified; so that the Beam of Light goes from C to E in the Direction parallel to the Floor, instead of falling on it in the Direction SG. By the Pulley 4, 5, the Glass is turn'd directly to the Sun, and by the jointed Wire and Screw at H it is elevated or depress'd, so as Fig. 2.

to

to bring the Glass into the Position AB required, where the Angle of Incidence ACS is equal to the Angle of Reflection BCE. Mr. *Liberklum*, a *Prussian* Gentleman, was the first who invented this Method of magnifying Objects, but without the Looking-Glass, which was afterwards added to it. The Theory of this Contrivance and the *Magic Lanthorn* is the same; only here we make use of Sun-Beams instead of Candle-Light, and the Object and magnifying Lens of the smallest Size.

20. Another most egregious Contrivance of this Sort we have from the late learned Dr. *s'Gravesande*, which he calls by the Name of HELIOSTATA, from its Property of *fixing* (as it were) *the Sun-Beam* in one Position, *viz.* in an *horizontal Direction* across the dark Chamber all the while it is in Use. It is an *Automaton*, or Piece of Clock-work, whose

Pl. LIII. Parts are as follow. AA is a Frame in which a metalline Speculum S is suspended, moveable about its Axis by means of two small Screws at *aa*. This Frame is fix'd to the Piece C, which being hollow is moveable upon the cylindric Shaft P about the Iron Pin *e*. (See the Part by itself.) This Pillar P is fix'd to a triangular Base or Foot set perpendicular by the three Screws B, B, B.

21. On the Back-part of the Speculum is fix'd a long cylindric Wire or Tail D, in a perpendicular Position. By this it is connected to the second Part of the *Heliostata*, which is a common Thirty-Hour Clock, represented at H; the Plane of which Clock is parallel to that of the Equator in any given Place. This Clock is sustain'd on the Column FG, in which it is moveable up and down by a thin *Lamina* or Plate that enters it as a Case, and fix'd to a proper Height by the Screws *d*, *d*, at the Side. The Whole is truly adjusted to a perpendicular Situation by means of the three Screws I, I, I, in the Tripod LLM, and the Plummet Q, whose *Cuspis* must answer to the Point *o* beneath.

22. The Axis of the Wheel, which moves the Index NO over the Hour-Circle, is somewhat large, and perforated with a cylindric Cavity verging a little to a conical Figure; and receives the Shank *pq* of the said Index NO very close and tight, that by its Motion the Index may be carried round. In the Extremity O of the Index is a small cylindric Piece *n*, with a cylindric Perforation to receive the Tail *t* of the Fork T, yet so as to admit a free Motion therein. In each Side of the Fork are several Holes exactly opposite to each other, in which go the Screws *r*, *r*, upon whose smooth cylindric Ends moves the tubular Piece R on its Auricles *m*, *m*.

23. When the Machine is to be fix'd for Use, another Part

is made use of to adjust it; which is call'd the *Positor*, and is denoted by the Letters VXYZ. The Cylinder C is removed with the Speculum from the Foot P, and the Brass Column VX put on in its stead, and adheres more strictly to the Pin *e*, that it may keep its Position while the Machine is constituted.

24. On the Top of the Column, about X as a Center, moves the Lever YZ, so that it may be any how inclined to the Horizon, and keep its Position. The Arm YX may be of any Length at Pleasure, but the Arm YZ is of a peculiar Construction, and of a determinate Length. To this Arm, which extends no farther than *y*, is adapted a Sliding-Piece Z*x* sharp-pointed at Z. By this the Arm XZ is determined to a given Length, the Piece Z*x* being fix'd by the Screws *zz*.

25. Upon this Arm is drawn the short Line *vx*, by which it may be lengthen'd in the Whole, and is $\frac{8}{100}$ of the whole Length XZ when shortest. The Reason is, this Arm is always to increase and decrease in Proportion to the Secant of the Sun's Declination to the Radius XZ when shortest; but the Radius is to the Secant of 23° 30′ (the Sun's greatest Declination) as 10000000 to 10904411, or as 100 to 109.

26. Now the Reason of this Construction of the Arm XZ is to find for any given Day the Distance of the Center of the Speculum S from the Top *l* of the Style *l*N, which must ever be equal to the Secant of the Sun's Declination; for it must always be equal to the Distance of the Top of the said Style *l* from the Center of the Cylinder R in the Fork T, and that is ever equal to the said Secant of Declination.

27. For since the Style *l*N and the Fork T are in a Position parallel to each other, therefore the middle Hole in the Sides of the Fork being (as they must be) of the same Height above the End of the Index O as is the Height of the Style NT, 'tis evident that on an equinoctial Day the Sun's Rays will pass directly through the Perforation of the Piece R, if it be put in a Position parallel to the Plane of the Ecliptic, or that of the Clock; and also that the Top of the Shadow of the said Style will fall exactly on the said Hole.

28. In this Case the Top of the Style is at the least Distance from the central Point of R, and therefore may be represented by *Radius*, while in any other Position above or below, the Distance will increase in Proportion to the Secant of the Angle which the Rays make with this first or middle Ray, that pass by the Top of the Style, and through the Hole R.

29. Now it may be demonstrated, that on any Day of the Year,

Year, if the Clock and its Pedestal be so fix'd that the Line of XII be exactly in the Meridian, and that the Position of R in the Fork be such that the Sun's Rays go directly through it, and the Shadow of the Style's Top fall just upon the Hole; moreover if the Distance of the Center of the Speculum S from the Top of the Style *l* be made equal (by the *Positor*) to the Distance of the central Point of R therefrom; and lastly, the Tail of the Speculum DE passing through R; if then the Clock be put into Motion, the Index NO shall carry about the Tail of the Speculum in such a Manner, that at all Times of that Day when the Sun can come upon the Speculum it will reflect the Rays constantly in one and the same Position and Direction all the time without Variation.

30. The Machine thus constituted is placed in a Box or Case, and set in a Window with one Side open, exposed to the Sun, and all the other Parts close; so that when the Room is made dark, and the Solar Microscope fix'd to the Fore-part of the Box in which the *Heliostata* is placed, just against the Center of the Speculum to receive the reflected horizontal Beam, all the Experiments of the Darken'd Room are then perform'd as usual. This is a very ingenious Construction of a *Solar-Microscope Apparatus*, and full of Art, but, I fear, too expensive and troublesome for common Use. However, 'tis easy to see that this Machine is capable of being greatly reduced; for it may be made to answer the End very well without a Clock; also the Speculum may be Glass instead of Metal, and all fix'd on one Foot or Pedestal: But this I leave to the Ingenuity of the Mechanical Reader.

LECTURE

LECTURE XI.

Of ASTRONOMY; *and the Use of the* ORRERY *and* COMETARIUM.

Of the UNIVERSE; *an* INFINITY *of* SYSTEMS; *of the* PTOLOMAIC SYSTEM; *the* TYCHONIC SYSTEM; *of the* COPERNICAN *or* SOLAR SYSTEM *of the World. The* Extent *and* Constituent PARTS *thereof.* ARGUMENTS *for the Truth thereof.* DEMONSTRATIONS *of its Truth. Of the* SUN; *the* PRIMARY PLANETS; *the* Secondary Planets, *or* MOONS. *The* COMETS. *Of the* Magnitude, Motion, Maculæ, *&c. of the* SUN. *Of the* Number, Order, Magnitude, Distances, *&c. of the* PLANETS; *their Periods*; *of the* Nodes, Inclination, *and* Aphelia *of their* ORBITS. *Of the* MOON, *its* Phases, Period, Distance, Magnitude, *and* Light. *Of the* SATELLITES *or* Moons *of* Jupiter *and* Saturn. *Of* Saturn*'s* RING. *The* MATHEMATICAL THEORY *of the* CELESTIAL MOTION, *with* CALCULATIONS *and* EXAMPLES. *Of the* ORRERY; *an historical Account of the* Invention *and* Improvements *thereof. A Description of the* ARMILLARY SPHERE. *Of the* MOTION *of the* EARTH *about its* Axis, *and about the* Sun. *The* VICISSITUDES *of the* SEASONS *ex-*

plained. Of the various LENGTHS *of* DAYS *and* NIGHTS. *The* Third Motion *of the* Earth; *the great* PLATONIC YEAR; *the* RECESSION *of the* EQUINOXES *explain'd. A Calculation of the hottest Time of the Day. The* Doctrine *of* Solar *and* Lunar ECLIPSES *fully explain'd, by Calculations on a* Mathematical Theory. *An Explanation of the* ASTRONOMY *of* COMETS. *A* new Method *for* Construction *of their* ORBITS. Calculations *relating to the whole* THEORY *of Comets. An* Analytical Investigation *of their* Elliptic Orbits. *Of their* TAILS, *and all other* Phænomena *accounted for on the* genuine Principles *of* Physics.

I SHALL in this Lecture endeavour to exhibit to you a *just and natural Idea of the Mundane or Solar System*, that is, the System of the World; consisting of the *Sun*; the *Primary Planets*, and their *Secondaries*, or *Moons*; the *Comets*; and the *Fixed Stars*; according to the Hypothesis of *Pythagoras* among the Ancients, and revived by *Copernicus:* Which System is fully proved, and establish'd on the justest Reasoning, and Physical and Geometrical Conclusions, by all our modern Astronomers (CXXXI).

(CXXXI) 1. By the UNIVERSE we are to understand the whole Extent of Space, which, as it is in its own Nature every way infinite, gives us an Idea of the Infinity of the Universe, which can therefore be only in Part comprehended by us: And that Part of the Universe which we can have any Notion of, is that which is the Subject of our Senses; and of this the Eye presents us with an Idea of a vast extended Pro-

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THE most celebrated Hypotheses, or Systems of the World, are three, *viz.* (1.) The *Ptolomean*, invented by *Ptolomy*, an ancient *Egyptian* Philosopher, which assigns such Positions and Motions to the heavenly Bodies, as they appear to

spect, and the Appearance of various Sorts of Bodies disseminated through the same.

2. The infinite Abyss of Space, which the *Greeks* call'd the τὸ πᾶν, the *Latins* the *Inane*, and we the *Universe*, does undoubtedly comprehend an Infinity of Systems of moving Bodies round one very large central one, which the *Romans* call'd *Sol*, and we the *Sun*. This Collection of Bodies is therefore properly call'd the SOLAR SYSTEM, and sometimes the MUNDANE SYSTEM, from the *Latin* Word *Mundus*, the *World*.

3. That the Universe contains as many Solar Systems or Worlds as there are what we call *Fix'd Stars*, seems reasonable to infer from hence, that our Sun removed to the Distance of a Star would appear just as a Star does, and all the Bodies moving about it would disappear entirely. Now the Reason why they disappear is because they are opake Bodies, and too small to be seen at so great a Distance, without an intense Degree of Light; whereas theirs is the weakest that can be, as being first borrow'd and then reflected to the Eye.

4. But the Sun, by reason of his immense Bulk and innate Light, which is the strongest possible, will be visible at an immense Distance; but the greater the Distance, the less bright it will appear, and of a lesser Magnitude: And therefore every Star of every Magnitude may probably be a Sun like our own, informing a System of Planets or moving Bodies, each of which may be inhabited like our Earth with various Kinds of Animals, and stored with vegetable and other Substances.

5. In this View of the Universe, an august Idea arises in the Mind, and worthy of the Infinite and Wise Author of Nature, who can never be supposed to have created so many glorious Orbs to answer so trifling a Purpose as the twinkling to Mortals by Night now and then; besides that the far greatest Part of the Stars are never seen by us at all, as will be farther shewn when we come to treat of those celestial Bodies.

6. When therefore *Moses* tells us, that *in the Beginning*

the Senses to have. (2.) The *Tychonic System*, or that of the noble *Danish* Philosopher, *Tycho Brahe*. (3.) The *Pythagorean*, *Copernican*, or *Solar System*, above-mention'd. Of all which in Order (CXXXII).

God created the Heavens and the Earth, it is to be understood in a limited Sense, and to mean only the *Making*, or rather *New-making*, of our Terraqueous Globe; for 'tis expresly said that the Earth in its first State was a *Chaos*, (in *Hebrew* תהו ובהו, *Shapeless and Void*) which probably might be only the Ruins of a pre-existent Globe, inhabited by rational Creatures in the same manner as since its Renovation. And though it be said, *God made two great Lights*, the *Sun* and the *Moon*, it does not follow they had no Existence before that Time, any more than it does that the Stars had not, which he is said to have made also.

7. Now if the Stars had no Existence before the *Mosaic* Creation, then were there no other Systems of Worlds before our own; then must all the Infinity of Space have been one eternal absolute *Inane* or *Empty Space* till that Time, and God who made the Worlds must be supposed to have made them all at once: Which Suppositions are too extravagant and unreasonable, and therefore cannot be the Sense of that Passage of Scripture; which I think can be no more than this, that when God had form'd the Earth into an habitable Globe, he gave it such a Position and Motion about the Sun, and about its own Axis, as should cause an agreeable Variety in the Length of Days and Nights, and in the Temperature of the Seasons of the Year: All which will be shewn to have their Existence and Distinction resulting from these Principles, and no other, in the Sequel of the Notes to this *Lecture*.

(CXXXII) 1. I have thought it expedient to illustrate the Idea of the three remarkable Systems of the World above-mention'd by proper Diagrams; in the First of which you
Pl. LIV. view the Disposition of the Heavenly Bodies according to
Fig. 1. the Hypothesis of *Claudius Ptolomæus*, a famous Mathematician and Astronomer of *Pelusium* in *Egypt*, who lived in the first Part of the second Century after *Christ*.

2. This was first invented and adhered to chiefly because it seem'd to correspond with the sensible Appearances of the Celestial Motions. They took it for granted that the Motions

THE

THE *Ptolomean System* ſuppoſes the Earth immoveably fix'd in the Center, not of the *World* only, but of the *Univerſe*; and that the *Sun*, the *Moon*, the *Planets*, and *Stars*, all moved about it from *Eaſt* to *Weſt* once in twenty-four Hours, in the Order following, *viz.* the *Moon*, *Mercury*, *Venus*, the *Sun*, *Mars*, *Jupiter*, *Saturn*, the *Fix'd Stars*, and, above all, the Figment of their *Primum Mobile*, or the Sphere, which gave Motion to all the reſt. But this was too groſs and ab- Pl. LIV. Fig. 1.

which thoſe Bodies appear'd to have were ſuch as they truly and really perform'd; and not dreaming of any Motion in the Earth, nor being apprized of the Diſtinction of *abſolute*, *relative*, or *apparent Motion*, they could not make a proper Judgment of ſuch Matters, but were under a Neceſſity of being miſled by their very Senſes, for want of proper Aſſiſtance which After-Ages produced.

3. 'Tis eaſy to obſerve they had no Notion of any other Syſtem but our own, nor of any other World but the Earth on which we live. They thought nothing leſs than that all Things were made for the Uſe of Man; that all the Stars were contain'd in one concave Sphere, and therefore at an equal Diſtance from the Earth; and that the *Primum Mobile* was circumſcribed by the *Cœlum Empyreum* of a cubic Form, which they ſuppoſed to be the *Heaven*, or bliſsful Abode of departed Souls.

4. It would ſcarce have been worth while to have ſaid ſo much about ſo abſurd an Hypotheſis, (as this is now well known to be) were it not that there are ſtill numerous Retainers thereto, who endeavour very zealouſly to defend the ſame, and that for two Reaſons principally, *viz.* becauſe the Earth is apparently fixed in the Center of the World, and the Sun and Stars move about it daily; and alſo becauſe the Scripture aſſerts the Stability of the Earth, the Motion of the Sun, &c.

5. Theſe two Arguments merit no particular Anſwer. It is ſufficient, with reſpect to the firſt, to ſay, that we are aſſur'd Things may (yea muſt) appear to be, in many Caſes, what they really are not, yea, to have ſuch Affections and Properties as are abſolutely contrary to what they really poſſeſs. Thus a Perſon

ſurd to be received by any learned Philoſopher, after the Diſcoveries by Obſervations and Inſtruments which acquaint us with divers Phænomena of the heavenly Bodies, altogether inconſiſtent with, and, in ſome Things, exactly contradictory to, ſuch an Hypotheſis; as will be ſhewn by the Arguments adduced to prove the Truth of the *Copernican Syſtem.*

Pl. LIV. Fig. 2. THE *Tychonic Syſtem* ſuppoſed the Earth in the Center of the World, that is, of the Firmament

ſitting in the Cabin of a Ship under Sail, will, by looking out at the Window, ſee an apparent Motion of the Houſes, the Trees, &c. on the Strand the contrary way, but will perceive no Motion at all in the Ship. Alſo a Perſon ſitting in a Wind-Mill, if the Mill be turn'd about, he will ſee an apparent Motion of the upright Poſt the contrary Way, but will not perceive any in the Mill itſelf.

6. All thoſe Caſes are exactly parallel to that of the Earth, (the Reaſon of which has been ſhewn in the former Part of this Work, *Annot.* XX.) and it is as rational to aſſert the Ship and the Mill are really quieſcent, and the other Bodies poſitively in Motion, as it is to inſiſt on the Motion of the Sun, and the Earth's being at Reſt in the Center.

7. As to the Scripture, as it was never intended for an Inſtitution of Aſtronomy or Philoſophy, ſo nothing is to be underſtood as ſtrictly or poſitively aſſerted in relation thereto, but as ſpoken only agreeably to the common Phraſe or vulgar Notion of Things. And thus Sir *Iſaac Newton* himſelf would always ſay, the *Sun riſes*, and the *Sun ſets*; and would have ſaid with *Joſhua*, *Sun ſtand thou ſtill*, &c. though he well knew it was quite contrary in the Nature of the Thing.

8. How ridiculous a Thing does Popery appear to be to all rational Minds, or to thoſe who are at liberty to *think*, by inſiſting on the literal Senſe of Scripture ſo rigidly in the Expreſſion, *This is my Body!* And is it not equally abſurd to maintain that the *Earth ſtands upon Pillars*, only becauſe we read ſo in the Bible? What an aukward Shift are thoſe celebrated Mathematicians Meſſ. *Le Seur* and *Jacquier* obliged to make, in their Commentary on Sir *Iſaac*'s *Principia!* The Editor, forſooth, is here the Commentator on all thoſe Parts

of

of Stars, and alſo of the Orbits of the Sun and Moon; but at the ſame Time it made the Sun the Center of the Planetary Motions, *viz.* of the Orbits of *Mercury*, *Venus*, *Mars*, *Jupiter*, and *Saturn*. Thus the Sun, with all its Planets, was made to revolve about the Earth once a Year, to ſolve the *Phænomena* ariſing from the *annual Motion*; and the Earth about its Axis from Weſt to Eaſt once in 24 Hours, to account for thoſe of the *diurnal Motion*. But this *Hypotheſis* is ſo monſtrouſly

that relate to the Earth's Motion, or *Copernican* Syſtem: And becauſe their Declaration is ſomething very ſingular in its Kind, I ſhall here give it in their own Words.

PP. LE SEUR & JACQUIER Declaratio.

Newtonus in hoc tertio libro Telluris motæ hypotheſin aſſumit. Autoris Propoſitiones aliter explicari non poterant, niſi eâdem quoque factâ hypotheſi. Hinc alienam coacti ſumus gerere perſonam; cæterum latis à ſummis Pontificibus contra Telluris Motum Decretis nos obſequi profitemur.

In *Engliſh* thus:

" *Newton* in this Third Book has aſſumed the Hypotheſis " of the Earth's Motion. The Author's Propoſitions are " not to be explain'd but by making the ſame Hypotheſis " alſo. Hence we are obliged to proceed under a feigned " Character; but in other Reſpects we profeſs ourſelves ob- " ſequious to the Decrees of the Popes made againſt the Mo- " tion of the Earth."

9. By this it appears how well many People underſtand the Truth, who yet dare not to profeſs it. But to conclude this Head: There is no Authority equal to that of Truth; the common Opinion, the literal Expreſſion of Scripture, the Decrees of Popes, and every thing elſe muſt give way to plain and evident Demonſtration; of which we have abundantly ſufficient for eſtabliſhing the true Syſtem of the World againſt all Oppoſition.

10. The TYCHONIC SYSTEM is repreſented in the next Diagram. This had its Original from *Tycho Brahe*, a Nobleman of *Denmark*, who lived in the latter Part of the laſt Century; he built and made his Obſervations at *Uraniburg*, Pl. LIV. Fig. 2.

strously absurd, and contrary to the great Simplicity of Nature, and in some respects even contradictory to Appearances, that it obtain'd but little Credit, and soon gave way to

Plate LV. THE *Copernican System* of the World, which supposes the Sun to possess the central Part; and that about it revolve the *Planets* and *Comets* in different Periods of Time, and at different Distances therefrom, in the Order following, *viz.* (CXXXIII).

(i. e. *Celestial Tower*) in the Island of *Weer* or *Huena*. This Philosopher, though he approved of the *Copernican* System, yet could he not reconcile himself to the Motion of the Earth; and being, on the other hand, convinced the *Ptolemean* Scheme in Part could not be true, he contrived one different from either, which is represented by the next Diagram.

11. In this the Earth has no Motion allowed it, but the Annual and Diurnal Phænomena are solved by the Motion of the Sun about the Earth, as in the *Ptolemaic* Scheme; and those of *Mercury* and *Venus* are solved by this Contrivance, though not in the same Manner, so simply and naturally, as in the *Copernican* System; as is easy to observe in the *Figure*.

12. After this Scheme had been proposed some time, it received a Correction, by allowing the Earth a Motion about its Axis, to account for the Diurnal Phænomena of the Heavens; and so this came to be call'd the *Semi-Tychonic System*. But this was still wide of the Truth, and encumber'd with such Hypotheses as the true Mathematician and genuine Philosopher could never relish. Therefore both these Systems, and all others, at length gave way to the True Solar System, to be more fully described in the following Notes.

(CXXXIII) 1. The SOLAR SYSTEM, as it is now taught, was in some part invented by the Ancients, perhaps by *Pythagoras* himself; for though *Diogenes Laertius* in writing his Life says no more of him than *his asserting the Antipodes of the Earth*, yet *Aristotle* tells us that the Sect of the *Pythagoreans* taught *that the Earth was carried about the Center* (viz. the Sun) *among the Stars*, (*i. e.* the Planets) *and by turning about* (its Axis) *caused Day and Night*. Hence it came to be

I. MER-

I. MERCURY, at the Distance of about 32 Millions of Miles, revolves about the Sun in the Space of 87 Days, 23 Hours, and 16 Minutes.

II. VENUS, at the Distance of 59 Millions of Miles, in 224 Days, 16 Hours, 49 Minutes.

III. THE EARTH, at the Distance of about 82 Millions of Miles, in 365 Days, 6 Hours, 9 Minutes, or *Sydereal* Year.

call'd the PYTHAGOREAN HYPOTHESIS or SYSTEM *of the World*.

2. But some of these, 'tis said, allow'd only one Motion to the Earth, *viz.* the *diurnal*; while others, as *Philolaus*, *Aristarchus* the *Samian*, *Plato* in his advanced Age, also *Seleucus* the Mathematician, and others, maintain'd the Earth had two Motions, the *diurnal* about its Axis, and the *annual* Motion about the Sun. Hence it is also call'd the PHILOLAIC SYSTEM.

3. But the Astronomy of these early Ages died in its Infancy, and was buried in Oblivion for many Ages after; till at length it began to be revived by Cardinal *Cusa*, who wrote in Defence of it, but to no great Purpose, till after him it was espoused by the celebrated *Nicholas Copernicus*, a Canon of *Thorn* in *Polish Prussia*, where he was born *A. D.* 1473. This Gentleman undertook to examine it thoroughly, and explain'd by it the Motions and Phænomena of the Heavenly Bodies so well to the Satisfaction of the Learned, that he was generally follow'd therein by the principal Astronomers of that and the following Age; as *Rheticus*, *Rothmannus*, *Lausbergius*, *Schickardius*, *Kepplerus*, *Galileus*, and numberless others. From this Time it was call'd the COPERNICAN SYSTEM.

4. After this arose divers great Men, as *Gassendus*, *Hevelius*, *Bullialdus*, *Ricciolus*, the two *Cassini's*, Mr. *Hugens*, *Horrox*, Bishop *Ward*, Mr. *Flamsteed*, Dr. *Halley*, Dr. *Gregory*, Dr. *Keil*, and, above all, that superlative Genius Sir *Isaac Newton*; who all of them, with the greatest Pains and Diligence, applied themselves to make Observations, to invent Instruments, and to investigate the Physical Causes of Celestial Phænomena; in which they so happily succeeded, especially the last great Man, that the Nature, Extent, Order, and Constitution of all and every Part of the Solar System,

IV. MARS,

IV. MARS, at the Distance of 123 Millions of Miles, in 686 Days, 23 Hours, 27 Minutes, or 1 Year, 321 Days, 17 Hours, and 18 Minutes.

V. JUPITER, at the Distance of 424 Millions of Miles, in 4332 Days, 12 Hours, 20 Minutes, or almost 12 Years.

VI. SATURN, at the Distance of 777 Millions

both of Planets and Comets, became so well defined, stated, and established, as to admit of no Contest or Scruple, with any Man properly qualified to understand it; and which therefore ought for the future to be call'd the NEWTONIAN SYSTEM *of the World*.

Plate LV. 5. This SYSTEM (no longer now to be call'd an *Hypothesis*) is represented in a Plate by itself, with the Orbits of all the Planets and Comets (hitherto determin'd) and at their proper Distances from the Sun, represented by the central Point; it being impossible to represent, either by an Instrument or Diagram, the true Proportion both of Magnitudes and Distances of the Sun and Planets, as will appear by what follows.

6. For it must be allowed, that to render any Machine or Delineation useful, the least Part ought to be visible; and one cannot well assign a less Bulk for the Globe of the Moon, than what is here represented in this Plate; which being fix'd upon, the Magnitudes of the Planets *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, and *Saturn* and its Ring, must be such as are shewn under the respective Names in the Plate; and with respect to these the Sun's Bulk or Face will be represented by the exterior Circle of the Diagram, which here represents the Ecliptic in the Heavens, and is nearly 9 Inches in Diameter.

7. Now the Diameter of the Earth in this Scheme is $\frac{1}{10}$ of an Inch, its Semidiameter is therefore $\frac{1}{20}$; and the Distance of the Earth from the Sun's Center is about 20000 Semidiameters. But $20000 \times \frac{1}{20} = 1000$ Inches $= 83\frac{1}{3}$ Feet; and since the Distance of *Saturn* is near ten times as great, it is evident *the Extent or Diameter of a Machine to exhibit the several Parts of the Solar System in their due Proportion of Distances and Magnitudes (though no bigger than those here assign'd) will be at least* 1600 *Feet, or more than a Quarter of a Mile: And consequently the Circumference of* Saturn's *Orbit will measure very near a Mile.*

of

of Miles, in 10759 Days, 6 Hours, 36 Minutes, or nearly 30 Years.

VII. THE COMETS, in various and vastly eccentric Orbits, revolve about the Sun in different Situations and Periods of Time, as represented in the Scheme of Mr. *Whiston*'s Solar System (CXXXIV).

8. In a much less Compass indeed the Distances might be represented very well in Proportion, but the respective Magnitudes can no otherwise be shewn than by such Globes or graphical Delineations as is the Plate of the Diagram under Consideration. Another Thing which cannot be properly represented in such a Plate is the Inclination of any Planetary Orbit to the Plane of the Ecliptic, especially the Orbits of the Comets, of whose Positions we can by no means this way get any Idea. The several Parts therefore of the *Solar System* must be explain'd and illustrated by distinct Theories, with proper Figures adapted to each: And this will be the Subject of the following Notes.

(CXXXIV) 1. The Periodical Times of the primary Planets Sir *Isaac Newton* has stated in Days and Decimal Parts of a Day, as follows:

☿	♀	⊕	♂	♃
87,9692.	224,6176.	365,2565.	686,9785.	4332,514.

♄
10759,275.

2. The mean Distances of the Planets from the Sun are thus stated by Sir *Isaac*:

☿	♀	⊕	♂	♃	♄
According to *Kepler*,					
38806.	72400.	100000.	152350.	519650.	951000.
According to *Bullialdus*,					
38585.	72398.	100000.	152350.	522520.	954198.
According to the Periodical Times,					
38710.	72333.	100000.	152369.	520096.	954006.

3. Before we can shew how the Periodical Times and Distances of the Planets are found, it will be necessary to premise the following Things, viz. *The Orbit of a Planet is not in the Plane of the Ecliptic.* Thus let ANLO be the Orbit of Pl. LVI. Fig. 1.

of a Planet P, and let BCET be the Earth's Orbit, which is in the Plane of the Ecliptic; then will one Half of the Planet's Orbit lie above the Plane, as NLO, and the other Half NAO below it.

4. *The two Planes, therefore, of the Planet's Orbit and of the Ecliptic will intersect one another*, which Intersection will be a Right Line, as NO; and this is call'd the *Line of the Nodes*, the *Nodes* being the two Points N and O, in which the Planet descends below, or ascends above, the Plane of the Ecliptic: Whence O is call'd the *Ascending Node*, and N the *Descending Node*.

5. Let the Curve N*m*O be described in the Plane of the Ecliptic perpendicularly under the Half-Orbit NLO; then is the Curve N*m*O said to be the *Projection of the Planet's Orbit* NLO *on the Plane of the Ecliptic*, and *p* the *projected Place* of the Planet P, or its Place *reduced to the Ecliptic*.

6. The Angle LO*m* measures the *Inclination of the Plane of the Planet's Orbit to that of the Ecliptic*; which is also call'd the *Obliquity* thereof. The perpendicular Distance P*p* is the *Latitude of the Planet* from the Plane of the Ecliptic; and E*m* is the greatest Latitude, if LO or LN be a Quarter of a Circle. Also the Distance of the Planet from the Node, *viz.* PO, is call'd the *Argument of Latitude*.

7. Draw SP, S*p*, and TP, T*p*, and join ST; then is the Angle PS*p* the true Latitude seen from the Sun at S, and therefore call'd the *Heliocentric Latitude*; and the Angle PT*p* is the apparent Latitude, or that which is seen from the Earth at T, and is therefore call'd the *Geocentric Latitude*.

8. The true Distance of the Planet from the Sun and Earth is measured by the Lines SP and PT; but S*p* and T*p* are call'd the *Curtate Distances*. Also in the Triangle S*p*T, the Angle ST*p* is call'd the *Angle of Elongation*, or Distance of the Planet from the Sun. The Angle S*p*T is call'd the *Parallactic Angle*, as being that under which the Semidiameter of the Earth's Orbit is seen; and the Angle *p*ST at the Sun is usually call'd the *Angle of Commutation*.

9. We may now proceed to shew the Methods of determining the *Periodical Time* of a Planet; which may be done either by the Conjunctions or Oppositions of the Planet to the Sun. Thus, for Example, observe well the Place of *Jupiter* in the Ecliptic at his Opposition to the Sun, and also when he comes to be in Opposition to the Sun again; and note well the Time that lapsed between. Then say, *As the Arch described between the two Oppositions is to the whole Circumference, so is the Time in which that Arch was described to the Periodical Time*, very nearly; for it will not be exactly so,

because

becauſe the Motion of a Planet is not quite uniform, as moving in an Ellipſis, and not in a Circle. In the ſame manner you proceed for an inferior Planet.

10. But a more accurate Method is by obſerving nicely the Time that elapſes between the Planet's being twice ſucceſſively in the ſame Node, (which may be eaſily known, becauſe in that Part of its Orbit the Planet has no Latitude) and that will be the Periodical Time of the Planet; for in one Revolution of a Planet, the Nodes (if they move at all) will not move ſenſibly, and may therefore be eſteem'd as quieſcent.

11. In order to eſtimate the Diſtances of the Planets, we proceed for *Venus* and *Mercury* in the following Manner. Let the Place of the Planet in its greateſt Elongation from the Sun be duly obſerved, the Difference between that and the Sun's Place (as ſeen from the Earth) will be the Quantity of the greateſt Elongation, or of the Angle ATS, with reſpect to the Planet *Venus* in her Orbit at A. And ſince the Orbit of *Venus* is nearly circular, the Line TA will touch the Orbit in the Point A, and ſo the Angle TAS will be a Right one. Suppoſe the Angle ATS = 47 Degrees by Obſervation; then if we put the Diſtance of the Earth ST = 100000, ſay, As Radius or Sine of 90° is to the Sine of 47°, ſo is TS = 100000 to SA = 73000, nearly the Diſtance of *Venus* from the Sun.

12. In like manner may the Diſtance of *Mercury* from the Sun be determined in the Groſs, but not ſo nearly as that of *Venus*, becauſe the Orbit is much more excentric or elliptical, and therefore the Angle TRS will not be a Right one. Its Quantity therefore muſt be found from the Theory of the Motions of *Mercury* founded on Obſervations; and from thence the third Angle TSR will be known, and conſequently the Side SR, which is the Diſtance of *Mercury* from the Sun.

13. In the Superior Planets this Matter is not quite ſo eaſy; however, there are divers Methods by which it may be done, by having the Theory of the Earth known, which gives the Side ST; and by Obſervation the Angle STP is known, which is the Difference of the Geocentric Place of the Sun and Planet; then there remains only the Angle SPT to be found, which Aſtronomers ſhew how to do ſeveral Ways; one of which is peculiar to *Jupiter*, being done by means of one of his *Satellites*, as will be ſhewn when we treat of them.

14. As I have in this Note mention'd the Inclination of the Planets Orbits to the Plane of the Ecliptic, I ſhall give the Quantity thereof for each Planet as follows:

The

		°	′	″
The Inclination of the Orbit of	*Mercury* is	6	59	20
	Venus — —	3	23	5
	Mars — —	1	52	0
	Jupiter —	1	20	0
	Saturn — —	2	33	30

15. Also the Line of the Nodes in the several Planetary Orbits is determined; and the Place in the Ecliptic of the Ascending Node for each Planet is as follows:

			°	′	″
For	*Mercury* in	♉	14	42	00
	Venus —	♊	14	25	54
	Mars —	♉	18	29	54
	Jupiter —	♋	7	19	54
	Saturn —	♋	21	49	54

16. The Distances of the Planets from the Sun as above determined are reducible to *English* Miles, by first finding the Earth's Distance in that Measure; and this is done by finding the Quantity of the Sun's Parallax, that is, of the Angle under which the Earth's Semidiameter appears at the Sun. Thus let S be the Center of the Sun, and C the Center of the Earth DEF in her Orbit AB; the Angle DSC is that which we speak of, as being that under which the Semidiameter CD of the Earth appears at the Sun. (Pl. LVI. Fig. 2.)

17. To find this Angle Astronomers have attempted Variety of Methods, but have as yet found none that will determine it exactly; however, by many repeated Observations of Dr. *Halley* it is found to be not greater than 12″, nor less than 9″. Wherefore $10\frac{1}{2}''$ (the Mean) has been fix'd upon as near the Truth, which we must be contented with till *May* 26, 1761, when *Venus* will transit the Sun's Disk, by which means the same Gentleman has shewn the Sun's Parallax may be determined to a great Nicety, *viz.* to within a 500dth Part of the Whole. See *Phil. Trans.* N° 348, abridged by *Jones*, Vol. IV.

18. Supposing therefore the Angle DSC $= 10'' \, 30'''$, and the Side DC $= 1$; then say,

As the Tangent of	DSC $10'' \, 30''' =$	5,706764
Is to Unity	DC $= 1 =$	0,000000
So is Radius	$90^\circ =$	10,000000
To the Side	SC $= 19657{,}8 =$	4,293236

Then 19657,8 Semidiameters of the Earth multiplied by 4000 gives 78631200 *English* Miles for the Distance of the Sun.

19. Not the Distances only, but also the Diameters of the Planets

Planets are to be investigated, by measuring their apparent Diameters with a Micrometer adapted to a good Telescope. Thus the Sun in his mean Distance will be found to subtend an Angle of $32' \, 12'' = 1932''$, and the Earth at the Sun subtends an Angle of $21''$ (being double the Angle DSC). Therefore the Sun's Diameter is to the Earth's Diameter as 1932 to 21, that is, as 10000 to 109.

20. Again: Mr. *Pound* (with the *Hugenian* Telescope of 123 Feet) found *Saturn* subtended an Angle of $16''$. Therefore if *Saturn* were brought to the mean Distance of the Earth from the Sun, his apparent Diameter would be increased in the Ratio of $\frac{954006}{100000}$ to 1; that is, its Diameter would be seen under an Angle equal to $\frac{954006}{100000} \times 16'' = 152,''64096$. Whence the Sun's Diameter is to *Saturn*'s as $1932'' : 152,''64096 :: 10000 : 790$.

21. The same Gentleman measured *Jupiter*'s apparent Diameter, and found it subtend an Angle of $37''$; wherefore *Jupiter* at the Distance of the Earth would subtend an Angle equal to $\frac{520096}{100000} \times 37'' = 192,''417$. Hence the Sun's real Diameter is to that of *Jupiter* as $1932'' : 192,''417 :: 10000 : 996$.

22. *Hugenius* measured the Diameter of *Mars* when nearest the Earth, and found it did not exceed $30''$; and that the Distance of *Mars* from the Earth was then to the Sun's mean Distance as 15 to 41. (See his *Systema Saturnium*.) Therefore *Mars* removed to the Distance of the Sun would subtend an Angle equal to $\frac{15}{41} \times 30'' = 10,''9756$. Whence the Diameter of the Sun is to that of *Mars* as $1932''$ to $10,''9756 :: 10000 : 57$.

23. Dr. *Halley* collected from the Appearance of *Venus* in the Sun's Disk, *May* 26, 1761, that *Venus* seen from the Sun at her mean Distance would appear under an Angle of $30''$; consequently, at the Sun's mean Distance she would appear under an Angle equal to $\frac{72333}{100000} \times 30'' = 21,''6999$. Therefore the Sun's real Diameter is to that of *Venus* as $1932'' : 21,''6992 :: 10000 : 112$.

24. The same learned Gentleman by the like means finds *Mercury* at his mean Distance subtend an Angle of $20''$, and therefore at the Sun an Angle of $\frac{38710}{100000} \times 20'' = 7,''742$.

Where-

Wherefore the Diameters of the Sun and *Mercury* are as $1932'' : 7,''742 :: 10000 : 40$.

25. There are other Phænomena of the Planets to be observed, from whence several important Discoveries have been made in the Physical Part of Astronomy. Thus the Sun and some Planets, when view'd with a good Telescope, appear to have dark Spots on their Surface; by these Spots those Bodies are found to have a Motion about their Axis, and the Position of their Axis with respect to the Plane of the Ecliptic is by this means determined.

26. These Spots are most numerous and easily observed in the Sun. It is not uncommon to see them in various Forms, Magnitudes, and Numbers, moving over the Sun's Disk. They were first of all discover'd by the lyncean Astronomer *Galileo*, in the Year 1610, soon after he had finish'd his new-invented Telescope.

27. That these Spots adhere to or float upon the Surface of the Sun, is evident for many Reasons. (1.) For many of them are observed to break out near the Middle of the Sun's Disk; others to decay and vanish there, or at some Distance from his Limb. (2.) Their apparent Velocities are always greatest over the Middle of the Disk, and gradually slower from thence on each Side towards the Limb. (3.) The Shape of the Spots varies according to their Position on the several Parts of the Disk; those which are round and broad in the Middle grow oblong and slender as they approach the Limb, according as they ought to appear by the Rules of *Optics*.

28. By comparing many Observations of the Intervals of Time in which the Spots made their Revolutions, by *Galileo*, *Cassini*, *Scheiner*, *Hevelius*, Dr. *Halley*, Dr. *Derham*, and others, it is found that 27 Days, 12 Hours, 20 Minutes is the Measure of one of them at a Mean: But in this Time the Earth describes the angular Motion of $26^\circ\ 22'$ about the Sun's Center; therefore say, As $360^\circ + 26^\circ\ 22'$ is to 360°, so is 27 d. 12 h. 20′ to 25 d. 15 h. 16′; which therefore is the Time of the Sun's Revolution about its Axis.

29. Had the Spots moved over the Sun in right lined Directions, it would have shewn the Sun's Axis to have been perpendicular to the Plane of the Ecliptic; but since they move in a curvilinear Path, it proves his Axis inclined to the Axis of the Ecliptic; and it is found by Observation, that
Pl. LVI. that Angle is equal to $7^\circ\ 30'$; that is, if BD passing through
Fig. 3. the Center of the Sun C be perpendicular to the Plane of the Earth's Equator HI, then will the Axis of the Sun's Motion AE contain with that Perpendicular the Angle ACB $= 7^\circ$

30′

$30' =$ GCI, the Angle which the Equator of the Sun GF makes with the Plane of the Ecliptic.

30. And the Points in which a Plane passing through the Perpendicular BD and Axis AE cuts the Ecliptic are in the 8th Degree of *Pisces* on the Side next the Sun's North Pole A, and consequently in the 8th Degree of *Virgo* on the other Side next the South Pole E. *Scheiner* had determined the Angle BCA to be 7 Degrees, and *Cassini* made it 8 by his Observations; which is the Reason why $7^{\circ}\ 30'$ is chosen for a Mean.

31. As to the Magnitude of the Spots, it is very considerable; as will appear if we observe that some of them are so large as to be plainly visible to the naked Eye. Thus *Galileo* saw one in the Year 1612, and I know two Gentlemen who have thus view'd them within 20 Years past: These Spots must therefore subtend at least an Angle of 1 Minute. Now the Diameter of the Earth, if removed to the Sun, would subtend an Angle of but $20''$; hence the Diameter of a Spot just visible to the naked Eye is to the Diameter of the Earth as 60 to 20, or as 3 to 1; and therefore the Surface of the Spot, if circular, to a Great Circle of the Earth as 9 to 1. But 4 Great Circles are equal to the Earth's Superficies; whence the Surface of the Spot is to the Surface of the Earth as 9 to 4, or as $2\frac{1}{4}$ to 1.

32. *Gassendus* says he saw a Spot whose Diameter was equal to $\frac{1}{20}$ of that of the Sun, and therefore subtended an Angle at the Eye of $1'\ 30''$; its Surface was therefore above 5 times larger than the Surface of the whole Earth. What those Spots are, I believe no body can tell; but they seem to be rather thin Surfaces than solid Bodies, because they lose the Appearance of Solidity in going off the Disk of the Sun. They resemble something of the Nature of Scum or Scoria swimming on the Surface, which are generated and dissolved by Causes little known to us.

33. But whatever the Solar Spots may be, 'tis certain they are produced from Causes very inconstant and irregular: For *Scheiner* in his *Rosa Ursina*, which contains near 2000 Observations upon these Spots, says he frequently saw 50 at once, but for 20 Years after (*viz.* between the Years 1650 and 1670) scarce any appear'd. And in this Century the Spots were frequent and numerous till the Year 1741, when for three Years successively very few appeared. I saw but one in all that Time; and now since the Year 1744 they have again appeared as usual.

34. These *Maculæ* or dark Spots are not peculiar to the Sun; they have been observed also in the Planets. Thus *Venus* was observed to have several by Signior *Blanchini*, the Pope's Domestic Prelate, in the Year 1726; by which he determined her Revolution about her Axis to be perform'd in 24 Days and 8 Hours; and that her Axis is inclined to the Plane of the Ecliptic in an Angle of 15 Degrees; and lastly, that the North Pole of this Planet faces the 20th Degree of *Aquarius*.

35. As in *Venus*, so in *Mars*, both dark and bright Spots have been observed by *Galileo* first, and afterwards by Signior *Cassini*, Dr. *Hook*, *Miraldi*, Mr. *Roemer*, and others. By these Spots the diurnal Revolution of *Mars* about its Axis is determined to be 24 Hours and 40 Minutes; and that the Axis is nearly perpendicular to the Plane of its Orbit.

36. There seems to be good Reason to conclude *Mars* is encompassed with a large Atmosphere; for *Cassini* observed a Fix'd Star, at the Distance of 6 Minutes from the Disk of *Mars*, became so faint before its Occultation, that it could not be seen with the naked Eye, nor with a Telescope of 3 Feet; though Stars of that Magnitude are plainly visible even in Contact with the Moon, which for that Reason seems to have no Atmosphere.

37. *Jupiter* has had his Spots observable ever since the Invention and Use of large Telescopes; and from repeated Observations they shew *Jupiter*'s Revolution about its Axis is in 9 Hours and 56 Minutes. Besides these Spots, *Jupiter* has the Appearance of three Zones or Belts encompassing his Body, sometimes more, so that his Disk seems clouded with them: What they are, no body yet can tell. The Axis of this Planet also is nearly perpendicular to the Plane of his Orbit.

38. Considering the large Magnitude of *Jupiter*, and his short diurnal Rotation, the Equatorial Parts of his Surface must have a prodigious Velocity, which of consequence must cause him to be of a spheroidical Figure (as was shewn of the Earth). Accordingly *Cassini* found the Axis of the Equator to be to that of the Poles as 14 to 15; but Mr. *Pound* afterwards more exactly determin'd them to be as 12 to 13, agreeable to Sir *Isaac Newton*'s Computation.

39. *Saturn* by reason of his great Distance on one hand, and *Mercury* by reason of his Smallness and Vicinity to the Sun on the other, have not as yet had any Spots discover'd on their Surfaces; and consequently nothing in relation to their diurnal Motions, and Inclinations of their Axis to the Planes of their Orbits, can be known.

THESE

THESE are all the heavenly Bodies yet known to circulate about the Sun, as the Center of their Motions; and among the Planets, there are three which are found to have their *secondary Planets*, *Satellites*, or *Moons*, revolving constantly about them, as the Centers of their Motions, (CXXXV) *viz.*

(CXXXV) 1. Of the six Primary Planets, we find but *three* that are certainly attended with Moons, *viz.* the *Earth*, *Jupiter*, and *Saturn*; for though Mr. *Short* has given an Account of a Phænomenon that he observed some Years ago, which seems extremely like a Moon about *Venus*, yet as it was never observed before nor since through the best of Telescopes, I can by no means think it was a real Moon: However, that the Reader may use his own Judgment, I refer him to the Account given of it in the *Philosophical Transactions*.

2. The Distance of our Moon from the Earth is determined by her horizontal Parallax, or the Angle which the Semidiameter of the Earth subtends at the Moon, *viz.* the Angle AOC, which is the Difference between the true Place of the Moon's Center O when in the Horizon, and the apparent Place thereof as view'd from the Surface of the Earth at A. The former is known by Astronomical Tables, the latter by Observation: And the Quantity of this Difference or Angle at a Mean is $57' \, 12'' = AOC$. Pl. LVI. Fig. 4.

3. If therefore we say, As the Tangent of $57' \, 12''$ is to Radius, so is $AC = 1$ to $CO = 60{,}1$; this will be the mean Distance of the Moon in Semidiameters of the Earth. Therefore since one Semidiameter of the Earth contains 3982 Miles, we have $3982 \times 60{,}1 = 239318{,}2 = CO$ the mean Distance of the Moon.

4. The Moon's apparent Semidiameter MO measures (at her mean Distance) $15' \, 38'' = 938''$ by the Micrometer, which is the Quantity of the Angle MCO. The Earth's Diameter therefore is to the Moon's as $3432''$ to $938''$, that is, as 109 to 30, or as 3,63 to 1. Wherefore $\frac{30}{109} \times 7964 = 2192$ Miles in the Moon's Diameter.

5. Therefore the Face of the Earth, as it appears to the *Lunarians*, is to the Face of the Moon as it appears to us,

THE EARTH, which has only *one Moon* revolving about it, in 27 Days, 7 Hours, 43 Minutes, at the mean Distance of about 240000 Miles.

as 169 x 109 to 30 x 30, *viz.* as 11881 to 900, or as 13,2 to 1. And the real Bulk of the Earth is to that of the Moon as 109 x 109 x 109 to 30 x 30 x 30, *viz.* as 1295029 to 27000, that is, as 1295 to 27, or as 48 to 1 very nearly.

6. Sir *Isaac Newton* mentions the Atmosphere about the Moon, but other Astronomers think there is Reason (not to say a Demonstration) for the contrary: For were there an Atmosphere of Air like ours, it must necessarily obscure the Fix'd Stars in the Moon's Appulse to them; but it has been observed that this never happens; on the contrary they preserve all their Splendor to the Moment of their Occultation, and then disappear instantaneously, and in the same Manner they recover their Light when they appear again on the other Side. And this I am very certain of from the late remarkable Occultation of *Jupiter*, which I observed with a good reflecting Telescope from the Beginning to the End with all the Attention possible, because I was very desirous to be satisfied about that Matter; and all the Phænomena conspired to convince me, there was nothing like an Atmosphere about the Moon.

7. That the Surface of the Moon is not smooth or even, but diversified with Hills and Vales, Continents and Seas, Lakes, &c. any one would imagine who views her Face through a large Telescope. That she has Variety of Hills and Mountains is demonstrable from the Line which bounds the light and dark Parts not being an even regular Curve, as it would be upon a smooth spherical Surface, but an irregular broken Line, full of Dents and Notches, as represented in the Figure: Also because many small (and some large) bright Spots appear in the dark Portion, standing out at several small Distances from the boundary Line; which Spots in a few Hours become larger, and at last unite with the enlighten'd Portion of the Disk.

8. On the other hand we observe many small Spots interspersed all over the bright Part, some of which have their dark Sides next the Sun, and their opposite Sides very bright and circular, which infallibly proves them to be deep, hollow, round Cavities; of which there are two very remarkable ones near together on the upper Part, and may be view'd exceeding plain when the Moon is about four or five Days old.

JUPITER

JUPITER is observed with a Telescope to have four *Satellites*, which move about him in the Times and Distances following, *viz.*

9. To measure the Height of a Lunar Mountain is a curious Problem, and at the same time very easy to effect in the following Manner. Let C be the Moon's Center, EDB a Ray of the Sun touching the Moon's Surface in D, and the Top of a Mountain in B. Draw CB and CD; the Height of the Mountain AB is to be found. With a Micrometer in a Telescope find what Proportion the Distance of the Top of the Mountain B, from the Circle of Illumination at D, bears to the Diameter of the Moon, that is, the Proportion of the Line DB to DF; and because DF is known in Miles, DB will be also known in that Measure. Pl. LVI. Fig. 5.

10. Now admit that DB : DC :: 1 : 8, as in one of the Hills it will be; then $\overline{DC}^2 + \overline{DB}^2 = 64 + 1 = 65 = \overline{CB}^2$; whence $\sqrt{65} = 8{,}062 = BC$; wherefore $BC - AC = 8{,}062 - 8 = 0{,}062 = AB$, the Height of the Mountain required. Wherefore AC : AB :: 8 : 0,062 :: 8000 : 62. And since the Moon's Semidiameter AC = 1096 Miles, therefore 8000 : 62 :: 1096 : 8,5 nearly. This Mountain then being $8\frac{1}{2}$ Miles high, is near three times higher than the highest Mountain on the Earth.

11. Again, the Cavities are proportionably large and deep. I have observed Cavities in the Moon more than the 100dth Part of the Moon's Diameter in Breadth, which is about 200 Miles upon the Moon's Surface; their Depths appear likewise proportional. The Lunar Cavities therefore prodigiously exceed the Height of the Mountains; and consequently the Surface of the Moon has but little Similitude to the Surface of the Earth in these Respects.

12. Since the Moon's Surface appears to be so very mountainous and irregular, it has been a Question, how it comes to pass that the bright circular Limb of the Disk does not appear jagged and irregular, as well as the Curve bounding the light and dark Parts? In Answer to this, it must be consider'd, that if the Surface of the Moon had but one Row of Mountains placed round the Limb of the Disk, the said bright Limb would then appear irregularly indented; but since the Surface is all over mountainous, and since the visible Limb is to be consider'd not as a single curve Line, but a large Zone, having many Mountains lying one behind another from the Observer's Eye, 'tis evident the Mountains in some Rows be-

THE *First* in 1 Day 18 Hours 27 Minutes, at the Distance of $5\frac{6}{10}$ Semidiameters of *Jupiter*'s

ing opposite to the Vales in others, will fill up the Inequalities in the visible Limb in the remoter Parts, which diminish to the Sight and blend with each other, so as to constitute (like the Waves of the Sea) one uniform and even Horizon.

13. Whether there be Seas, Lakes, &c. in the Moon, has been a Question long debated, but now concluded in the Negative: For in those large darker Regions (which were thought to be Seas) we view through a good Telescope many permanent bright Spots, as also Caverns and empty Pits, whose Shadows fall within them, which can never be seen in Seas or any liquid Substance. Their dark and dusky Colour may proceed from a kind of Matter or Soil which reflects Light less than that of the other Regions.

14. These Spots in the Moon have continued always the same unchangeably since they were first view'd with a Telescope; though less Alterations than what happen in the Earth in every Season of the Year, by Verdure, Snow, Inundations, and the like, would have caused a Change in their Appearance. But indeed, as there are no Seas nor Rivers in the Moon, and no Atmosphere, so of course there can be no Clouds, Rain, Snow, or other Meteors, whence such Changes might be expected.

15. Since (as we have shewn) the mean Distance of the Moon is about 60 Semidiameters of the Earth, at the Distance of the Moon one Degree of the Earth's Surface will subtend an Angle of one Minute, and will therefore be visible; but such a Degree is equal to $69\frac{1}{2}$ Miles, therefore a Spot or Place 70 Miles in Diameter in the Moon will be just visible to the naked Eye.

16. Hence a Telescope that magnifies about 100 times will just discover a Spot whose Diameter is $\frac{1}{100}$ of 70 Miles, or $\frac{7}{10}$ of a Mile, or 3698 Feet: And a Telescope that will magnify 1000 times will shew an Object that is but $\frac{7}{100}$ of a Mile, that is, whose Diameter is but 370 Feet, or little more than 120 Yards; and therefore will easily shew a small Town or Village, or even a Gentleman's Seat, if any such there be.

17. The Time which the Moon takes up in making one Revolution about the Earth, from a Fix'd Star to the same again, is 27 d. 7 h. 43′, which is call'd the *Periodical Month*. But the Time that passes between two Conjunctions, that is, from one New Moon to another, is equal to 29 d. 12 h.

Body

Body from his Center, as measured with a Micrometer.

44′ 3″, which is call'd a *Synodical Month:* For after one Revolution is finish'd, the Moon has a small Arch to describe to get between the Sun and the Earth, because the Sun keeps advancing forwards in the Ecliptic. Now this Surplus of Motion takes up 2 d. 5 h. 1′ 3″, which added to the Periodical Month makes the Synodical, according to the mean Motions.

18. The Moon moves about its own Axis in the same Time that it moves about the Earth, from whence it comes to pass that she always shews the same Face to us: For by this Motion about her Axis just so much of her Surface is turn'd towards us constantly, as by her Motion about the Earth would be turn'd from us.

19. But since this Motion about the Axis is equable and uniform, and that about the Earth (or common Center of Gravity) is unequal and irregular, as being perform'd in an Ellipsis, it must follow, that the same Part of the Moon's Surface precisely can never be shewn constantly to the Earth; and this is confirm'd by the Telescope, through which we often observe a little Gore or Segment on the Eastern and Western Limb appear and disappear by turns, as if her Body librated to and fro; which therefore occasion'd this Phænomenon to be call'd the *Moon's Libration.*

20. The Orbit of the Moon is elliptical, more so than any of the Planets, and is perpetually changing or variable, both in respect of its Figure and Situation; of which we shall treat more largely in another Place. The Inclination of the Moon's Orbit to the Plane of the Ecliptic is also variable, from 5 Degrees to 5° 18′. The Line of Nodes likewise has a variable Motion from East to West, contrary to the Order of the Signs, and compleats an entire Revolution in a Space of Time a little less than 19 Years. Also the Line of the *Apsides*, or of the *Apogee* and *Perigee*, has a direct Motion from West to East, and finishes a Revolution in the Space of about 9 Years. All which will be more copiously treated of when we come to explain the Physical Causes thereof.

21. The Phases of the Moon in every Part of the Orbit are easily accounted for from her different Situation with respect to the Earth and Sun: For though to an Eye placed in the Sun she will always exhibit a compleat illuminated Hemisphere, yet in respect to the Earth, where that Hemisphere is view'd in all Degrees of Obliquity, it will appear in every Pl. LVI. Fig. 6.

The *Second* in 3 Days, 13 Hours, 13 Minutes, at the Distance of 9 Semidiameters.

The *Third* in 7 Days, 3 Hours, 42 Minutes

Degree from the greatest to the least; so that at E no Part at all of the enlighten'd Surface can be seen. At F a little Part of it is turn'd towards the Earth, and from its Figure it is then said to be *horned*. At G one Half of the enlighten'd Surface is turned to the Earth, and she is then said to be *dichotomised*, and in her first Quarter or *Quadrature*. At H a Part more than half is turned to the Earth, and then she is said to be *gibbous*. At A her whole illumined Hemisphere is seen, being then in *Opposition* to the Sun; and this is call'd the *Full Moon*. At B she is again *gibbous*, but on the other Part; at C she is again *dichotomised*, and in her last Quarter; at D she is *horned*, as before; and then becomes *new* again at E, where she is in *Conjunction* with the Sun.

22. If MN be drawn perpendicular to the Line SL joining the Centers of the Sun and Moon, and OP perpendicular to the Line TL joining the Centers of the Earth and Moon, 'tis evident the Angle OLM in the first Half of the Orbit, and PLN in the second, will be proportional to the Quantity of the illuminated Disk turn'd towards the Earth; and this Angle is every where equal to the Angle ETL, which is call'd the *Elongation of the Moon* from the Sun.

23. To find what Quantity of the Moon's visible Surface is illustrated for any given Time, we are to consider that the Circle of Illumination BFC is oblique to the View every where (but at G and A), and therefore by the Laws of the Orthographic Projection (which see in my *Elements of all Geometry*) it will be projected into an Ellipse whose longest Axis is the Diameter of the Moon BC, and the Semi-conjugate is FL = Cosine of the Angle of Elongation FBP. Hence FP = Versed Sine of the said Angle. But from the Nature of the Circle and Ellipse we have LP in a constant Ratio to EP, wherever the Line PO is drawn perpendicular to B; therefore also 2LP = PO has a constant Ratio to FP. *But* (by *Euclid* V. 12.) *the Sum of all the Lines* OP = *Area of the Circle is to the Sum of all the Lines* FP = *Area of the illuminated Part, as the Diameter of the Circle* OP *to the Versed Sine of the Elongation* FP.

Pl. LVI. Fig. 7.

24. As the Moon illuminates the Earth by a reflex Light, so does the Earth the Moon; but the other Phænomena will be different for the most part, I shall recount them for the

at

at the Distance of 14 $\frac{3}{10}$ Semidiameters.

The *Fourth* in 16 Days, 16 Hours, 32 Minutes, at the Distance of 25 $\frac{3}{10}$ Semidiameters (CXXXVI).

Reader's Curiosity as follows. (1.) The Earth will appear but to little more than one Half of the Lunar Inhabitants. (2.) To those to whom the Earth is visible, it appears fix'd, or at least to have no circular Motion, but only that which results from the Moon's *Libration*. (3.) Those who live in the Middle of the Moon's visible Hemisphere see the Earth directly over their Heads. (4.) To those who live in the Extremity of that Hemisphere the Earth seems always nearly in the Horizon, but not exactly there, by reason of the *Libration*. (5.) The Earth in the Course of a Month would have all the same Phases as the Moon has. Thus the *Lunarians* when the Moon is at E, in the Middle of their Night, see the Earth at *Full*, or shining with a full Face; at G it is *dichotomised*, or half light and half dark; at A it is wholly dark, or *New*; and at the Parts between these it is *gibbous*. (6.) The Earth appears variegated with Spots of different Magnitudes and Colours, arising from the Continents, Islands, Oceans, Seas, Clouds, &c. (7.) These Spots will appear constantly revolving about the Earth's Axis, by which the *Lunarians* will determine the Earth's diurnal Rotation, in the same manner as we do that of the Sun.

(CXXXVI) 1. GALILEO first discover'd the *Satellites* or *Moons* of *Jupiter*, in the Year 1610; and call'd them *Medicea Sidera*, or *Medicean Stars*, in honour of the Family of the *Medici*, his Patrons. The famous Piece call'd *Sidereus Nuncius*, in which he particularly describes the Discovery of these Stars, he dedicated to COSMUS MEDICES II, the fourth Great Duke of *Hetruria*.

2. The Orbits of *Jupiter*'s Moons lie nearly in the Plane of the Ecliptic, which is the Reason why their Motion is apparently in a right Line, and not circular, as it really is. To understand this, let S be the Sun, T the Earth in its Orbit TH, I the Planet *Jupiter* in his Orbit AIB, and in the Center of the four Orbits of his Moons. Then, because the Plane of those Orbits does nearly pass through the Eye, the real Motion of the *Satellite* in the Periphery will be apparently in the Diameter of the Orbit, which is at Right Angles to the Line joining the Center of the Earth and *Jupiter*. Pl. LVI, Fig. 8.

SATURN

SATURN has *five Moons*; and besides them a stupendous RING surrounding his Body, whose *Width* and *Distance* from *Saturn*'s Body are equal, and computed at upwards of 20000 Miles. The Periodical Times and Distances of the *Saturnian* Moons, in Semidiameters of the Ring, are as follow.

3. Thus supposing the Earth at R, if DC be drawn through the Center of *Jupiter* perpendicular to RI, the Motion of each Moon and their Places will appear to be in that Line. Thus if the exterior Moon be at E or F, it will appear to be at I, either upon or behind the Center of *Jupiter*; if the Moon move from E to K, it will appear to have moved from I to L; and when it moves from K to C, it will appear to move from L to C. Again, while the Satellite moves from C to M, it will appear to move from C to L; and as it goes from thence to F, it apparently moves from L to I. Thus also on the other Side the Orbit, while the Satellite describes the Quadrant FD, its apparent Motion will be from I to D; and then from D to I again, as it comes from D to E.

4. Whence, since this is the Case of each Satellite, it appears that while each Satellite describes the remote Half of its Orbit CFD, its apparent Motion will be direct, or from West to East along the Line CD; and while it describes the other Half DEC, its apparent Motion is retrograde, or from East to West back again along the same Line from D to C. So that each Satellite traverses the Diameter of its Orbit twice in each Revolution.

5. The Moons of *Jupiter* severally shew the same Phases to him as ours does to us. They disappear from our Sight sometimes, so that 'tis very rare to have all the four in View at once; nor is it possible to know which Satellite in Order you see, but from the Knowledge of the Theory and Calculation, because the remotest Satellite may appear nearest to *Jupiter*, and the contrary, as is evident from a View of the Figure.

6. These Moons, like our own, suffer an Eclipse every time they come to the Shadow of *Jupiter*, as at F. Also, supposing the Earth at T, the Satellite at G will undergo an Occultation behind the Body of *Jupiter*, as is evident from the Scheme. Again, a Satellite will sometimes lose its Lustre as it passes over the enlighten'd Disk of its Primary; as when

THE

THE *First*, or inmost, revolves about *Saturn* in 1 Day, 21 Hours, 18 Minutes, at the Distance of near 2 Semidiameters of the Ring.

THE *Second* in 2 Days, 17 Hours, 41 Minutes, at the Distance of 2 $\frac{2}{3}$ Semidiameters.

THE *Third* in 4 Days, 12 Hours, 25 Minutes,

it is at E and N, and the Earth in R and T. Lastly, one Satellite at O may disappear behind another at K, or cause another to disappear behind it at M.

7. The Observations by Telescopes have been carried so far as to make it very probable that all the Satellites do really revolve about their own Axis, by means of Spots which they have discover'd to belong to them, and which by their Motion cause a great Variety in the Brightness of the Satellites, and sometimes do almost obscure them: For which see Mr. *Pound's Observations* on *Jones's Abridgement of the Philosophical Transactions*, Vol. IV. p. 307.

8. By means of *Jupiter's* Satellites several noble Problems in Natural Philosophy have an easy and elegant Solution; the First of which is, *to determine the Ratio of the Velocity of Light*. The Manner how this is done I have elsewhere shewn: See *Annot.* CXII. The Second is, *to determine the Longitude of a Place from any proposed Meridian*; which is easily done by the following Method. Let the Moment of Time in which the Satellite enters the Shadow of *Jupiter* be calculated for the given Meridian from Tables of its Motion; then let the Moment of Time be well observed when this Immersion happens at the proposed Place; the Difference of these two Moments turn'd into Motion will give the Longitude of the Place, allowing 15 Degrees for every Hour, 1 Degree for every 4 Minutes of Time, or 15 Minutes of a Degree for every Minute of Time.

9. The Third Problem is, *to find the Distance of* Jupiter *from the Sun*. This is done as follows: Let the middle Moment of the Occultation of a Satellite as at G be observed, and again the middle Moment of the following at F; this will give the Time in which the Arch GF is described. Then say, As the Time of the whole Revolution is to the Time now found, so is the whole Circle or 360 Degrees to the Degrees and Minutes contained in the Arch FG; which is therefore the Measure of the Angle FIG, or its equal TIS, which is the Parallactic Angle at *Jupiter*; which being known, at

at the Distance of 3 $\frac{4}{5}$ Semidiameters.

THE *Fourth* in 15 Days, 22 Hours, 41 Minutes, at the Distance of 8 Semidiameters.

The *Fifth* in 70 Degrees, 22 Hours, 4 Minutes, at the Distance of 23$\frac{3}{15}$ Semidiameters, (CXXXVII).

the Distance of *Jupiter* from the Sun IS is known, by what has been shewn in *Annot*. CXXXIV.

(CXXXVII) 1. Though *Galileo*'s Telescope was sufficient to discover all *Jupiter*'s Moons, it would not reach *Saturn*'s, they being at two great a Distance. But yet this sagacious Observer found *Saturn*, by reason of his Ring, had a very odd Appearance; for his Glass was not good enough to exhibit the true Shape of the Ring, but only a confused Idea of that and *Saturn* together, which in the Year 1610 he advertised in the Letters of this Sentence transposed: *Altissimum Planetam tergeminum observavi*; i. e. *I have observed* Saturn *to have three Bodies*.

2. This odd Phænomenon perplex'd the Astronomers very much, and various Hypotheses were form'd to resolve it; all which seem'd trifling to the happy *Hugenius*, who applied himself purposely to improve the Grinding of Glasses, and perfecting long Telescopes, to arrive at a more accurate Notion of this Planet and its Appendage. Accordingly in the Year 1655 he constructed a Telescope of 12 Feet, and viewing *Saturn* divers times he discover'd something like a Ring encompassing his Body; which afterwards with a Tube of 23 Feet he observed more distinctly, and also discover'd a Satellite revolving about that Planet. This *Hugenian* Satellite is the fourth in Order from *Saturn*.

3. In the Year 1671 *Cassini* discover'd the third and fifth, and in the Year 1686 he hit upon the first and second, with Tubes of 100 and 136 Feet; but could afterwards see all five with a Tube of 34 Feet. He call'd these Satellites *Sidera Lodoicea*, in honour of *Louis le Grand*, in whose Reign and Observatory they were first discover'd.

4. In the Year 1656 *Hugens* publish'd his Discovery in relation to *Saturn*'s Ring in the Letters of this Sentence transposed, *viz*. *Annulo cingitur tenui, plano, nusquam cohærente, ad Eclipticam inclinato*; that is, *Saturn is encompass'd by a thin Plane or Ring, no where cohering to his Body, and inclined to the Plane of the Ecliptic*. This Inclination of the Ring to the

THESE

THESE are the constituent Parts of the *Solar System*, which is now received and approved as the only *true System of the World*, for the following Reasons (CXXXVIII).

Ecliptic is determined to be about 31 Degrees by *Hugens*, *Roemer*, *Picard*, *Campani*, &c. though by a Method not very definitive.

5. However, since the Plane of the Ring is inclined to the Plane of the Earth's Motion, it is evident when *Saturn* is so situated that the Plane of his Ring passes through the Earth, we can then see nothing of it; nor yet can we see it when the Plane passes between the Sun and the Earth, the dark Side being then turn'd to us, and only a dark List appears upon the Planet, which is probably the Shadow of the Ring. In other Situations the Ring will appear elliptical more or less; when it is most so, the Heavens appear through the elliptic Space on each Side *Saturn* (which are call'd the *Ansæ*); yea, a Fix'd Star was once observed by Dr. *Clarke*'s Father in one of them.

6. The Nodes of the Ring are in 19° 45′ of *Virgo* and *Pisces*. During *Saturn*'s Heliocentric Motion from 19° 45′ to the opposite Node, the Sun enlightens the Northern Plane of the Ring, and *vicissim*.

7. Since *Saturn* describes about one Degree in a Month, the Ring will be visible through a good Telescope till within about 15 or 20 Days before and after the Planet is in 19° 45′ of *Virgo* or *Pisces*. The Time therefore may be found by an Ephemeris, in which *Saturn* seen from the Earth shall be in those Points of the Ecliptic; and likewise when he will be seen from the Earth in 19° 45′ of *Gemini* and *Sagittarius*, when the Ring will be most open, and in the best Position to be view'd.

8. There have been some Grounds to conjecture that *Saturn*'s Ring turns round an Axis, but that is not yet demonstrable. This wonderful Ring in some Situations does also appear double; for *Cassini* in the Year 1675 observed it to be bisected quite round by a dark elliptical Line, dividing it as it were into two Rings, of which the inner one appear'd brighter than the outer. This was oftentimes observed afterwards with Tubes of 34 and 20 Feet, and more evidently in the Twilight or Moon-Light than in a darker Sky. See *Phil. Trans.* abridged, Vol. II. p. 221, 222.

(CXXXVIII) 1. The sagacious *Kepler* was the first who discover'd this great Law of Nature in all the Primary Pla-

I. It is most simple, and agreeable to the Tenor of Nature in all her Actions; for by the two Motions of the Earth all the *Phænomena* of the Heavens are resolved, which by other *Hypotheses* are inexplicable without a great Number of other Motions, contrary to philosophical Reasoning by Rule I.

II. It is more rational to suppose the Earth moves about the Sun, than that the huge Bodies of the Planets, the stupendous Body of the Sun, and the immense Firmament of Stars, should all move round the inconsiderable Body of the Earth every twenty-four Hours.

III. The Earth moving round the Sun is agreeable to that general Harmony, and universal Law,

nets, and afterwards the Astronomers observed that the Secondaries did likewise regulate their Motions by the same Law. I have already exhibited the Mathematical Theory thereof in *Annot*. XXXIV. 11, and given an Example in the Earth and *Venus*. And that the same Law holds in the System of *Jupiter*'s and *Saturn*'s Moons, will appear from the following Instances.

2. The first of *Jupiter*'s Moons is at the Distance of $2\frac{5}{6}$ of *Jupiter*'s Diameters from his Center, and revolves in 42 Hours. The outermost describes its Orbit in 402 Hours; therefore say, As 1764 (the Square of 42) is to 161604, (the Square of 402) so is $\frac{4913}{216}$ (the Cube of $2\frac{5}{6}$) to nearly $\frac{450000}{216}$, the Cube of $\frac{76}{6}$ or $12\frac{2}{3}$, the Distance of the fourth Satellite; which answers to Observations.

3. Or thus analytically by Logarithms. Let L = Logarithm of the Period of the first Satellite, l = Logarithm of any other Satellite's Period, and D and d the Logarithms of their Distances; then will it be $2L : 2l :: 3D : 3d$, and therefore $2L + 3d = 2l + 3D$; whence we have $d = D + \frac{2}{3}l - \frac{2}{3}L$. For Example; in the first and second Sa-

which

which all other moving Bodies of the System observe, viz. *That the Squares of the periodical Times are as the Cubes of the Distances:* But if the Sun move about the Earth, that Law is destroy'd, and the general Order and Symmetry of Nature interrupted; since according to that Law the Sun would be so far from revolving about the Earth in 365 Days, that it would require no less than 5196 Years to accomplish one Revolution.

IV. AGAIN: Did the Sun observe the universal Law, and yet revolve in 365 Days, his Distance ought not to be above 310 Semidiameters of the Earth; whereas it is easy to prove it is really above 20000 Semidiameters distant from us.

tellites of *Jupiter*, *Cassini* observed the Distance of the first in Semidiameters of *Jupiter* to be $5\frac{2}{3}$, whose Logarithm is 0,753353. The Periods of those Satellites give $\frac{2}{3}l =$ 2,32459, and $\frac{2}{3}L = 2,122851$; from whence we get $d =$ 0,95509, the Number corresponding to which is 9,07, the Distance of the second Satellite, agreeing wonderfully with Observation.

4. Now since the Moon turns round the Earth, if the Sun did likewise perform his Circuit about it, their Motions would undoubtedly be regulated by the same Law with all the rest. But the Period of the Moon is 27 Days, that of the Sun 365; the Distance of the Moon 60 Semidiameters of the Earth; therefore say, As 729 (the Square of 27) is to 133225 (the Square of 365;) so is 216000 (the Cube of 60) to 39460356, the Cube Root whereof is 340, which ought to express the Sun's Distance in Semidiameters of the Earth. But we have shewn the Sun is really distant from the Earth near 20000, (see *Annot.* CXXXIV. 18.)

5. Admitting the Sun to be at the Distance of 20000 Semidiameters, his *Periodical Time* would then be more than 450 Years, if its Motion were govern'd by *Kepler*'s Law, and compared with that of the Moon; for as 216000 ($=60^3$) is to 8000000000000 ($=20000^3$) so is 729 ($=27^2$) to a

V. THE

V. The Sun is the Fountain of Light and Heat, which it irradiates through all the Syſtem; and therefore it ought to be placed in the Center, that ſo all the Planets may at all times have it in an uniform and equable Manner: For,

VI. If the Earth be in the Center, and the Sun and Planets revolve about it, the Planets would then, like the Comets, be ſcorched with Heat when neareſt the Sun, and frozen with Cold in their *Aphelia*, or greateſt Diſtance; which is not to be ſuppoſed.

VII. If the Sun be placed in the Center of the Syſtem, we have then the rational Hypotheſis of the Planets being all moved about the Sun by the univerſal Law or Power of Gravity ariſing from his

Number; the ſquare Root of which is 164320 Days = 450 Years nearly, which is the Periodical Time of the Sun's Revolution at that Diſtance, and moving according to the Univerſal Law.

6. This beautiful and harmonious Syſtem, or Frame of the World, ſufficiently recommends itſelf from the Principles of right Reaſon only; ſuppoſing there were no ſuch Thing as abſolute Demonſtration attainable in the Caſe. It is therefore very ſurprizing, to obſerve, how few among thoſe who are not Mathematically learn'd, can be induced to believe, and acquieſce in this Doctrine of the Earth's Motion, and Stability of the Sun. *Copernicus*, above 200 Years ago, mentions the zealous Father *Lactantius*, as ridiculing thoſe who aſſerted the Spherical Figure of the Earth. Therefore, ſays he, it is not to be wonder'd at if ſuch Sort of People ſhould ridicule Us. And whatever the Popes may have ſince decreed, 'tis certain, this Doctrine was ſo far from being then reputed heretical and damnable, that this great Man dedicated his Book to Pope *Paul* III. becauſe by his Holineſs, Authority, and Learning, he might be ſecured againſt the Calumnies of ignorant Gainſayers; yea, and appealed to his Holineſs at the ſame Time for the Uſefulneſs of his Doctrine even to the Eccleſiaſtical Republick. His Words are, *Mathe-*

His vast Body; and every thing will answer to the Laws of circular Motion, and central Forces: But otherwise we are wholly in the dark, and know nothing of the Laws and Operations of Nature.

VIII. But happily we are able to give not only *Reason*, but *demonstrative Proofs*, that the Sun does possess the Center of the System, and that the Planets move about it at the Distance and in the Order above assign'd: The *first* of which is, That *Mercury* and *Venus* are ever observed to have *two Conjunctions* with the Sun, but no *Opposition*; which could not happen, unless the Orbits of those Planets lay within the Orbit of the Earth (CXXXIX).

mata Mathematicis scribuntur, quibus & hi nostri Labores, si me non fallit opinio, videbuntur etiam Reipublicæ Ecclesiasticæ conducere aliquid, cujus Principatum tua Sanctitas nunc tenet.

(CXXXIX) 1. What relates to the Conjunctions and Oppositions of the Planets will be easily understood by a Diagram. Let S be the Sun; T the Earth, V *Venus*, and M *Mercury*, in their several Orbits. Now 'tis evident that when *Venus* and *Mercury* are at V and M, they will be seen from the Earth T in the same Part of the Heavens with the Sun, *viz.* at W, because they are all posited in one Right Line TW; and this is call'd the *Lower* or *Inferior Conjunction*. Pl. LVII.

2. Again: When *Venus* and *Mercury* come to the Situations D and O, they are again in the same Right Line joining the Centers of the Earth and Sun, and are therefore again seen in the same Part of the Heavens with him; and this is call'd the *Upper* or *Superior Conjunction*. Here 'tis evident, those two Planets must appear twice in Conjunction with the Sun in each Revolution, to a Spectator on the Earth at T, which we at present will suppose to be at Rest.

3. Hence we have an infallible Proof that the Orbits of *Venus* and *Mercury* lie both within the Orbit of the Earth: Also the Orbits of *Mars*, *Jupiter*, and *Saturn* must lie without the Orbit of the Earth; for otherwise they could not exhibit the Appearance they do of alternate Conjunctions and

IX. The *second* is, That *Mars*, *Jupiter*, and *Saturn*, have each their *Conjunctions* and *Oppositions* to the Sun, alternate and successively; which could not be, unless their Orbits were exterior to the Orbit of the Earth.

X. In the *third* Place, The *greatest Elongation* or *Distance* of *Mercury* from the Sun is but about 20 Degrees, and that of *Venus* but about 47; which answers exactly to their Distances in the

Oppositions. Thus let *Mars* be in his Orbit at Y, 'tis evident when the Earth is at T, that Planet will be seen in Conjunction with the Sun, and will be then at its greatest Distance from the Earth.

4. But when the Earth is at *t* between the Sun and *Mars*, 'tis plain they must appear in opposite Parts of the Heavens, because a Person at *t* viewing the Sun at S must look directly to the contrary Part to view the Planet at Y; and in this Opposition to the Sun *Mars* is nearest to the Earth: All which is so evident from the Scheme, and so exactly agreeable to the Phænomena of those Planets in the Heavens, that any Person must be strangely obstinate, and incapable of any Sort of Conviction, who cannot see the Constitution of Nature, and the Disposition of the Planetary Orbits, are such as are above described.

5. But farther: If we divide the Distance of the Earth from the Sun, *viz.* the Line ST, into a hundred or a thousand equal Parts, and place the Orbits of *Venus* and *Mercury* at the Distance of SV = 724, and SM = 388, and then draw TA, TR, to touch those Orbits in the Points A and R; then 'tis plain the Angles ATS and RTS will measure the greatest Distance at which either of those Planets can be seen from the Sun; because the visual Ray passing to the Planet in any other Part of its Orbit will lie nearer to the Line TSW, and therefore shew the Planet nearer the Sun than when at A or R.

6. Now 'tis found by measuring those Angles geometrically in the Diagram, that the Angle ATS = 47 Degrees, and RTS = 20, very nearly; and this agrees exactly with their observed greatest Distances or Elongation from the Sun in the Heavens. Hence it is that *Mercury* is so rarely seen, and *Venus* but at certain Times of the Year; whereas if the

System

System above assign'd: But in the *Ptolomean* System, they might and would sometimes be seen 180 Degrees from the Sun, *viz.* in Opposition to him.

XI. FOURTHLY, In this Disposition of the Planets they will all of them be sometimes much nearer to the Earth than at others; the Consequence of which is, that their Brightness and Splendor, and also their *apparent Diameters*, will

Earth were at Rest, and in the Center of the Planetary Orbits, those Planets would be seen in all Positions and Distances from the Sun, in every respect like the Moon; and therefore 'tis perfectly surprizing, how any Man can resist such glaring Evidence of Truth on one hand, and Falshood on the other.

7. We have already shewn, that the apparent Magnitude and Brightness of an Object decreases as the Square Distance increases; therefore the Magnitude of *Venus* seen at V is to that as it appears from D in the Proportion of $\overline{TD}^2$ to $\overline{TV}^2$, that is, as $\overline{1724}^2$ to $\overline{276}^2$, or as to 1 nearly. And when *Venus* is measured in both those Distances with a Micrometer in a Telescope, the Numbers shew the perfect Agreement of this System with Nature itself.

8. Thus also the apparent Magnitude of *Mars* when his Distance is tY, is to that when his Distance is TY, as $\overline{TY}^2$ to $\overline{tY}^2$; that is, as $\overline{2523}^2$ to $\overline{523}^2$, or very nearly as to 1. And this we know is true in Fact, by measuring the Planet in both those Distances. It is likewise obvious to common Sense; for *Mars* in his nearest Distance appears so large that he has been often mistaken for *Jupiter*, whereas in his greatest Distance he appears so small as scarcely to be distinguish'd from a Fix'd Star.

9. From what has been said of the Phases of the Moon, 'tis easy to understand that *Venus* and *Mercury* must have nearly the like Appearances. Thus when *Venus* is at V, all her illumined Hemisphere will be turn'd directly from the Earth, and she will then be *New*. As she passes from V to A she will appear *horned*. At A she will shew just half her enlighten'd Surface to the Earth, and appear *bisected*, or *dichotomised*. From A to D she will appear more and more *gibbous*;

be proportionally greater at one Time than another: And this we observe to be true every Day. Thus the apparent Diameter of *Venus*, when greatest, is near 66 Minutes, but when least not more than 9 Minutes and a half; of *Mars*, when greatest, it is 21 Minutes, but when least no more than 2 Minutes and a half; whereas by the *Ptolomean* Hypothesis they ought always to be equal.

XII. The *fifth* is, That when the Planets are

and at D would appear a *Full* enlighten'd Hemisphere, were it not that she is then lost in the Sun's Blaze, or hid behind his Body: All which Phases return again in the other Half of the Orbit. The same Thing is obvious in *Mercury*, and *Mars* shews part of those Phases; but *Jupiter* and *Saturn* appear always with a Full Face, by reason of their very great Distance.

10. The Appearance of *Venus* in the Day-time for several Days together, in some certain Years, put the sagacious Dr. *Halley* on resolving the following Problem, *viz.* *To find the Situation of* Venus *in respect of the Earth, when the Area of the illuminated Part of her Disk is a Maximum.* I shall here give the Solution as he has proposed it in the *Philosophical Transactions*, N° 349; and also the Demonstration, which the Doctor omitted.

Plate LVIII. Fig. 1.

11. In order to this, let S be the Sun, V the Planet *Venus* in the Situation required, T the Earth, and TV her Distance sought. Put $TS = a$, $SV = b$, $TV = x$, and on the Point V with the Radius VT describe the Quadrant TA; from T let fall the Perpendicular TB, and put $BV = d$; then $AB = x - d = v$, the Versed Sine of the Angle TVA. Now (by *Euclid* II. 12.) we have $a^2 = b^2 + x^2 + 2bd$, whence $a^2 - 2bd = b^2 + x^2$; and by adding $2bx$ on each Side, $a^2 + 2bx - 2bd = b^2 + 2bx + x^2$; that is, $a^2 + 2bv = b^2 + 2bx + xx = r$. Then $r - a^2 = 2bv = s$; and multiplying by $2x$ we have $4bxv = 2xs$, whence $4bx : s :: 2x : v$; that is, $4bx : b^2 + 2bx + xx - a^2 :: 2TV : AB ::$ the Diameter of a Circle to the Versed Sine of the exterior Angle TVA.

12. But in any Situation B of the Planet *Venus* the Arch of Illumination af is equal to the Arch bd, which measures the exterior Angle bBd. And it has been shewn, that *the*

view'd

view'd with a good Telescope they appear with *different Phases*, or with different Parts of their Bodies enlighten'd. Thus *Venus* is sometimes *new*, then *horned*, after that *dichotomized*, then *gibbous*, afterwards *full*; and so increases and decreases her Light, in the same manner as the Moon, and as the *Copernican System* requires.

XIII. THE *sixth* is, That the Planets, all of them, do sometimes appear *direct* in Motion,

Area of the whole Disk of the Planet is to the Area of the enlighten'd Part as the Diameter of a Circle to the Versed Sine of the Arch of Illumination, and therefore as $4bx$ to $b^2 + 2bx + x^2 - a^2$.

13. But the Area of the whole Disk is every where as $\frac{1}{xx}$; therefore, as $4bx : b^2 + 2bx + x^2 - a^2 :: \frac{1}{xx} : \frac{b^2 + 2bx + x^2 - a^2}{4bx^3}$, which in all Cases will be proportional to the enlighten'd Area of the Disk. And to determine this a *Maximum* its Fluxion must be $= o$, or the negative Parts thereof be equal to the affirmative, that is, that $\overline{2b\dot{x} + 2x\dot{x}} \times 4bx^3 = 12bx^2\dot{x} \times \overline{b^2 + 2bx + xx - a^2}$; and dividing all by $4bx^2\dot{x}$, the Equation becomes $2bx + 2x^2 = 3b^2 + 6bx + 3xx - 3a^2$. Consequently $3bb + 4bx + xx = 3aa$; whence we get $x = \sqrt{3aa + bb} - 2b = 427$.

14. If therefore we take 427 from the Scale of equal Parts ST, and set from T to the Orbit of *Venus*, it will intersect it in the Point x; and drawing Tx, it will give the Angle xTS $= 40$ Degrees nearly; which shews that when *Venus* is 40 Degrees distant from the Sun, before and after her *Inferior Conjunction* with him, she then shines with the greatest Lustre possible.

15. In this Position we see not much more than $\frac{1}{4}$ of her Disk enlighten'd, and yet she shines with so great a Lustre as to surpass the united Light of all the Fix'd Stars that appear with her, and casts a very strong Shade on the horizontal Plane, and may be seen in the full Sun-shine of the Day; a Phænomenon very extraordinary, and which returns but once in eight Years.

ſometimes *retrograde*, and at other times *ſtationary*. Thus *Venus*, as ſhe paſſes from her greateſt Elongation Weſtward to her greateſt Elongation Eaſtward, will appear *direct in Motion*, but *retrograde* as ſhe paſſes from the latter to the former; and when ſhe is in thoſe Points of greateſt Diſtance from the Sun, ſhe ſeems for ſome time *ſtationary*: All which is neceſſary upon the *Coperni-*

16. The different Directions in which the Planets appear to move in the Heavens is an irrefragable Argument of the Truth of the Solar Syſtem; for in the *Ptolomean* Syſtem they would be ſeen to move with their true or real Motion, and in their Direction according to the Order of the Signs from Weſt to Eaſt, in every Part of their Orbits, and that always in an equable Manner; whereas now we obſerve them move ſometimes from *Weſt to Eaſt*, when they are ſaid to be *direct in Motion*; ſometimes from *Eaſt to Weſt*, when they are ſaid to be *retrograde*, or to go backwards; and ſometimes they appear not to move at all for a certain Time, when they are ſaid to be *ſtationary*: And laſtly, the Motion of a Planet when *direct* is always much ſlower than when it is *retrograde*.

17. Now all theſe Phænomena are not only explicable by, but neceſſarily follow from, the *Copernican* Theory. Thus with reſpect to the Planet *Mercury*, when at R he will appear at his greateſt Diſtance from the Sun among the Stars at Q, being ſeen in the Line TQ; but as the Planet paſſes from R by N to O, the viſual Line TQ will continually approach the Line TW, in which the Sun appears at W; and when the Planet is come to D it will be in Conjunction with the Sun, and will have apparently deſcribed the Arch QW in the Heavens. After this, while the Planet moves from O to Z, it will appear to go in the Heavens from W to X, ſtill the ſame Way as before; and becauſe its apparent Motion agrees with the true, it is all this while *direct*.

18. But when the Planet moves from Z to M, the Ray TX will return, and deſcribe the Arch XW back again; and as the Planet moves from M to R, the viſual Ray will keep moving on from W to Q; and ſo in the Paſſage of the Planet through the Part of its Orbit ZMR it will appear to move in the Heavens through XWQ, the ſame Arch as before, but in a *retrograde* Direction.

can

can Hypothesis, but cannot happen in any other.

XIV. THE *seventh* is, That the Bodies of *Mercury* and *Venus*, in their lower Conjunctions with the Sun, are *hid behind the Sun's Body*; and, in the upper Conjunctions, are seen to pass over the Sun's Body or Disk in form of a *black round Spot*: Which is necessary in the *Copernican*, but impossible in the *Ptolomean* System.

19. Now because the Tangent Line or visual Ray TQ or TX coincides as to Sense with the Orbit of the Planet for a small Distance on each Side the Points R and Z, as from *a* to *b*, and from *c* to *d*; therefore the Planet when it arrives at *a* will appear to move in the Tangent from *a* to *b*, during which Time it will be seen in the same Right Line TQ, and consequently in the same Point Q in the Heavens: So that in its Motion from *a* to *b* it must appear *stationary*, or without any Motion; and the same is to be observed in moving from *c* to *d*, when the Planet is in that Part of its Orbit.

20. Hence we observe, that in *Mercury* and *Venus*, the Places R, Z, and A, G, of their greatest Elongation are those in which they are *stationary*. It is in these two Points that we can at any time see *Mercury*; and it is in those Points that we see *Venus* such a glorious Morning-Star or *Phosphorus* at A, and such a splendid Evening-Star or *Hesperus* at G. Hence we observe, that from the Time *Venus* is a Morning-Star in her greatest Elongation at A, to the Time of her being an Evening-Star in her greatest Elongation at G, she is *direct* in Motion: Consequently, half the Time of her being a Morning or Evening Star she is *direct*, and the other half *retrograde*.

21. Also it is easy to observe, that since the same Arch QX is described in Times very unequal, *viz.* in the Times the Planet describes the very unequal Parts of its Orbit ROZ and ZMR, the Velocity of the Motion in the former Case must be much less than that in the latter; that is, the Planet when *direct* moves apparently much slower than when it is *retrograde*.

22. If we consider the Dispositions of the Orbits of the superior Planets, we shall observe the same Phænomena of them also. Let S be the Sun, ACH the Earth's Orbit, IMK that of *Mars*, and OLQ the Firmament of Stars. Plate LVIII. Fig. 2.

XV. The *eighth* is, That the Times in which these *Conjunctions*, *Oppositions*, *Stations*, and *Retrogradations* of the Planets happen, are not such as they would be, were the Earth at Rest in its Orbit; but precisely such as would happen, were the Earth to move, and all the Planets in the Periods above assign'd them: And *therefore this, and no other, can be the true System of the World*;

Through *Mars* at M draw QMG and OMC, to touch the Earth's Orbit in G and C. Then because the Earth and *Mars* do both move the same Way, but the Earth very quick in respect of *Mars*, all the Phænomena will be the very same if we suppose *Mars* to be at Rest, and the Earth to move with the Difference of their Velocities.

23. Let *Mars* then be at Rest in M, and the Earth begin her Motion from G. At G the Planet will be seen in the Line GQ, among the Stars at Q. When the Earth is at H, *Mars* will be seen in the Line HP, among the Stars at P. In the same manner at A, B, and C, the Planet will be projected to the Points L, N, O, in the Heavens. Therefore while the Earth describes the Part of its Orbit GAC, *Mars* will appear to move through the Arch of the Heavens QLO; which being from West to East is according to the Order of the Signs, and the Planet will be *direct in Motion*.

24. But as the Earth proceeds from C to D, *Mars* will appear to move from O to N; and as the Earth goes on through E, F, to G, *Mars* will appear to return by L, P, to Q, and so measure back again the same Arch as before: And thus during the Earth's Passage from C to G, this Planet will appear *retrograde*; which therefore must always be the Case when he is in Opposition to the Sun and nearest to the Earth, as in Conjunction he is always *direct in Motion*; and when the Earth is in G or C, the Planet must appear for some Time *stationary*, for the Reasons mention'd in *Art.* 19. The same may be shewn of *Jupiter* and *Saturn*; but as the Earth has a much greater relative Velocity in respect to *Jupiter* than it has with respect to *Mars*, the Times of the *Conjunctions* and *Oppositions*, as also of the *progressive* and *regressive Motions*, will be more frequent in *Jupiter* than in *Mars*, and for the same Reason will happen oftener in *Saturn* than in *Jupiter*.

25. Again: Another Phænomenon, which infallibly proves

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and it will stand the eternal Test of future Ages, for, MIGHTY IS THE FORCE OF TRUTH, AND SHALL PREVAIL.

BUT though the Planets all move round the Sun in Orbits commonly supposed *circular*, yet are they not exactly so, but *elliptical*, or in form of an ELLIPSIS, which Figure is vulgarly call'd

the Truth of the *Copernican* System, is, that *Venus* and *Mercury* suffer an Occultation behind the Sun's Disk, when they are in the remotest Parts of their Orbits, as at D and O; but this can never happen in the *Ptolomean* Hypothesis, because there the Orbit of the Sun is supposed exterior to the Orbits of those two Planets.

26. All these Phænomena of the Planets plainly prove, that the Earth holds that Place in the Heavens which the present Philosophy assigns her; but to shew moreover that she has not only a Place among the Planets, but likewise that she is carried in the same Manner with them about the Sun, we need only observe, that the Times in which these Phænomena happen to the Planets are no ways such as they would be were the Earth at Rest, but such as they must necessarily be supposing the Earth's Period about the Sun to be in $365\frac{1}{4}$ Days.

27. For Example: Suppose *Venus* at any time in Conjunction with the Sun at V, then were the Earth at Rest at T, that very Conjunction would happen again when *Venus* had made just one Revolution, that is, in 225 Days; but every one knows this is contrary to Experience, for a much longer Time than that lapses between two Conjunctions of the same Kind; as there evidently must, if we suppose the Earth to have a Motion towards the same Parts in the same Time; because then, 'tis plain, when *Venus* comes again to V, the Earth will have pass'd in that Time from T to some other Part of the Orbit, and from this keeps moving on, till *Venus* gets again between it and the Sun.

28. What this Surplus of Time is may be easily estimated, by supposing the Earth to be at Rest in her Orbit, and *Venus* to move with the Difference of their mean Motions. Thus the daily mean Motion of the Earth is 59′ 8″, and the daily mean Motion of *Venus* is 1° 36′ 8″. The Difference of these mean Motions is 37′; therefore say, As 37′ is to the whole

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an *Oval*, as ABPD, described about two Centers S, F, call'd the *Foci*, or *Focal Points* of the Ellipse. The Point C is the Center; A P the Axis, or longest Diameter; and B D the shortest Diameter: And in one of these Focus's. *viz.* S, the Sun is placed, about which the Planet moves in the Orbit ABPD (CXL).

Circle or $360^{\circ} = 21600'$, so is 1 Day to 583 Days, the Time between two Conjunctions as required, *viz.* 1 Year and 218 Days, in which Time *Venus* performs a little more than $2\frac{1}{2}$ Revolutions. In the same Manner the Time may be found for any of the other Planetary Conjunctions, Oppositions, Stations, Retrogressions, &c.

27. These Arguments are plain, and easy to be understood; most of them require no more than common Observation, that is, in other Words, *common Sense*. To be ignorant of the Truths here specified, is to shew an unaccountable Inattention to the most obvious and glaring Phænomena of Nature: And if People are not convinced by these Proofs, it is not because they *cannot*, but because they *will not*; and therefore, *Si Populus vult decipi, decipiatur*.

(CXL) 1. We have hitherto consider'd the Phænomena of the Heavenly Bodies without regard to the accurate Form of their Orbits, which is not *circular*, but *elliptical*; yet that it is very little so, even in the most eccentric Orbit, as that of *Mercury*, will appear by comparing their Eccentricities with their mean Distances from the Sun. Thus suppose the mean Distance of the Earth from the Sun be divided into 1000 equal Parts, then in those Parts we have,

In *Mercury*,	CS : DS ::	80 :	387 :: 1 :	4,84
Venus,	CS : DS ::	5 :	723 :: 1 :	144,6
Earth,	CS : DS ::	17 :	1000 :: 1 :	19
Mars,	CS : DS ::	141 :	1524 :: 1 :	10,8
Jupiter,	CS : DS ::	250 :	5201 :: 1 :	20,8
Saturn,	CS : DS ::	547 :	9538 :: 1 :	17,4

2. It is found by Experience that the Orbits of the Planets are quiescent, or that the Line of the *Apsides* A P always keeps one and the same Position with respect to the Fix'd Stars: And the *Aphelium*, or Point A, possesses different Points in the Ecliptic in the several Orbits as follows.

Plate LVIII. Fig. 3.

HENCE

HENCE, when the Planet is in the Point P, it is nearest the Sun, which Point is, for that Rea-

		°	′	″			°	′	″
In *Mercury*,	♐	12	44	00	In *Mars*,	♍	0	31	54
Venus,	♒	4	19	54	*Jupiter*,	♎	9	9	54
Earth,	♑	8	1	10	*Saturn*,	♐	27	49	54

3. That the Earth's Orbit is elliptical is well known from common Experience; for were the Orbit circular, the Sun's apparent Diameter would always be the same; but we find it is not, for if it be measured with a Micrometer in Winter-time, it will be found considerably larger than in the Summer, and it will be greatest of all when the Sun is in the 8° of ♑, (which shews that is the Place of the *Aphelium*) it being then 32′ 47″; whereas when the Sun is in the 8° of ♋, his Diameter is but 31′ 40″.

4. Hence it is evident that the Sun is really nearer to us in the Midst of Winter than in the Midst of Summer; but this seems a Paradox to many, who think the Sun must needs be hottest when it is nearest to us, and that the Sun is apparently more distant from us in *December* than in *June*. As to the Sun's being hotter, 'tis true it is so to all those Places which receive his Rays directly or perpendicularly, but we find his Heat abated on account of the Obliquity of the Rays, and his short Continuance above the Horizon at that Time. And as to his Distance, it is only with respect to the Zenith of the Place, not the Center of the Earth; since it is plain, the Sun may approach the Center of the Earth, at the same time that it recedes from the Zenith of any Place.

5. Agreeable to the Sun's nearer Distance in the Winter, we observe his apparent Motion is then quicker than in Summer; for in the 8° of ♑ it is about 61′ *per* Day, but in the 8° of ♋ his Motion is but 57′ *per* Day. Accordingly we find the Summer Half-Year 8 Days longer than the Winter Half-Year, as appears by the following Computation.

SUMMER *Half-Year includes*		WINTER *Half-Year includes*	
In *March*	$21\frac{1}{2}$ Days.	In *September*	18 Days.
April	30	*October*	31
May	31	*November*	30
June	30	*December*	31
July	31	*January*	31
August	31	*February*	28
September	12	*March*	$9\frac{1}{2}$
Summer-Half	$186\frac{1}{2}$		$178\frac{1}{2}$
Winter-Half	$178\frac{1}{2}$		
The Difference	8 Days.		

son,

fon, call'd the *Perihelion:* Here, therefore, the Attraction of the Sun is strongest, his Light and

6. For the Sun's attracting Force being one Part of the Cause of the Planet's Motion, and this Force always increasing and decreasing in the inverse Ratio of the Squares of the Distances, 'tis evident the Velocity of the Planet will always be greater the nearer it is to the Sun, and *vice versa.* Hence the Motion of a Planet is every where unequable, being constantly accelerated as it passes from A by D to P, and in the other Half from P to A it is retarded.

7. Yet is this unequal Motion of a Planet regulated by a certain immutable Law, from which it never varies, which is, *That a Line drawn from the Center of the Sun to the Center of the Planet does so move with the Planet about the Sun, that it describes elliptic Areas always proportional to the Times.* That is, if when the Planet moves slowest it describes the Arch A*a* in a given Time, and when it moves quickest it describes the Arch *b*P in the same Time, then will the trilineal Area AS*a* be equal to the other trilineal Area *b*SP.

Plate LVIII. Fig. 4. 8. To demonstrate this, let the Time in which the Planet moves through the Periphery of its Orbit be divided into equal Parts, and suppose that in the first Part it described any Right Line AB, by the Projectile Force in any Direction and the Centripetal Force conjointly; then in the second Part of Time it would proceed in the same Right Line to *c*, if nothing prevented; so that B*c* = AB, as is manifest from the first *Law of Motion.*

9. Draw the Right Lines SB, S*c*, and the Triangles ABS and B*c*S will be equal, as having equal Bases AB, B*c*, and the same Altitude of the Vertex S. But when the Body comes to B, let the centripetal Force act with a new Impulse either equal to the former or unequal, and let it cause the Body to decline from the Right Line B*c*, and describe the Right Line BC; draw C*c* parallel to BS, meeting BC in C; and at the End of the second Part of Time the Body will be at C, and in the same Plane with the Triangle ASB. Join SC, and because of the Parallels SB, C*c*, the Triangle SBC will be equal to the Triangle SB*c*, and therefore equal to the Triangle SAB. By the same Way of Reasoning, if the centripetal Force act successively in the Points C, D, E, causing the Body in each equal Part of Time to describe the Right Lines CD, DE, EF, &c. the Triangles SCD, SDE, SEF, &c. will be equal, and all in the same Plane.

10. In equal Times, therefore, equal Areas are described;

Heat

Heat greatest, and his apparent Diameter largest; and in this Point the Planet must consequently

and, by Composition of Ratios, any Sums of Areas SADS, SAFS, are to each other as the Times in which they are described. Let now the Number of Triangles be increased, and their Breadth be diminish'd *in infinitum*; then will their Perimeter ADF be ultimately a Curve: And therefore the centripetal Force, by which the Body is drawn perpetually from the Tangent of this Curve, acts incessantly; and the Areas described are also in this Case proportional to the Times of their Description.

11. Hence the Velocity of the revolving Body or Planet is every where inversely as the Perpendicular let fall from the Center S to the Tangent of the Orbit in the Place of the Planet. For the Velocities in the Points A, B, C, &c. are as the Bases of the Triangles AB, BC, CD, &c. as being the Spaces described in the same Time; and the Bases of equal Triangles are reciprocally as their perpendicular Altitudes; and therefore since in the evanescent Triangles ASB, ASC, &c. the Right Lines A*c*, B*d*, C*e*, &c. become Tangents to the Curve in the Points A, B, C, &c. 'tis manifest the Velocity in those Points will be inversely as a Perpendicular from S let fall upon those Tangent Lines produced.

12. Hence also it follows, that the Times in which equal Arches are described in any Planetary Orbit are directly as those Perpendiculars, because they are inversely as the Velocities.

13. If two Chords of very small Arches described in the same Time AB, BC, and DE, EF, be compleated into the Parallelograms ABCV and FEDZ, and the Diagonals BV and EZ be drawn; then will those Lines tend to the Sun or Center S, and be proportional to the centripetal Force: For the Motion BC and EF is compounded of BV, B*c*, and EZ, E*f*; but BV $=$ C*c*, and EZ $=$ F*f*; but C*c* and F*f* were generated by the Impulses of the centripetal Force in B and E, and are therefore proportional to them; and consequently so are BV and EZ.

14. Draw the Diagonal AC, and it will bisect the Line BV in *b*; consequently the *Sagitta* B*b* is as the centripetal Force by which the Arch ABC is described, whose Chord is AC.

15. Hence, if a Body revolve in any Curve AP*q* about an immoveable Center S, the Force in any Point P will be to that in any other Point *p* as $\frac{QR}{SP^2 \times QT^2}$ to $\frac{qr}{Sp^2 \times qt^2}$; Plate LVIII. Fig. 5.

move

move with the greatest Velocity. But in the Point A, where the Planet is farthest distant from

for the *Sagittæ* QR, qr, (which call S, s,) are as the centripetal Forces (F, f,) in P and p, when the Times (T, t,) are given, (by the last) that is, $S : s :: F : f$. But when the Forces are given, the *Sagittæ* will be as the Squares of the Times, *viz.* $S : s :: TT : tt$. Therefore when neither the Times nor the Forces are the same, it will be $S : s :: F \times T^2 : f \times t^2$; and so $\frac{S}{T^2} : \frac{s}{t^2} :: F : f$. And because the elliptic Areas SQP and Sqp are as the Times in which they are described, therefore when the Arches PQ and pq are indefinitely small, we have $T : t :: \frac{1}{2}SP \times QT : \frac{1}{2}Sp \times qt :: SP \times QT : Sp \times qt$. Consequently we have, as $F : f ::$ $\frac{QR}{SP^2 \times QT^2} : \frac{qr}{Sp^2 \times qt^2}$.

16. Let SY be a Perpendicular let fall from S upon the Tangent PR produced; then will the centripetal Force be as $\frac{QR}{SY^2 \times QP^2}$, because the Rectangle $SY \times QP = SP \times QT$; for the evanescent Arch QP is coincident with the Tangent PR, and may therefore be esteem'd as the Base of the Triangle SPQ, whose Area is either $\frac{1}{2}SP \times QT$, or $\frac{1}{2}QP \times SY$; therefore $SP \times QT = QP \times SY$. Which was to be shewn.

Plate LVIII. Fig. 6.

17. If the Orbit were a Circle, as PQVF, and PV a Chord drawn through the Center of Force S; then drawing the Chord QM in such manner as it may be bisected in K by the Chord PV, we have $QK^2 = VK \times PK$, (by *Euclid*, III. 35.) but in the vanishing State of PK it will be $VK = VP$, and $QR = PK$ (by *Art.* 13.); also $QK = QP$, therefore $QP^2 = VP \times QR$, and $PV = \frac{QP^2}{QR}$; whence, in this Case, the central Force will be inversely as $SY^2 \times PV$.

18. Wherefore, since the Velocity is as $\frac{1}{SY}$, we have SY^2 as the Square of the Velocity inversely; therefore the centripetal Force is as the Square of the Velocity directly, and the Chord PV inversely.

19. Hence if the curvilinear Figure APQ be given, and any Point S to which the centripetal Force is continually directed; the Law of the centripetal Force may be found, by which any Body P perpetually drawn from a right-lin'd Course shall be detain'd in the Perimeter of that Figure, and by re-

the

the Sun, (for that Reason call'd the *Aphelion*) every thing is just the reverse: And in the

volving shall describe it, *viz.* by computing the Value of the Expression $\frac{QR}{SP^2 \times QT^2}$, or of $SY^2 \times PV$.

20. For Example: *Let a Body P revolve in the Circumference of a Circle, 'tis required to find the Law of the centripetal Force tending to any given Point S.* Let PY be a Tangent in the Point P, and SY the Perpendicular, and VP the Chord passing through S. Let VA be the Diameter of the Circle, and join AP. Then is the Triangle SYP similar to the Triangle VAP; as may be shewn from *Eucl.* III. 32. Therefore $AV : PV :: SP : SY$; consequently $\frac{SP \times PV}{AV} = SY$, and so $\frac{SP^2 \times PV^2}{AV^2} = SY^2 \times PV$, which therefore is as the centripetal Force inversely; but because AV^2 is a given Quantity, we have the said Force reciprocally as $SP^2 \times PV^3$. Plate LVIII. Fig. 6.

21. Again: *Let it be required to find the Law of the centripetal Force by which a Body is moved, so as to describe the equiangular Spiral* PQS *about the Center* S. In this Case all the Angles are given in every trilineal Area SQP, and therefore also the Ratio of all the Sides in the Figure SPRQT; therefore the Ratio of $\frac{QT}{QR}$ is given, whence $\frac{QT}{QR} \times QT$ is as QT; that is, (because of the given Ratio of QT to PS) $\frac{QT^2}{QR}$ is as SP. And this Ratio will be constant, let the Angle PSQ be changed in any Manner whatsoever: For let $QR = a$, when the Angle PSQ is constant, and $QT = b$; but when it is variable, let $QR = x$, and $QT = y$; then (by *Lem.* 11. of *Princip.*) it will be $a : x :: b^2 : y^2$, whence $\frac{b^2}{a} = \frac{y^2}{x} = \frac{QT^2}{QR}$; which shews that $\frac{QT^2}{QR}$ will always remain the same as at first, *viz.* as SP. Therefore $\frac{QT^2 \times SP^2}{QR}$ will become SP^3; consequently the centripetal Force QR will be inversely as SP^3. Fig. 7.

22. *Let a Body revolve in an Ellipsis* APQ, *by a Force every where directed to the Center* C; *it is required to find the Law of that Force.* Let Qv be drawn parallel to the Tangent PR, and PF perpendicular to KC; and parallel to PF join Fig. 8.

Points

Points B or D it is in its mean Distance from the Sun.

CQ; the rest as before. The right-angled Triangles QTV and PFC are similar; for the Angle $QvC = PCF$, (by *Euclid*, XXIX. 1.) therefore $QT : Qv :: PF : PC$; and $QT \times PC = Qv \times PF$. But $QT \times PC$ is equal to twice the Triangle PQC, which is a constant Quantity, as being proportional to the constant Particle of Time in which it is described. Also in the Ellipsis $DK \times PF$ is a constant Quantity (*per Conics*). Therefore $DK \times PF$ is to $QT \times PC$, or $Qv \times PF$, that is, DK to Qv, in a given Ratio, wherever the Point P is taken in the Ellipsis. Hence also the Ratio of DK^2 to Qv^2 is a constant one: But in the Ellipsis $DK^2 : Qv^2 :: PG^2 : Pv \times vG$ (*per Conics*). Now because $Qv = QR$, and the Difference between Gv and GP is infinitely small, therefore $Pv \times vG = QR \times PG$; whence PG^2 is in a constant Ratio to $PG \times QR$, that is, QR *or the centripetal Force is every where in a constant Ratio to* PG, *or to* PC, *the Distance from the Center*.

23. Hence if the Center C of the Ellipsis were to go off to an infinite Distance, the Ellipsis would be changed into a *Parabola*, in which the Body would move, and the Force now tending to a Center at an infinite Distance would become equable, or the same with *Gravity*, according to the Theory of *Galileo*. And if the *Parabola* should be changed into an *Hyperbola*, the Body would move in that Curve by the same Law of the Force now changed from a centripetal to a centrifugal one, because now it causes the Body to recede from the Center.

Pl. LIX. Fig. 1. 24. Lastly: *Let it be required to find the Law of the Force tending to one of the Foci of an Ellipsis.* Draw SP to the Focus S, and PH to the Focus H, and HI parallel to DK. Now because $CS = CH$, we have $SE = EI$; and because the Angle $HPZ = SPR$, (*per Conics*) and HI parallel to PR, therefore the alternate Angle $PHI = PIH$, and so $PI = PH$; consequently $EP = \frac{PS + PH}{2} = AC$, from the Genesis of an Ellipsis. Let the *Latus Rectum* of the Ellipsis be $L = \frac{2BC^2}{AC}$, (because $2AC : 2BC :: 2BC : L$) and Qv intersect PS in x. Then because $QR = Px$, and the Triangle Pxv similar to the Triangle PEC, we have $Px : Pv :: PE (= AC) : PC$; therefore $QR : Pv :: AC : PC ::$

Now

Now though the Planetary Orbits are really *elliptical*, yet is the *Eccentricity* CS, in most of

$L \times QR : L \times Pv$. (*Theorem* I.) Again, $L \times Pv : Gv \times vP :: L : Gv$. (*Theorem* II.) Also, $Gv \times vP : Qv^2 :: PC^2 : DC^2$, (*per Conics*) *Theorem* III. Again, $Qx^2 : QT^2 :: PE^2 : PF^2$; but when the Points P and Q coincide, it is $Qx^2 = Qv^2$, and $PE^2 = CA^2$; wherefore then $Qv^2 : QT^2 :: CA^2 : PF^2$. Now because $PF \times CD = AC \times BC$, (*per Conics*) therefore $PF^2 \times CD^2 = AC^2 \times BC^2$, and so $AC^2 : PF^2 :: CD^2 : BC^2$; consequently $Qv^2 : QT^2 :: CD^2 : CB^2$. (*Theorem* IV.)

25. These four Theorems set separately as below.

THEOREM I. $L \times QR : L \times Pv :: AC : PC$.

II. $L \times Pv : Gv \times vP :: L : Gv$.

III. $Gv \times vP : Qv^2 :: PC^2 : CD^2$.

IV. $Qv^2 : QT^2 :: CD^2 : CB^2$.

It is evident, by joining all the Ratios we have $L \times QR : QT^2 :: AC \times L \times PC^2 \times CD^2 : PC \times Gv \times CD^2 \times CB^2$; but because $AC \times L = 2\,BC^2$, we have $L \times QR : QT^2 :: 2\,PC : Gv$. Now when P and Q coincide, $2\,PC = Gv$, and then $L \times QR = QT^2$; and multiplying each Side by $\frac{SP^2}{QR}$, we shall have $L \times SP^2 = \frac{SP^2 \times QT^2}{QR}$. Therefore the centripetal Force is as $L \times SP$ inversely; or, because L is a given Quantity, it will be directly as $\frac{1}{SP^2}$.

26. I shall now shew what Ratio the projectile Force which causes a Body to describe a *Circle* has to that which (*cæteris paribus*) causes the Body to describe any *Conic Section*. Let us assume this Ratio to be that of n to 1; and putting $2a$ and $2b$ for the transverse and conjugate Diameters of the Conic Section AN, the Circle being AIH, suppose the Right Line EF to move parallel to itself, and the Points a and d therein so as to describe the Curves AI and AN; and let the Distance of that Line from AB be call'd x, *viz.* $AE = x$; and let $2d = AH$ the Diameter of the Circle. Pl. LIX. Fig. 2.

27. Now $\sqrt{2dx - xx} = Ed$ in the Circle, and $\frac{b}{a} \times \sqrt{2ax \mp x^2} = Ea$ in the Conic Section. The Fluxions of the Ordinates Ed and Ea, *viz.* $\frac{\overline{d-x} \times \dot{x}}{\sqrt{2dx - xx}}$ and $\frac{b}{a} \times$

them, ſo extremely ſmall, as to be almoſt inſenſible; and therefore their Motions may be look'd

$\frac{\overline{a \mp x} \times \dot{x}}{\sqrt{2ax \mp xx}}$, will be as the Velocities in every Point of the Curves in the Direction EF or AB. But theſe Fluxions are as $\frac{d-x}{\sqrt{2d-x}}$ and $\frac{b}{a} \times \frac{a \mp x}{\sqrt{2a \mp x}}$, (dividing by $\frac{\dot{x}}{\sqrt{x}}$) and therefore when EF arrives to AB, or $x = o$, the Ratio of thoſe Fluxions or Velocities will become that of $\frac{d}{\sqrt{2d}}$ to $\frac{b}{a} \times \frac{a}{\sqrt{2a}}$, or as $\sqrt{d}$ to $\frac{b}{\sqrt{a}}$ in the Point A. Wherefore $\sqrt{d} : \frac{b}{\sqrt{a}} :: 1 : n$; whence we have $nnad = bb$.

28. And when $x = d = AC$, the Diſtance of the Center of Force, we have $\frac{b}{a}\sqrt{2ax \mp xx} = p = \frac{bb}{a}$ become $2ad \mp dd = bb = nnad$. Whence we get $a = \frac{\pm d}{2 - n^2}$ and $b = \frac{\pm nd}{\sqrt{2-n^2}}$. Having therefore the Diameters $2a$ and $2b$, the Conic Section is given in Specie.

29. Now becauſe Unity, or 1, repreſents the projectile Force to deſcribe a Circle, the Force n may be any other Number greater or leſs to deſcribe a Conic Section. And firſt let $n^2 = 2$; then will $a = \frac{\pm d}{2 - n^2} = \frac{\pm d}{o} =$ Infinite, or the Center of the Curve will be at an infinite Diſtance from A, and conſequently be the *Parabola* AN.

30. If the Value of n^2 be between 1 and 2, or if n be any Number between 1 and $\sqrt{2}$, then will the Conic Section be an Ellipſe between the Circle AEFH and the Parabola AN, having the Center of Force C in the upper Focus next A, as the Ellipſe ALMK.

31. But if n be any Number leſs than 1, the Curve will ſtill be an Ellipſe, but within the Circle, having the Center of Force C in the lower or remote Focus, as the Ellipſis AIGO.

upon

upon as *circular*, and as such represented in Or-

32. Again; if n^2 be greater than 2, or n greater than $\sqrt{2}$, then will a be negative; consequently the Curve will be an *Hyperbola*, as AO.

33. Lastly; if $n^2 = o$, then $b = \frac{\pm nd}{\sqrt{2 - n^2}} = o$, and $a = \frac{1}{2}d$; that is, if the projectile Velocity be diminish'd *ad infinitum*, then the Curve or Trajectory will become the Right Line AC; or the Projectile will descend directly to the Center of Force C.

34. Let A = the Area of any Ellipse, S, *S*, *s*, the Areas of the Sectors ASB, BSC, CSD, &c. and T, *T*, *t*, the Times in which they are described; then we have S : *S* :: T : *T*, and S : *s* :: T : *t*, and so on for every Sector through the whole Area. Therefore S : T :: S + *S* + *s* : T + *T* + *t* :: Sum of all the Sectors : Sum of all the Times in which they are described; so is the whole Area A to the Periodical Time P of a whole Revolution. Consequently, S × P = T × A, and $P = \frac{A \times T}{S}$; and in a given Particle of Time T, we have P as $\frac{A}{S}$. Pl. LIX. Fig. 4.

35. By *Art.* 25. we have the principal *Latus Rectum* $L = \frac{QT^2}{QR}$, but in a given Time the centripetal Force QR is as $\frac{1}{SP^2}$; wherefore in a given Time L : $QT^2 \times SP^2$; and so $L^{\frac{1}{2}} : QT \times SP : S$, the Sector ASB described in a given Time. Whence P : $\frac{A}{L^{\frac{1}{2}}}$; therefore A : P × $L^{\frac{1}{2}}$, that is, *The Area of an Ellipse is in the Subduplicate Ratio of the* Latus Rectum *and Periodical Time conjointly.*

36. Now let a = Transverse Axis, and b = Conjugate; then (by *Conics*) $a : b :: b : L$, and so $b^2 = aL$, and $b = a^{\frac{1}{2}} \times L^{\frac{1}{2}}$; whence $ab = a^{\frac{3}{2}} \times L^{\frac{1}{2}}$. But the Rectangle $a \times b$: A, the Area of the Ellipse, (by *Conics*) therefore $a^{\frac{3}{2}} \times L^{\frac{1}{2}}$: A : P × $L^{\frac{1}{2}}$, (by *Art.* 35.) that is, $a^{\frac{3}{2}}$: P; or, *The Periodical Time is in the Sesquiplicate Ratio of the Transverse or greater Axis of the Ellipse.*

37. Hence the Periodical Time will be the same in all the

reries and Diagrams, without any ſenſible Error.

Species of an Ellipſis from a Right Line to a Circle deſcribed upon the ſame tranſverſe Diameter; or, more particularly, the Time of deſcribing the Semi-Ellipſe AED will be the ſame as that of the Semi-Ellipſe AOD; and the ſame alſo as the Time of deſcribing the Semi-Circle APD, which is only one Species of an Ellipſis, where the Foci coincide with the Center N, and the Semi-Conjugate NO becomes the Semi-Diameter NP. Laſtly, when the Semi-Ellipſe AOD degenerates into a Right Line AD by diminiſhing the Semi-Conjugate NO *in infinitum*, and the Focus receding to the End of the Axis at D, it is plain the Time of deſcribing the Line AD is ſtill the ſame.

Pl. LIX. Fig. 5. 38. The Velocity of the revolving Body P is as $\frac{SY^2}{L}$, SY being a Perpendicular let fall on the Tangent PY from the Center of Force S; for the Velocity is ever as the ſmall Arch QP deſcribed in a given Time. But QP = PR, in its evaneſcent State; and becauſe of the Right Angles at T and Y, and the Angle QPT = YPS when the Points Q, P, coincide, the evaneſcent Triangle QPT will be ſimilar to PSY; and therefore give QP (= PR) : QT :: PS : SY; whence $PR = \frac{SP \times QT}{SY}$. But $SP \times QT : L^{\frac{1}{2}}$; therefore PR : $\frac{L^{\frac{1}{2}}}{SY}$. That is, *The Velocity is in the Subduplicate Ratio of the* Latus Rectum *directly, and the Perpendicular inverſely.*

39. Hence the Velocities in the greateſt and leaſt Diſtances A and D are in the Ratio compounded of the Diſtances SA and SD inverſely, in the ſame Figure where L is a given Quantity; becauſe in that Caſe the Diſtances are the Perpendiculars.

40. Therefore if a Circle DECF be deſcribed at the ſame Diſtance SD, becauſe the Circle is that Species of Ellipſis whoſe *Latus Rectum* is equal to the Diameter 2 DS, and ſince in this Point D the perpendicular Diſtance is the ſame in both, the Velocity of the Body in the Ellipſis at the Point D is to that of a Body deſcribing the Circle in the Subduplicate Ratio of L to 2 DS, or as $\sqrt{L}$ to $\sqrt{2\,DS}$; and the ſame may be ſhewn with reſpect to the Velocities at the other Point A.

41. To compare the Velocity in the Ellipſe at the mean

THE

THE ORRERY is, therefore, an adequate Representation of the TRUE SOLAR SYSTEM, and

Distance B with that of a Body describing a Circle EF at the same Distance CB from the common Focus S, let R = Radius of the Circle = AC = CD = SB, and let B = lesser Semi-axis BC, which is here equal to the Perpendicular SY to the Tangent in the Point B. Let the Velocity in the Ellipse be V, and in the Circle v; and as L = *Latus Rectum* of the Ellipse, so 2 A is that of the Circle; therefore (*Art.* 38.) Pl. LIX. Fig. 6.

$V : v :: \frac{L^{\frac{1}{2}}}{B} : \frac{\overline{2A}^{\frac{1}{2}}}{A}$, or $V^2 : v^2 :: \frac{L}{B^2} : \frac{2A}{A^2} :: L \times A : 2B^2$.

But because (by *Conics*) A : B :: 2 B : L, therefore $2B^2 = A \times L$; consequently $V^2 = v^2$, and so $V = v$. That is, *The Velocity of the Body in the Ellipse in the Point* B *is equal to that in the Circle* EF *described with the mean Distance* SB.

42. It has been already shewn (*Art.* 29.) that the Velocity of a Body in the Vertex of a *Parabola* is to that in a *Circle* at the same Distance from the Focus, as $\sqrt{2}$ to 1. And because every thing that has been shewn relating to the Motion in an Ellipse may be demonstrated also of the *Parabola* and *Hyperbola*, (See *Princip.* Lib. I. Prop. XII, XIII.) therefore in the *Parabola* the Velocity will be every where at P as a Perpendicular SY let fall upon the Tangent PY reciprocally. And (by *Conics*) $SY^2 : SP$, and so $SY : \sqrt{PS}$; therefore, Fig. 7.

The Velocity in the Parabola will be every where as $\frac{1}{\sqrt{SP}}$, *or in the Subduplicate Ratio of the Distance inversely.*

43. We have also shewn (*Annot.* XXXIV. 13.) in a Circle whose Radius is a, P = Periodical Time, V = Velocity, that $VP = a$, and $V = \frac{a}{P}$, and therefore $V^2 = \frac{a^2}{P^2}$; but also $P^2 : a^3$, (*ibid.* 11.) whence $V^2 : \frac{a^2}{a^3} : \frac{1}{a}$; therefore $V : \sqrt{\frac{1}{a}}$. Therefore the Velocity (V) in the Circle AGHI is to the Velocity in the Circle EPF described with the Radius SP, as $\sqrt{\frac{1}{AS}}$ to $\sqrt{\frac{1}{SP}}$; or $V : v :: \sqrt{SP} : \sqrt{AS} = \sqrt{\frac{1}{4}L}$. But the Velocities in the Points A and P in the *Parabola* also are in the same Ratio of $\sqrt{SP}$ to $\sqrt{\frac{1}{4}L}$ (by 42.);

 gives

gives a just Idea of the *Number*, *Motions*, *Order*, and *Positions* of the heavenly Bodies: But the Pro-

consequently, *The Velocity in the Parabola at the Vertex* A *is to the Velocity in the Circle in the same Distance* AS, *as the Velocity in the Parabola at* P *is to the Velocity in the Circle described at the same Distance* SP; *that is, in the Ratio every where of* $\sqrt{2}$ to 1.

44. Again; the Velocity in the Circle whose Radius is $\frac{1}{2}$SP is to the Velocity in a Circle whose Radius is SP, as $\sqrt{SP}$ to $\sqrt{\frac{1}{2}SP}$, or as $\sqrt{2}$ to 1; consequently, *The Velocity in the Parabola at* P *is equal to the Velocity in a Circle whose Radius is* $\frac{1}{2}$SP.

Pl. LIX. Fig. 8. 45. The angular Velocity of a Body P revolving in any Orbit, that is, the Angle which is made at the Center S, *viz.* PSQ, by the *Radius Vector* SP describing in a given Time the Arch PQ, is as QT directly, and as SP inversely; that is, the Angle $PSQ : pSq :: \frac{QT}{PS} : \frac{qt}{pS}$. This is easy to understand when we consider, that any Angle is greater as the Arch PQ or pq, described in a given Time, is so; and less in Proportion to the Distance SP and sp, because the Velocities with which those Arches are described are inversely as the Perpendiculars SY, Sy, to the Tangents in those Points; and when the Arches QP and qp are indefinitely small, we may esteem them equal to the Lines QT and qt. Whence the Proposition is evident.

46. Hence the angular Velocity at P and p is as $\frac{1}{SP^2}$ and $\frac{1}{Sp^2}$; for the Sectors PSQ and pSq, being described in the same Time, are equal; whence $QT \times SP = qt \times Sp$. Therefore $QT : qt :: Sp : SP$; and hence $\frac{QT}{SP} : \frac{qt}{Sp} :: \frac{Sp}{SP} : \frac{SP}{Sp} :: Sp^2 : SP^2 :: \frac{1}{SP^2} : \frac{1}{Sp^2}$.

Fig. 9. 46. From the Foci S, s, of the Ellipse ABD let fall the Perpendiculars SY, sy, to the Tangent Yy in the Point P; let the centripetal Force tend to the Focus S; and let CB be the lesser Semi-axis. Then will the Velocity (v) in B be to the Velocity (V) in P, in the Ratio of $\sqrt{sP}$ to $\sqrt{SP}$. For $V : v :: CB : SY$, (*Art.* 11.) whence $V^2 : v^2 :: CB^2 : SY^2$,

portion

portion of *Magnitude* and *Distances* of the Planets is not to be expected from the Orrery, but by

But (by *Conics*) $BC^2 = SY \times sy$; therefore $V^2 : v^2 :: SY \times sy : SY^2 :: sy : SY$. But because of the similar Triangles SPY and sPy, it is $sy : SY :: sP : SP$; wherefore $V^2 : v^2 :: sP : SP$; consequently $V : v :: \sqrt{sP} : \sqrt{SP}$.

47. From what has been said it appears, that the Motion of a Planet in its Orbit is very unequal and anomalous; and this Anomaly or Irregularity of the Planet's Motion is in itself very irregular also, being sometimes more, and sometimes less than at others. And in order to explain this, it will be requisite to compare it with an equal and uniform Motion of a Body moving in a Circle. Let therefore the Ellipse AEBF be the Orbit of a Planet, whose Focus is S, its greater Axis AB, and lesser OQ. On the Center S, and with the Distance SE, (which is a mean Proportional between AK and OK, the two Semi-axes) describe the Circle CEGF. The Area of this Circle will be equal to the Area of the Ellipse, as I have shewn in my *Elements of Geometry*. Pl. LIX. Fig. 10.

48. In this Circle let us suppose a Point to move with an uniform or equal Motion through the Periphery CEGF, in the same Time that the Planet describes the Ellipse; and when the Planet is in its *Aphelium* A, let the circulating Point be in C, and the Motion of this Point will represent the equal or mean Motion of the Planet; and the Point will describe round S Areas proportional to the Times, and equal to the elliptic Areas the Planet at the same time describes.

49. Let now the equal Motion or angular Velocity in the Circle be CSM, and take the Area ASP equal to the Sector CSM; and then the Place of the Planet in its Orbit will be P; and the Angle MSD, the Difference between the true Motion of the Planet and its mean Motion, is the Equation, and is call'd the *Prosthaphæresis*, from its being *added to* or *taken from* the mean Motion, to obtain the true or equated Anomaly.

50. Hence the Area ACDP will be equal to the Sector DSM, and therefore proportional to the *Prosthaphæresis*; and consequently where this Area is biggest, there the *Prosthaphæresis* or Equation will be greatest, or a *Maximum*; which evidently happens when the Planet arrives at E, where the Ellipse and the Circle cut each other. For when the Planet descends farther to R, the Equation becomes proportional to the Difference of the Areas ACE and mER, or to the Area GBRm; for when the Planet is at R, let the Point be at V,

Delineation, as in Mr. *Whiston*'s *Solar System*;

and the Sector CSV will be equal to the elliptic Area ASR, that is, ACE + CERS = CERS + mER + mSV; consequently ACE — mER = mSV = mRBG.

51. In the *Perihelion* the equal Motion and the true Motion of the Planet coincide, because the Semicircle CEG and Semi-ellipse AEB are equal, and are described in the same Time. As the Planet descended from the *Aphelium* A to the *Perihelium* B, its Motion was slower, or less than the mean Motion; in which Case the Equation or *Prosthaphæresis* is to be *subtracted* from the mean Motion, to get the true Motion and Place of the Planet.

52. But during the Ascent of the Planet from the *Perihelium* B to the *Aphelium* A, its Motion will be quicker than the mean Motion, as might be shewn in the same Manner as above. In A the Velocity is least of all, and in B greatest, as we have shewn; and in E it is equal to the mean Velocity in the Circle. For when the Planet is in E, let the Point be in m, and let the Area ESn and Sector mSi be described in the same infinitely small Particle of Time, and therefore equal to each other; for Eb × Sn = (Eb × SE =) mi × mS; but SE = mS, therefore Eb = mi; therefore the angular Velocity ESb at E is equal to the angular Velocity mSi, which is the mean Velocity.

53. In order therefore to find the Equated or true Anomaly from the Mean, we are to find the Position of a Line SP that shall cut off the elliptic Area ASP, to which the whole Area of the Ellipse has the same Proportion as the whole Periodical Time of the Planet has to the Time given in which the elliptic Sector was described. Or if AQB be a Semicircle described on the longer Axis of the Ellipse, we must draw from S the Line SQ, which shall cut off the Area ASQ, to which the Area of the whole Circle is in the above-mentioned Ratio; for then a Perpendicular QH will cut the Ellipse in P, so that the Line PS being drawn, the elliptic Area ASP will be to the Sector ASQ as the whole Area of the Ellipse to that of the Circle, as is shewn.

Pl. LIX. Fig. 11.

54. To cut an Ellipse or Circle in this Proportion was the famous Problem long since proposed by *Kepler*, which is solved as follows. Upon QC, produced if required, let fall the Perpendicular SF; the Area ASQ is equal to the Sector ACQ and the Triangle QSC, that is, equal to $\frac{1}{2}$QC × AQ + $\frac{1}{2}$QC × SF; and because $\frac{1}{2}$QC is a constant Quantity, the Area ASQ will be proportional to AQ + SF. Hence if

where

where the ſeveral Orbits of the Planets are laid down in their proportional Diſtances from

we take the Arch $QN = SF$, we have the Arch AN proportional to the Time or mean Anomaly of the Planet; which we can eaſily find by having the true Anomaly given.

55. For Example; in the Orbit of *Mars* we have $QC : SC :: 152369 : 14100$; and becauſe the Length of an Arch equal to Radius is $57°,29578$, ſay,

As the Radius	$QC =$	$152369 =$	5.182985
Is to the Eccentricity	$SC =$	$14100 =$	4.149219
So is the Length of the Arch		$57°,29578 =$	1.758078
To the Length of an Arch B,		$5°,302 =$	0.724312

Then ſay,

As Radius SC	$90° \; 00 =$	10.00000
Is to the Sine SF of the Angle $SCF = ACQ$, which ſuppoſe	$30° \; 00 =$	9.698970
So is the Length of the Arch $B =$	$5°,302 =$	0.724312
To that of the Arch $QN = SF =$	$2°,651 =$	0.423282

56. Therefore $AQ + QN = 30° + 2°,651 = 32° \; 39' \; 3''$. Thus from the eccentric Anomaly ACQ we gain the mean Anomaly $AQ + QN = AN$, which is proportional to the Time; and the Reverſe of this, *viz.* from the mean Anomaly AN given, to find the eccentric Anomaly ACQ, is to be done by the Method of *Infinite Series*, as follows. Let the Arch $NQ = y$, the Sine of the Arch AN be $= e$, the Co-ſine $= f$, and the Eccentricity $SC = g$. The Sine of the Arch AQ is equal to the Sine of the Arch $AN - NQ$, equal to the Sine of the Arch $AN - y$, which Sine is thus expreſſed by a Conv. Series, $e - \frac{fy}{1} - \frac{ey^2}{1.2} + \frac{fy^3}{1.2.3} - \frac{ey^4}{1.2.3.4}$, &c. as Dr. *Keill* has ſhewn in his *Trigonometry*.

57. Call that Series s, then Radius (1) : Sine of $AQ \; (s)$ :: $SC \; (g) : SF = (y) \; NQ$; therefore $y = gs = ge - \frac{gfy}{1} - \frac{gey^2}{1.2} + \frac{gfy^3}{1.2.3} + \frac{gey^4}{1.2.3.4}$, &c. Conſequently we have $ge = y + \frac{gfy}{1} + \frac{gey^2}{1.2} - \frac{gfy^3}{1.2.3} - \frac{gey^4}{1.2.3.4}$, &c. Let $ge = z$,

the

the Sun; and their Magnitudes comparatively with each other, and with that of the Sun, ex-

$\overline{1+fg} = a$, $\frac{ge}{2} = b$, $\frac{gf}{1.2.3} = c$, $\frac{ge}{1.2.3.4} = d$; and the Equation will become $z = ay + by^2 - cy^3 - dy^4$, &c. which reverted gives $y = \frac{1}{a}z - \frac{b}{a^3}z^2 + \frac{2b^2 + ac}{a^5}z^3 - \frac{5abc - 5b^3 + a^2 d}{a^7}z^4$, &c. Or, by substituting the Values of b and d, $y = \frac{1}{a}z - \frac{1}{2a^3}z^3 + \frac{c}{a^4}z^3 - \frac{5c}{2a^6}z^5$, &c.

58. But if the Arch AN be greater than 90 Degrees, and less than 270, then $ge = z = y - \frac{gfy}{1} + \frac{gey^2}{2} + \frac{gfy^3}{6} - \frac{gey^4}{24}$, &c. And then $a = 1 - fg$, and $y = \frac{z}{a} - \frac{z^3}{2a^3} + \frac{cz^3}{a^4}$, &c. This Series expresses the Arch QN in Parts, whereof the Radius contains 100000; but to have it in Degrees and Parts of a Degree, say, As Radius (1) is to this Series (s), so is the Radial Arch 57°,29578 (R) to QN $= y$ in Degrees; that is, $y = s\text{R} = \frac{\text{R}}{a}z - \frac{\text{R}}{2a^3}z^3 + \frac{\text{R}c}{a^4}z^3$, &c.

59. Now the very first Term of this Series $\frac{\text{R}}{a}z$ is sufficient to determine the Anomaly of the Eccentricity in almost all the Planets nearly enough; for in the Earth's Orbit, where CQ : CS :: 1 : 0,01691, the Error is only a 10000 Part of a Degree. For Example, Let the Arch AQ $= 30°$;

Then	The Log. of the Eccentricity CS $= g =$	8.228144
	The Log. of the Sine of AN $= e = 30° =$	9.698970
	The Log. of Radial Arch R $= 57°,295 =$	1.758122
	The Sum is the Log. $ge \times$ R, or Rz $=$	9.685236
	Subduct the Log. of $a = 1 + fg$ $=$	0.006314
	There remains the Log. of $\frac{\text{R}z}{a} = y = 0,4774 =$	9.678922

But 0,4774 Parts of a Degree are equal to 28′ 38″; therefore AN − NQ $= 30° - 28' 38'' = 29° 31' 22'' =$ AQ

press'd

prefs'd by the outmoft Circle of the Scheme (CXLI).

or Angle ACQ, the eccentric Anomaly. In the Triangle QCS, having two Sides QC and CS, and the included Angle given, we find the Angle $CSQ = 29^\circ\ 3'\ 7''$.

60. Now making $zCS = SH$ Radius, we have $QH : PH$ (:: CE ($= AC$) : CD) :: Tangent of ASQ : Tangent of $ASP = 29^\circ\ 2'\ 54''$, the equated or co-equated Anomaly required. And that this is fufficiently near the Truth, let us fee the Value of the fecond Term of the Series, *viz.* $\frac{Rz^3}{2a^3}$.

Thus, the Logarithm of $\frac{z}{a}$	$= .7.920800$
Multiply by — — —	2
The Product is the Logarithm of $\frac{z^2}{a^2}$	$= .5.841600$
Add the Logarithm of $\frac{Rz}{a}$	$= 9.678922$
The Sum is the Logarithm of $\frac{Rz^3}{a^3}$	$= .5.520522$
Subduct the Logarithm of 2	$= 0.301030$
The Logarithm of the fecond Term $\frac{Rz^3}{2a^3}$	$= .5.219492$

To which Logarithm anfwers the Number 0,000016, or the $\frac{16}{1000000}$ Part of a Degree; too fmall to be regarded. And in the Orbit of *Mars* and *Mercury* the two firft Terms $\frac{Rz}{a} - \frac{Rz^3}{2a^3}$ will determine the Value of y to more than any neceffary Degree of Exactnefs.

(CXLI) 1. The ORRERY (though a modern Name) has fomewhat of Obfcurity in refpect of its Origin, or Etymology; fome Perfons deriving it from a *Greek* Word which imports to *fee* or *view*, becaufe in it the Motions of the Heavenly Bodies are all reprefented to the View, or made evident by Infpection: But others fay that Sir *Richard Steele* firft gave this Name to an Inftrument of this Sort, which was made by Mr. *Rowley* for the late Earl of *Orrery*, and fhew'd

THE

The principal Use of the Orrery is to render the Theory of the *Earth* and the *Moon* easy and intelligible; and to evidence to our Senses how all those Appearances happen, which depend on the *annual* or *diurnal Rotation* of the *Earth*, and the *monthly Revolutions* of the *Moon:* As, the Variety of Seasons, the Vicissitudes and various Lengths of Days and Nights, the Manner of Solar and Lunar Eclipses, the various Phases of the Moon, *&c.*

In my *Orrery*, which is of a peculiar and most elegant Structure, the *Earth* in its annual Motion passes round by a Circle, on which is engraved the *Calendar*, and the *Ecliptic*; and the Plate which carries the Earth about has an Index on the opposite Part from the Earth, to shew the apparent

only the Movement of one or two of the Heavenly Bodies. From hence many People have imagined that this Machine owed its Invention to that Noble Lord.

2. But the Invention of such Machines as we now call ORRERIES, and PLANETARIUMS, is of a much earlier Date. The first we have any Mention of is that of *Archimedes*, generally call'd *Archimedes*'s SPHERE; though it was more than what we now-a-days call a SPHERE, which is an Instrument consisting only of large and small Circles artfully put together; but this famous Machine of *Archimedes* was of a more complex Nature, and consisted of a Sphere, not of Circles, but of an hollow globular Surface of Glass, within which was a Piece of Mechanism to exhibit the Motions of the Moon, the Sun, and the Five Planets. This *Cicero* asserts in his *Tusculan Questions*.

3. But the most copious and accurate Description of this Sphere is that of *Claudian*, in *Latin* Verse. Thus the Poet sings:

Jupiter in parvo cum cerneret æthera vitreo,
Risit, & ad Superos talia dicta dedit.
Huccine mortalis progressa potentia curæ?
Jam meus in fragili luditur orbe labor.

Place

Place of the Sun in the *Ecliptic*, for every Day of the Year; and one Turn of the Winch carries the Earth once round its Axis, and the said Index over the Space of one Day in the Calendar: So that by this means the true Place of the Earth, and the apparent Place of the Sun, also the Place and Phases of the Moon, may be readily shewn for any Day required.

THE *Orrery-Part*, containing the *Wheel-Work*, is placed within a large and most beautiful ARMILLARY SPHERE, which turns about upon its Axis. with a fairly-engraved and silver'd Horizon, which is also moveable every way upon a most elegant Brass Supporter, with four Legs richly wrought; at the Bottom of which is a noble large silver'd Plate, with a BOX and NEEDLE, and COMPASS,

Jura poli, rerumque fidem, legesque Deorum,
Ecce Syracusius transtulit arte senex.
Inclusus variis famulatur spiritus astris,
Et vivum certis motibus urget opus.
Percurrit proprium mentitus Signifer annum,
Et simulata novo Cynthia mense redit.
Jamque suum volvens audax industria mundum
Gaudet, & humanâ sidera mente regit.
Quid falso insontem tonitru Salmonea miror?
Æmula Naturæ parva reperta manus.

4. This Machine appears from hence to have been sufficiently grand and universal, as comprehending all the Heavenly Bodies, and exhibiting all their proper Motions; which is all that can be said of our common modern Orreries. 'Tis true, this Orrery of *Archimedes* was contrived to represent the *Ptolemaic* System; but the Mechanism and Nature of the Instrument is the same, whether the System of *Ptolemy*, or *Copernicus*, or any other be represented by it.

5. The next Orrery we have any Mention of is that of *Posidonius* the *Stoic*, in *Cicero*'s Time, 80 Years before our Saviour's Birth: Concerning which the Orator, in his Book *De Nat. Deorum*, has the following Passage.—*Quid si in Scy-*

COMPASS, with the Names of all the *Points* finely engraven in Words at Length. The Circles of the Sphere are as follow.

THE EQUINOCTIAL, which divides the Sphere into two Parts, *viz.* the *Northern* and the *Southern Hemisphere*; and is so call'd, because when the Sun comes to pass over it, (as it does twice every Year) the *Days and Nights are then equal.* This Circle is divided into 360 Degrees, call'd the *Right Ascension* of the Sun or Stars.

THE ECLIPTIC is that great Circle which represents the apparent annual Path of the Sun through the Heavens. It is divided into 12 equal Parts call'd *Signs*, consisting of 30 Degrees each; whose *Names* and *Characters* are as follows: 1. *Aries*, the Ram, ♈; 2. *Taurus*, the Bull ♉;

thiam, aut in Britanniam, Sphæram aliquis tulerit hanc, quam nuper familiaris noster effecit Posidonius, cujus singulæ conversiones idem efficiunt in Sole, & in Luna, & in quinque Stellis errantibus, quod efficitur in Cælo singulis diebus & noctibus; quis in illa barbarie dubitet, quin ea Sphæra sit perfecta Ratione? That is, "If any Man should carry this Sphere of *Posidonius* into "*Scythia* or *Britain*, in every Revolution of which the Mo-"tions of the Sun, Moon, and Five Planets were the same "as in the Heavens each Day and Night, who in those bar-"barous Countries could doubt of its being finish'd (not to "say actuated) by perfect Reason?" What can be a more genuine Account of a compleat Orrery than this? And, by the way, what would *Cicero* say, were he now to rise from the Grave, and see his *Barbarous Britain* abounding in Orreries of various Kinds and Sizes?

6. From this Time we hear no more of Orreries and Spheres, till about 510 Years after *Christ*, when the famous *Severinus Boethius*, the *Christian* (though *Roman*) Philosopher, is said to have contrived one, which *Theodoric* King of the *Goths* wrote to him about, and desired it for his Brother-in-Law *Gandibald* King of *Burgundy*; in which Letter he calls it *Machinam Mundo gravidam,—Cælum gestabile,—Rerum Com-*

3. Ge-

3. *Gemini*, the Twins, ♊; 4. *Cancer*, the Crab, ♋; 5. *Leo*, the Lion, ♌; 6. *Virgo*, the Virgin, ♍; 7. *Libra*, the Scales, ♎; 8. *Scorpio*, the Scorpion, ♏; 9. *Sagittarius*, the Bowman, ♐; 10. *Capricorn*, the horned Goat, ♑; 11. *Aquarius*, the Waterer, ♒; 12. *Pisces*, the Fishes, ♓. The *Ecliptic* intersects the *Equinoctial* in the Beginning of *Aries* and *Libra*, in an Angle of 23 Degrees, 29 Minutes. In this Circle the Longitude of the heavenly Bodies is reckon'd. The *Ecliptic* is the Middle of

THE ZODIAC, which is a broad silver'd *Zone*, encompassing the Sphere to five Degrees on each Side the *Ecliptic*; so call'd from the Figures of the several *Animals*, or *Constellations of the Signs*, with which it is adorned and embellish'd.

pendium; that is, *a Machine pregnant with the Universe,—a portable Heaven,—a Compendium of all Things.* What more can be said of our Orreries?

7. After this succeeded a long Interval of Barbarism and Ignorance, which so deluged the Literary World, that we find no Instances of Mechanism of any Note till the Sixteenth Century, when the Sciences began again to revive, and the Mechanical Arts to flourish. Accordingly we meet with many Pieces of curious Workmanship about this Time; and in the Astronomical Way particularly is the stately Clock in his Majesty's Palace at *Hampton-Court*, made in *Henry* the Eighth's Time, *A. D.* 1540, by one *N. O.* This shews not only the Hour of the Day, but the Motion of the Sun and the Moon through all the Signs of the Zodiac, with other Matters depending thereon; and is therefore to be esteem'd a Piece of Orrery-Work.

8. Such another is mention'd by *Heylin* at the Cathedral Church of *Landen* in *Denmark*; but the most considerable at this Time is that Piece of Clock-Work in the Cathedral of *Strasburg* in *Alsace*; in which, besides the Clock-Part, is the Celestial Globe or Sphere, with the Motions of the Sun, Moon, Planets, and Fix'd Stars, &c. It was finish'd in the

This

This Zone comprehends within it the *Paths* or *Orbits* of all the *Planets*.

THE MERIDIAN is a great Circle passing through the *Poles*, and cutting the *Equinoctial* at Right Angles; so call'd, because when the Sun is upon any Meridian, it makes the *Meridies*, Mid-Day, or Noon, to all Places under it. Of these, there is one call'd

THE GENERAL MERIDIAN, within which the whole Sphere turns, and upon which are engraven the *Degrees of Latitude*, beginning and proceeding each way from the Equinoctial to the Poles. To this Circle the Sphere is suspended; and being moveable within the Horizon, the Sphere may be *elevated* or *rectified for the Latitude of any Place*.

Year 1574, and is much superior to that pompous Clock at *Lyons*, which also contains an Orrery-Part.

9. About the Beginning of the Seventeenth Century this Sort of Mechanism began to be greatly in Vogue, and Spheres and Orreries were now no uncommon Things; though Orreries bore an excessive Price till very lately. The first large one made in *London* by Mr. *Rowley* was purchased by King *George* I. at the Price of 1000 Guineas; nor has any of that large Sort, which contains all the Movements of Primaries and Secondaries, been sold for less than 300*l*. at any Time since.

10. There have been various Forms invented for this noble Instrument, two of which have principally obtain'd, *viz.* the *Hemispherical Orrery*, and the *Whole Sphere*; though the Orrery at first was made without any Sphere, and with only the Sun and the Earth and Moon revolving about it; but this was too imperfect a State, and they soon began to invest it, some with a *Half-Sphere*, some with a *Whole* or *Compleat Sphere*; for otherwise it could not be an adequate Representation of the Solar System.

11. The Hemispherical Orrery has been made in greater Numbers than any other, on account of their being made

THE HORIZON is that broad ſilver'd Frame, or Circle, which contains the whole Machine, moveable every way within it. It is ſo call'd becauſe it bounds our Sight in the Heavens, and divides the Sphere into *the upper and lower Hemiſphere*. Upon this Circle are curiouſly engraven the *Ecliptic Signs* and the *Calendar*, for readily finding the Sun's Place for any given Day or Time. On this Circle is alſo reckon'd the *Amplitude of the Sun*, &c.

THE Points where the Ecliptic interſects the Equinoctial are call'd the *Equinoctial Points*, or EQUINOXES, becauſe when the Sun is in them, *the Days and Nights are equal*. As the Sun is in one of them in the *Spring*, it is call'd the *Vernal*

much cheaper and eaſier than thoſe in a Sphere of the ſame Size; there being a vaſt Difference between *placing an Hemiſphere on the Box of an Orrery*, and *diſpoſing an Orrery in a large moveable Sphere*. But then the Idea given us by the former is very unnatural and imperfect; and 'tis ſurprizing to think they ſhould have ſuch a Run as they had, Mr. *Wright* having made between forty and fifty of that Sort ſince the Death of Mr. *Rowley* his Maſter. And though I incline to think few more of that Form will be made, yet as they have had ſo great a Name, I have thought proper to give the Reader a View of one in a Print.

12. This ill-judged and erroneous Form of an Orrery had this Effect with thoſe who knew the Nature of ſuch Machines very well, that ſome applied themſelves to conſtruct Orreries in a Compleat Sphere, others invented ſuch Inſtruments as ſerved to exhibit the Motions of the Heavenly Bodies ſeparately, which they accordingly call'd PLANETARIUMS, LUNARIUMS, &c. and others declared againſt all Orreries in general, as giving falſe Ideas of the Syſtem of the World; eſpecially as the Magnitudes and Diſtances of the Heavenly Bodies could not be repreſented by them in their proper Proportions.

13. But they muſt be ſuppoſed to reaſon very weakly, who

Equinox; and in the other at *Autumn*, it is call'd the *Autumnal Equinox*.

THE Beginning of *Cancer* and *Capricorn* are call'd the *Solſtitial Points*, or the SOLSTICES; which is as much as to ſay, *the Stations of the Sun*, becauſe when the Sun is in thoſe Points, he ſeems *ſtationary*, or *not to move* for ſome Days: The firſt is the *Summer*, the other the *Winter Solſtice*.

THE Meridians which paſs through the *Points* above-mention'd are call'd the *Equinoctial* and *Solſtitial* COLURES reſpectively. They divide the Sphere into *four Quarters*, in the Middle of the *four Seaſons of the Year*.

THE Leſſer Circles of the Sphere are the TROPICS and POLAR CIRCLES; which are all parallel

object an inconſiderable Deficiency in any Inſtrument, againſt its moſt important Uſes. No one ever decried an Air-Pum[p] becauſe an abſolute *Vacuum* was impoſſible by it; or th[e] of a Teleſcope, becauſe we cannot ſee ... habitants ... Planets. And on the other hand, to re... e Solar ... by Parts, or in a piece-meal Manner, ... than m... one of the moſt noble and uniform ... tural ... phy. The ...*nctarian Scheme* th... r. I... ſoon beca... tinct.

14. A... ery then, adapted ... ary ... the only ... e that can ex... ea... Syſtem ... orld, with th... an... of the ... Bodies; an... Kind ... : Yea furth... ble o... the *third* ... the A... here; I ... whi... s of the ... the ... d occaſio... ... *uinoxes* ...

to the Equinoctial, and are two on either Side. The *Northern Tropic* is that of *Cancer*; the *Southern*, that of *Capricorn*; as passing thro' the Beginning of those Signs. They are distant from the Equinoctial 23 Degrees, 29 Minutes, and include that Space or Part of the Sphere which is call'd the *Torrid Zone* on the Terrestrial Globe; because the Sun is at one Time or other perpendicular over every Part, and extremely torrifies or heats it.

WITHIN 23 Deg. 29 Min. of each Pole lie the *Polar Circles*; of which that about the North Pole is call'd the *Arctic Circle*, because of the Constellation of the *Bear* in that Part; and the other about the South Pole, the *Antarctic Circle*. They include those Spaces which are call'd the *Frigid Zones*, by reason of the intense Cold which reigns

follows. Let DCH be a Part of the Earth's Orbit, C its Center, EC the Axis of the Ecliptic, E its Pole, CP the Axis of the Earth, P its Pole; through the Points E and P draw the great Circle EPA, meeting the Ecliptic AL in A; the Arch PA measures the Inclination of the Axis of the Earth to the Plane of the Ecliptic, *viz.* the Angle PCH, which is foun[illegible]bservation to be about 66° 30′, and therefo e its [illegible]ental Arch EP or the Angle PCE = 23° 30′. Plate LXV Fig. 1

16. Th[illegible]ole P from the Point E describe a lesser Circle P[illegible]vill be parallel to the Ecliptic; then if the Axi[illegible] be directed at any particular Time to P, it i[illegible]bservations of many Years, that it will not be[illegible]ected to the Point P in the Heavens, but will i[illegible]e be directed to some other Point Q, so that [illegible] = 1 Degree; and therefore in the Space of [illegible]5920 Years, the Point P or Pole of the W[illegible]e the Circle PFG about the Pole of the E[illegible] Revolution is call'd the *Great Year*.

[illegible] of this *Conical Motion* of the Earth's Axis [illegible] all the Astronomers and Philosophers before [illegible]'s Time, none of them being able to guess

in those Regions the greatest Part of the Year. Those Spaces which lie between the Tropics and Polar Circles, on either Side, are call'd the *Temperate Zones*, as enjoying a mean or moderate Degree of Heat and Cold.

THE Circles above are essential to the Sphere; besides which there is the *Quadrant of Altitude*, for shewing the Height of any Luminary above the Horizon; and a large and most beautiful *Horary Circle and Index*, shewing the Time corresponding to the Motion of the Sphere: Also the *Solar Label*, for fixing the Sun to its proper Place in the *Ecliptic*.

IT is easy to conceive, that the Sun will always enlighten *one Half of the Earth*; and that when the Sun is in the Equinoctial, the Circle which

from whence it could proceed: But this divine Geometer soon investigated the Cause thereof, and demonstrated it to result from the Laws of Motion and Gravity, that is, from the *Spheroidical Figure of the Earth*; for were the Earth a perfect Globe, its Axis would always remain parallel to itself, and have no such Motion. See the *Principia*.

18. From this Motion of the Earth's Axis follow several remarkable Phænomena; as *First*, a *constant Change of the Pole-Star*; for 'tis evident, if any Star should chance to coincide with the Pole P at any time, it will after 72 Years be left at the Distance QP, or one Degree Westward, and the Star at Q becomes then the North Pole-Star.

19. *Secondly*, The present Polar Star will in time be on the South Part of our Meridian; that is, the Star, which suppose at present at P, will after 12960 Years be at G, which being 47 Degrees (in the Arch of a great Circle) distant from P, will be on the South Part of the Meridian of *London*, which suppose on the Earth's Surface at *b*. For if TR be the Equator, then the Latitude of *London* $Tb = 51° 30'$; and its Complement $bp = 38° 30'$; therefore $gp - bp = 47° - 38° 30' = 8° 30' = gb$, the Distance of the present North Star towards the South at that Time.

terminates

terminates *the enlighten'd and darken'd Hemiſpheres* (which is call'd *the Circle of Illumination)* will paſs thro' the *Poles of the Earth*, and alſo divide all the *Parallels of Latitude* into two equal Parts. But ſince the Earth moves not in the Plane of the *Equinoctial*, but that of the *Ecliptic*, the Axis of the Earth will be inclined to that of the *Ecliptic* in an Angle of 23 Degrees 29 Minutes; and therefore the *Circle of Illumination* will, at all other Times, divide the *Parallels of Latitude* into two unequal Parts.

Now ſince any Parallel is the Path or Tract which any Place therein deſcribes in one Revolution of the Earth, or 24 Hours; therefore that Part of the Parallel which lies in *the enlighten'd Hemiſphere* will repreſent *the Diurnal Arch*, or

20. *Thirdly*, The Circle EPA paſſing through both the Pole of the Ecliptic and Equator will be the *Solſtitial Colure*, and A the *Solſtitial Point*, when the Axis of the Earth points to P; but after 72 Years, when it points to Q, then the great Circle EQB will be the *Solſtitial Colure*, and B the *Solſtice*, for the ſame Reaſon. And hence alſo the *Equinoctial Points* (which are always 90 Degrees diſtant from the *Solſtices*) muſt move in the ſame Time through the ſame Arch, the ſame Way, *viz.* Weſtward.

21. *Fourthly*, Hence 'tis evident, all the Points of the Ecliptic do move backwards, or Weſtwards, through one Degree every 72 Years; which Motion is ſaid to be *in Antecedentia*, and is contrary to the Order of the Signs: As the other Motion, by which the Planets are carried round the Sun, is ſaid to be *in Conſequentia*, or according to the Order of the Signs, *viz.* from *Aries* ♈ to *Taurus* ♉, *Gemini* ♊, *&c.* And this retrograde Motion of the Equinoctial Points is call'd the *Receſſion of the Equinoxes.*

22. *Fifthly*, This Receſſion of the Equinoctial Points, and indeed of the whole Ecliptic, is the Cauſe of the ſlow apparent Motion of the Fix'd Stars forwards; for ſince the ſeveral Circles of Longitude by which they are referr'd to the

Length of the Day; and that Part in *the dark Hemisphere* will be the *Nocturnal Arch*, or *Length of the Night*, in that Parallel of Latitude.

Hence, when the Orrery is put into Motion, the Earth moving with its Axis always *parallel to itself*, yet always *inclined to the Plane of the Ecliptic*, will sometimes have the *Northern* Parts turn'd more directly to the Sun, and most enlighten'd; and at other times the *Southern* Parts will be so. Hence various Alterations of *Heat and Cold*, and *Length of Days and Nights*, will ensue in the Course of the Revolution of the Earth about the Sun, which will constitute all *the Variety of Seasons*, as will most naturally and evidently be shewn in the Orrery, as follows (CXLII.)

Ecliptic are continually shifting backwards, the Stars, which are immoveable, must with respect to those Circles have their Distance, that is, their Longitude, constantly increasing from the first Point of *Aries*. Thus all the Constellations do continually change their Places at the Rate aforesaid: The bright Star of *Aries*, for Instance, which in *Hipparchus*'s Time was near the Vernal Equinox, is now removed near a whole Sign or 30° Eastward, and is in the Beginning of *Taurus* ♉, and *Taurus* is got into *Gemini* ♊; and thus all the Constellations of the Zodiac have changed their Places, and possess different Signs from what they formerly did.

(CXLII) 1. Though these Things are plain to a Person who has his Eye on an Orrery, while he hears or reads this Account of the Nature and Manner of the Seasons, and the Variety of Day and Night, yet Ideas of this Sort are not so easy to be obtain'd by mere Reading and Cogitation only, unless assisted by a proper Diagram or Representation; which therefore I shall here subjoin and explain.

Pl. LX. 2. Let S be the Sun, ABCD the *Orbis Magnus*, or annual Path of the Earth about the Sun. In this Orbit the Earth is represented in *four* several Positions, in the midst of the

WE will first give the Earth Motion in the first Point of *Libra*; the Sun will then appear to enter *Aries*, and this will be the *Vernal Equinox*; for now, *the Sun being in the Equinoctial*, all Parts of the Earth will be equally enlighten'd from Pole to Pole, and all the Parallels of Latitude divided into two equal Parts by the *Circle of Illumination*. Hence the Days and Nights will be equal, and the Sun's Heat is now at a Mean between the

four Seasons respectively. On the Earth are drawn the several Circles and Lines as follow.

ÆCQ	The Equator.
TOR	The Tropic of *Cancer*.
PML	The Tropic of *Capricorn*.
abc	The North Polar or *Arctic* Circle.
def	The South Polar or *Antarctic* Circle.
EGD	The Parallel of *London*.
NCS	The Earth's Axis.
aCf	The Axis of the Ecliptic Plane.

3. As the Sun is supposed to be at so great a Distance, that the Rays coming from it do arrive at the Earth nearly *parallel*, they will therefore illuminate very nearly one Half of the Globe of the Earth, abstracting from the Refraction of the Air. And if we are supposed to view the Earth circulating about the Sun at a very great Distance in the Positions represented in the Scheme, we shall have all the enlighten'd Part turn'd to the Eye on the Equinoctial Day in the Spring, but on that in the Autumn we see only the dark Part; as on the Summer and Winter Solstices we see only half the light and dark Hemispheres respectively: And accordingly the Earth is thus represented in the Figure.

4. But (as I find by Experience) the best Way to convey an Idea of the Seasons, and Day and Night, is to represent the Earth also in Positions exhibiting the visible Hemisphere equally divided into the light and dark Parts, or semicircular Areas, as in the next Plate; and to compare these both together in the Description. To begin therefore with the Situation of the Earth in the Spring and Autumn. Pl. LXI.

5. In either of these Cases, 'tis evident the Sun is in the Plane of the Equator ÆQ, and therefore equally distant from each Pole of the World; consequently the *Circle of Illumi-* Fig. 2.

 greatest

greateſt and the leaſt: All which Particulars conſtitute that agreeable Seaſon we call the SPRING; the Middle of which is ſhewn by the Index to be the 11*th* of *March*.

As the Earth paſſes on from Weſt to Eaſt, through *Libra*, *Scorpio*, and *Sagittarius*, to the Beginning of *Capricorn*, the Sun will appear from the Earth to move through the oppoſite Signs of the Ecliptic, *viz.* *Aries*, *Taurus*, *Gemini*, to the

nation will paſs through both the Poles, N, S; and therefore every Place at an equal Diſtance on either Side will have an equal Degree of the Sun's Light and Heat. And as the Earth revolves upon its Axis, every Place muſt deſcribe a Circle parallel to the Equator, one Half of which will be in the *light*, the other Half in the *dark* Hemiſphere; and as Parts of the Circle meaſure the Day and Night, it is plain they muſt then be equal. Thus in the Equator, the Diurnal Arch QC is equal to the Nocturnal Arch CÆ; in the Tropics RO and LM are equal to OT and MP; in the Latitude of *England* the Day EG is equal to the Night GD; and ſo in all other Parts.

6. Hence, by the way, we may obſerve, that had the Sun always moved in the Equator, there could have been no Diverſity of Day and Night, and but *one Seaſon* of the Year for ever to all the Inhabitants of the Earth. No Alteration of Heat or Cold, ſo agreeable now both to the Torrid and the Frozen Zones; but the ſame uniform eternal Round of unvariable Suns had been our uncomfortable Lot, every way contrary to that Diſpoſition we find all Mankind form'd with, of being delighted and charm'd with Variety to an extreme Degree. The Obliquity of the Ecliptic is therefore not to be look'd upon as a Matter of Chance or Indifferency, but an Inſtance of Wiſdom and Deſign in the adorable Author of Nature, who does nothing in vain.

7. If we conſider the Earth moving on in its Orbit, with its Axis NS always parallel to itſelf, till it comes into the Summer Situation, we ſhall there ſee, that by this Paralleliſm of the Axis all the Northern Parts of the Earth will be brought towards the Sun, which will in this Caſe be in the
Pl. LX. Pl. LXI. Fig. 1. Plane of the Northern Tropic, and his Rays perpendicular upon it, as at R. The Circle of Illumination aCf will now

Begin-

Beginning of *Cancer*; during which Time, by the inclined Position of the Earth's Axis, the *Northern* Parts will be gradually turn'd towards the Sun, and the *Southern* Parts from it; whence the Sun's Rays will fall more and more directly on the former, and pass through a still less Quantity of the *Atmosphere*; but in the *Southern* Parts, the reverse. Also in the *Northern* Parts the Arches of the Parallels in the *enlighten'd Hemisphere* will

be in such a Site, as to include the North Pole and all about it to the Distance NA $= 23^\circ\ 30'$; and on the contrary to exclude the South Pole S, and Southern Regions to the same Distance Sf. The Northern Climates must therefore now have their *Summer*, and the Southern Climates their *Winter*; as will appear more particularly if we consider,

8. *First*, The Sun-Beams fall more perpendicularly upon any Northern Parallel than upon the same Southern Parallel, and have therefore a shorter Passage through the Atmosphere. Thus, for Instance, in the Parallel of *England* E, let the Rays *i*E, *kg*, be incident on the Atmosphere *mn* in *b* and *l*; then will their Passage *b*E, *lg*, be shorter than it would be in the same Latitude Southwards, and therefore will not be so much refracted, blended, and absorb'd; and consequently their Effect will be more considerable and sensible. Again, as Rays are more perpendicular, they will strike with a greater Force; also the more will fall on a given Space; on both which Accounts their Effect, in respect of Light and Heat, will be greater.

9. *Secondly*, As the Earth revolves about its Axis, every Place in North Latitude will describe a greater Part of its Parallel in the enlighten'd than in the dark Hemisphere; or, in other Words, the Day will be longer than the Night. Thus in the Northern Tropic the Diurnal Arch is RY, the Nocturnal YT, which is less than the other by the Difference YQ. Again, in the Parallel of *London* the Length of Day is shewn by the Arch EZ, of the Night by ZD, which is shorter than the Day by the Difference GZ. And lastly, at the Polar Circle cha it is all Day, no Part of that Parallel lying within the dark Hemisphere aÆf. On which Account it is evident the Light and Heat of the Sun is greater in any Place of North Latitude now than at any other Time of the Year.

conti-

continually increase, and those in the *dark* one decrease, shewing the constant Increase of the Days, and Decrease of the Nights: All which will be in their greatest Degree when the Sun is arrived to *Cancer*; and therefore that will be the Middle of that Season we call SUMMER, in *Northern Latitude*; but in *Southern Latitude* every thing will be the reverse, and their Season *Winter*.

THE *North Frigid Zone* is now wholly en-

lighten'd,

It is therefore now the Midst of the Summer Season in all the Northern Climates.

10. In the Southern Part of the World it is *Winter*, for the same Reasons reversed; *viz.* because the Sun's Rays fall more obliquely there; they therefore pass through a greater Quantity of the Atmosphere, on which account they are more refracted, blunted, and stifled, and their Effect weaken'd. Also a less Quantity of the Solar Rays will fall on a given Space, and each Ray strike with a less Force. And lastly, the Duration of their Presence will be shorter than that of their Absence, or the Day will be shorter than the Night; as in the Southern Tropic the Day is LX, but the Night XP, longer by the Difference MX; which Difference is still greater the farther you go, till you come to the *Antarctic* Circle def, where there is no Day at all, and all within to the South Pole S is involved in Night, of greater or less Duration.

11. For the same Reasons, when the Earth arrives to the opposite Part of its Orbit, it will be SUMMER to all the Southern Climates, and WINTER in the Northern. It is evident this must necessarily happen by the Parallelism of the Earth's Axis, and the Change of her Place in the Orbit: By which means the Sun now illumines that very Half of the Globe which in the other Position was dark; and whence it follows, that in all North Latitudes the Length of the Days *now* are equal to the Length of the Nights *then*, and *vice versa* in South Latitudes. Thus the Day (in the Parallel of *England*) EZ = DZ, the Night in the Summer Season; and the Night now, *viz.* ZD = ZE, the Day at that Time. All which Things are too plain from the Schemes to want farther Explication.

Pl. LX. Pl. LXI. Fig. 3.

12. Thus the Vicissitudes and Variety of the Seasons, and of Day and Night, appear in general; but to exhibit the same

lighten'd, and the Pole turn'd towards the Sun as far as possible; but now as the Earth moves on, the *North* Pole returns, the Diurnal Arches begin gradually to decrease, and the Nocturnal to increase; and of consequence the Sun's Rays fall more and more obliquely, and his Heat proportionally diminishes till the Earth comes to *Aries*, when the Sun will appear in *Libra*; and thus produce an Equality of Light and Heat, of Day

in an especial Manner for any particular Place, as *London*, another Scheme is necessary, wherein the Sphere shall have the same Position with respect to that Place, as the Earth itself has. Thus let ÆNQS be the Earth; Z will be the highest Point, or Place of *London*; HO the Horizon, and N the lowest Point or *Antipodes*; and ÆQ the Equator, TR and PL the two Tropics, ac and df the two Polar Circles, as before. Pl. LXII. Fig. 1.

13. Then when the Sun is in the Plane of the Equator at *e*, the *Semi-diurnal Arch*, or half the Length of the Day, will be represented by ÆC; and that of the Night by CQ, which is equal to the former. In this Case the Angle *e*C*b*, which measures the Altitude of the Sun above the Horizon *b*O, is 38° 30′ = *be*.

14. Again: When the Sun is in the Tropic TR, and consequently nearest to the Zenith of *London*, the Semi-diurnal Arch is then TI, which is longer than the former in the Proportion of the Right Angle ÆNC = 6 Hours, to the obtuse Angle ÆNF = 8 Hours 16 Minutes; NES being an Hour-Circle drawn through the Point I, and intersecting the Equator in E. The Semi-nocturnal Arch is IR, and equal in Time to the Angle ENQ = 3 Hours 44 Minutes, the Complement of the other to 12 Hours.

15. Lastly: When the Sun appears in the Southern Tropic at P, and most remote from the Zenith of *London*, the Semi-diurnal Arch is then PK, equal to the Angle ÆND = 3 Hours 44 Minutes nearly, equal to the Night when the Sun was in the other Tropic; and the Semi-nocturnal Arch KL at this Time is evidently equal to the Semi-diurnal Arch TI at the opposite Time of the Year.

16. Whenever the Sun comes upon the Line NS, representing the Hour-Circle of Six, it is then *Six o'Clock*, as at

and

and Night, to all Parts of the World. This will be the Middle of the Season call'd AUTUMN, and that Day the *Autumnal Equinox.*

BUT as the Earth goes on through *Aries, Taurus,* and *Gemini,* you will see the Sun pass through the opposite Signs of *Libra, Scorpio, Sagittarius.* The *North* Pole is now in the dark Hemisphere, and the *Frigid Zone* is now more and more obscured therein: All *Northern Latitudes* continue

X in the Summer Tropic, before Sun-set at I; and at B in the Winter Tropic, after Sun-set at K. Also when the Sun comes upon the Line ZCN, (which represents the *Prime Vertical,* or *Azimuth* of *East and West*) it is then due *East* and *West,* which happens at Y in the Northern Tropic, after Six in the Morning, and before Six in the Afternoon, and *vice versa* at W in the Southern Tropic.

17. It is found by Observation, that the Air is not absolutely dark, till the Sun is depress'd about 18 Degrees below the Horizon, *viz.* at *i,* that is, till the Angle $bCi = 18^\circ =$ HM; and drawing MV parallel to the Horizon HO, it will represent the Circle at which the *Crepusculum,* or Twilight, begins and ends, in the several Points where it cuts the Parallels of the Sun's Declination, as at G in the Tropic PL, and at F in the Equator. But since RO = PH = 15 Degrees, the Arch OR is less than OV, and so the Tropic TR will not touch the Circle MV at all; which shews that for some Time in the Middle of Summer there is *no dark Night:* And this happens between *May* 12 and *July* 11. See my SYNOPSIS SCIENTIÆ COELESTIS, on a large Imperial Sheet.

18. Moreover it is evident that CF = KG, because PL is parallel to ÆQ; the Time, however, of describing CF and KG will not be the same; from whence it appears there is a certain Parallel in which the Twilight will be the *least of all,* and another in which it will be a *Maximum* or *greatest.* The former is when the Sun has $6^\circ\ 7'$ South Declination, *viz.* in *Libra* ♎ or *Pisces* ♓ $17^\circ\ 30'$, which happens *February* 22, and *September* 27, in the present Age: And 'tis plain the Twilight is greatest of all in the Parallel which touches the Point V, on *May* 12 and *July* 11, as aforesaid. *Note,* How the Time of the least Duration of Twilight is investigated may be seen in the best manner in Dr. *Gregory*'s *Elements*

gradually

gradually turning from the Sun; and his Rays fall more and more obliquely on them, and pass through a larger Body of the Atmosphere: The *nocturnal Arches* continue to increase, and the *diurnal* to decrease: All which contribute to make the dismal dreary Season we call WINTER; the Midst whereof is shewn by the Sun's entering the first Scruple of *Capricorn* on the 10*th* of *December*, as by the Index may be seen.

of Astronomy; and I would have given it here, but that it is very tedious, and in itself a Matter of little Importance.

19. It is a Problem of much greater Consequence and Curiosity, to determine the Ratio or Proportion of Heat which any Place receives from the Sun in any Day of the Year. In order to this it must be considered, that *the Quantity of Heat will be as the Time*, if we suppose the Sun to have the same Altitude; and as the *Sine of the Altitude*, if the Time be the same. Therefore if neither the Time nor Sine of the Altitude be given, the Quantity of Heat will be as the Rectangle or Product of both.

20. Therefore let $a =$ Sine of the Latitude Æz; its Co-sine (or Sine of z N) $= b$; the Sine of NS $= c$, and of its Complement (or Declination) SD $= d$; the Sine of the Hour from Noon (or Angle ÆND) $= x$, its Arch ÆD $= z$, and Radius $= 1$; then is $\sqrt{1 - xx} =$ Co-sine of the Angle zNS, (*viz.* Sine of the Angle DNC) and (*per Spherics*) we have $bc\sqrt{1-xx} \pm ad =$ Sine of the Sun's Altitude SB; which multiplied by the Fluxion of the Arch of Time $= \dot{z}$ will produce the Fluxion of the Sun's Heat, *viz.* $\dot{z} \times \overline{bc\sqrt{1-x^2} \pm ad}$. Or, putting $bc = g$, $\sqrt{1-x^2} = h$, $ad = f$, we have the Fluxion of the Heat $= \dot{z} \times \overline{gh \pm f}$. Plate LXVI. Fig. 5.

21. Now to find the Value of $\dot{z}$, let AB $= z$, BE its Sine, EC the Co-sine, and Radius CB; and suppose FG drawn infinitely near to EB, and BD parallel to AC; then 'tis evident from the similar Triangles EBC and BDG, That EC : BC :: DG : GB, or $h : 1 :: \dot{x} : \dot{z}$, whence $\dot{z} = \frac{\dot{x}}{h}$; wherefore $\frac{\dot{x}}{h} \times \overline{gh \pm f} = \dot{x}g \pm \frac{\dot{x}f}{h} =$ Fluxion of Plate LXIII. Fig. 6.

LASTLY:

LASTLY: As the Earth journeys on from thence through *Cancer*, *Leo*, and *Virgo*, the Sun appears to pass through *Capricorn*, *Aquarius*, and *Pisces*; and all Things change their Face. The *Northern* Climes begin to return, and receive more directly the enlivening Beams of the Sun, whose Meridian Height does now each Day increase; the Days now lengthen, and the tedious Nights contract their respective Arches; and e-

the Heat, whose Fluent is $xg \pm fz$, which therefore is as the Quantity of Heat from Noon to the given Time, as required.

22. From this Theorem we may calculate the Heat of any Day in the Year in any given Latitude required; of which I shall give the several following useful Examples. Let it be required *to express the Heat of an Equinoctial Day under the Equator*. In this Case the Latitude of the Place is Nothing, therefore $a = 0$; consequently $fz = adz = 0$. In the other remaining Part $xg = xbc$, $b = 1$, $c = 1$; therefore the Heat will be as x; and since the Semi-diurnal Arch is 90 Degrees, the Heat of the Half-Day will be as $x = 1$, and of the whole Day the Heat is as 2.

23. *Let the Heat of an Equinoctial Day be required for the Latitude of* 51° 30′; then because in this Case there is no Declination of the Sun, $d = 0$, and so $adz = 0$. And since NS $= 90°$, we have $c = 1$; and for the Semi-diurnal Arch $= 90°$, $x = 1$ also; therefore the Heat is as $b = 0{,}6225$ = Co-sine of the Latitude, which for the whole Day is 1,245, and which is to that under the Equinoctial as $1\frac{1}{4}$ to 2 nearly. At the Pole $b = 0$, therefore the Heat of an Equinoctial Day at the Poles is Nothing. Lastly, in the Latitude of 60° the Heat of such a Day is half that under the Equator, or 1; because then $b = \frac{1}{2}$ Radius, or 0,5.

24. In the next Place, let us *calculate the Heat of the Summer Tropical Day*. Here we have the Time of $\frac{1}{2}$ the Day 8 Hours 12 Minutes nearly; therefore the Arch of the Equator which passes the Meridian in that Time is $123° = z$. And because when Radius is 1 the Circumference is 6,28318; therefore say, As $360 : 6{,}28318 :: z : \frac{6{,}28318\,z}{360} =$ $0{,}01745329\,z$, the Length of the Arch z in the Measure of

very

very thing conſpires to advance the delightful Seaſon of the SPRING, the Midſt whereof is ſhewn by the Earth's returning again to that Point, where firſt we gave it Motion.

ALL theſe Appearances of the Seaſons, &c. are ſhewn as well for the *Southern Latitudes*, where at the ſame Time they happen in Order juſt the reverſe to what we have now obſerved for the *Northern*. Thus, when it is *Summer* with

the Radius. Therefore we have

The Logarithm of	$z = 123° = 2.089905$
The Logarithm of	$0,01745 = 8.241795$
The Logarithm of	$a = 51° 30' = 9.893544$
The Logarithm of	$d = 23° 30' = 9.600700$
Total, the Logarithm of	$adz = 0,6698 = 9.825944$

25. Then for the other Part of the Theorem, viz. xbc, we have

The Logarithm of	$x = 57° 00' = 9.923598$
The Logarithm of	$b = 38° 30' = 9.794149$
The Logarithm of	$c = 66° 30' = 9.962398$
Total, the Logarithm of	$xbc = 0,4788 = 9.680138$

Therefore the Heat of half the Day is $0,6698 + 0,4788 = 1,1486$; and of the whole Day it is 2,2972, almoſt twice as great as that of the Equinoctial Day with us, and greater than the Heat of ſuch a Day to thoſe who live under the Equator.

26. To find the Expreſſion of the Winter Tropical Day, we have the Semi-diurnal Arch $z = 57°$, and the reſt the ſame as before. Therefore

The Logarithm of	$z = 57° = 1.755875$
The Logarithm of	$0,01745 = 8.241795$
The Logarithm of	$ad = 9.494244$
Total, the Logarithm of	$adz = 0,3104 = 9.491914$

Then $xbc - adz = 0,4788 - 0,3104 = 0,1684$, and ſo $2 \times 0,1684 = 0,3368 =$ Heat of the whole Day, which is almoſt 7 times leſs than that of the Summer Tropic.

27. The Sum of the Heat of the two Tropical Days is $2,2972 + 0,3368 = 2,634$; which is greater than the Heat

us,

us, it is *Winter* with them, and they have their Days shortest when ours are longest; and *vice versa*. All which is most distinctly seen in the Orrery.

At the same Time the Earth is going round the Sun, the Moon is seen constantly circulating round the Earth once in 29 Days and a half; which Days are number'd on a silver'd Circle, and shewn by an Index moving over them. Thus

of two Equinoctial Days with us, which is but 2,49. Hence by means of the Obliquity of the Ecliptic, we who live beyond the Tropic have much more of the Sun's Heat than we could have enjoy'd had the Sun moved always in the Equinoctial. And on the other hand, it will be found by Calculation, that for those who live between the Tropics and the Equator, the Sum of the Heat of any two opposite Days of the Year is less than the Heat of two Equinoctial Days; and therefore the Heat of the whole Year is less in the present Case, than it would be from a constant Equinoctial Sun.

28. Lastly, let it be required to calculate the Heat of a *Polar Day*, or that under the Pole, for the Tropical Sun. In this Case $b = 0$, and $xg = xbc = 0$; also $a = 1$. Whence the Heat of any Day under the Pole will be as d, or *Sine of Declination*, because z is here always the same, *viz.* a *Semicircle*, or 180 Degrees. And under the Pole the Value of dz is thus expressed for the Tropical Sun.

The Logarithm of	$d = 23^\circ\ 30' =$	9.600700
The Logarithm of	$z = 180 =$	2.255272
The Logarithm of	$0{,}01745 =$	8.241795
Total, the Logarithm of	$dz = 1{,}252 =$	0.097767

The Double of which is 2,504; which therefore expresses the Heat of a Tropical Day under the Pole, which is greater than the Heat of any Day in any other Latitude. Hence we see the Extreme of Heat, as well as of Cold, is found in the same Place, *viz.* under the Pole.

29. It is Problem of another Sort, *To find when the Heat is a Maximum, or greatest of all, in any given Day.* In order to solve this, let the Semi-diurnal Arch $= a$, $x =$ Arch of the Hour from Noon, $b =$ Rectangle of the Sines of Latitude and Declination, and $c =$ Rectangle of their Co-sines.

each

each Day of the Moon's Age, and the *Phasis* proper thereto, are shewn for any required Time; and alſo why we ſee always *one and the ſame Face of the Moon*, *viz.* on account of her *turning about her own Axis in the ſame Time ſhe takes to revolve about the Earth.*

AGAIN: By placing a Lamp in the Orrery, and making the Room dark, we ſee very naturally how the *Sun is eclipſed* by the New Moon, and

Then (*per Spherics* and *Infinite Series*) we have the Co-ſine of the Hour from Noon $= 1 - \frac{1}{2}z^2 + \frac{1}{24}z^4 - \frac{1}{720}z^6$, &c. and the Sine of the Sun's Altitude $= c - \frac{1}{2}cz^2 + \frac{1}{24}cz^4 - \frac{1}{720}cz^6$, &c. $+ b$. This multiplied by $a + z$ is $ac + cz - \frac{1}{2}acz^2 - \frac{1}{2}cz^3 + \frac{1}{24}acz^4 + \frac{1}{24}cz^5 - \frac{1}{720}acz^6 - \frac{1}{720}cz^7$, &c. $+ ab + zb$; which therefore is proportional to the Sun's Heat. And this is greateſt when its Fluxion is equal to Nothing, *viz.* $c\dot{z} + b\dot{z} - acz\dot{z} - \frac{3}{2}cz^2\dot{z} + \frac{1}{6}acz^3\dot{z} + \frac{5}{24}cz^4\dot{z} - \frac{1}{120}acz^5\dot{z}$, &c. $= 0$. Then dividing by $\dot{z}$, $c + b - acz - \frac{3}{2}cz^2 + \frac{1}{6}acz^3 + \frac{5}{24}cz^4 - \frac{1}{120}acz^5$, &c. $= 0$; whence $\frac{c+b}{ac} = z + \frac{3}{2a}z^2 - \frac{1}{6}z^3 - \frac{5}{24a}z^4 + \frac{1}{120}z^5 = A$. Now putting $r = \frac{3}{2a}$, $s = \frac{-1}{6}$, $t = \frac{-5}{24a}$, $v = \frac{1}{120}$; by reverting the Series we have $z = A - rA^2 + \overline{2rr - s} \times A^3 + \overline{5rs - 5r^3 - t} \times A^4$, &c.

30. From this Theorem it will be eaſy to compute the Value of z, or the Time from Noon when the Heat is greateſt on any given Day. For Example: Let it be required for the Day of the Summer Solſtice in the Latitude of 51° 30′, when the Declination is 23° 30′. Then ſince (by *Art.* 24, 25.) we have $a = 2{,}146$, $b = 0{,}3121$, $c = 0{,}5709$, $b + c = 0{,}883$, and $ac = 1{,}225$; therefore $\frac{b+c}{ac} = A = 0{,}7207$. Whence by the three firſt Terms of the Series we ſhall have $z = A - rA^2 + \overline{2rr - s} \times A^3 = 0{,}7862$. Therefore ſay, As the Circumference 6,283 : 360° :: 0,7862 : 45° nearly

the Shadow passing over the Disk of the Earth; and also how the Moon, at Full, is eclipsed by passing through the Shadow of the Earth. Here also we see the Manner how *Mercury* and *Venus* transit the Sun's Face in form of a *dark round Spot*; and also why they can never appear at a great Distance from the Sun; and various other *Phænomena*, of the like Nature (CXLIII).

Whence, by allowing 15° to an Hour, it appears that *the hottest Time of the Day is Three o'Clock in the Afternoon.*

(CXLIII) 1. The *Doctrine of* ECLIPSES is next to be explain'd. The Sun being a luminous Body, vastly larger than the Earth, will enlighten somewhat more than one Half of it, and cause the Earth to project a long *conical Shadow*, as
Pl. LXII. Fig. 2. is represented in the Figure, where S is the Sun, E the Earth, and HBD its Shadow.

2. In order to find the Extent or Magnitude of the Earth's Shadow, the Lines being drawn as in the Figure, in the Triangle SBM, the outward Angle SDA = DSB + DBS, the two inward and opposite Angles; but the first, *viz.* DSB, is that under which the Earth's Semidiameter CD appears at the Sun, which is not sensible; therefore DBS, the Semiangle of the Cone, is equal to ADS, which is the Angle under which the Sun's Semidiameter AS appears at the Earth, which in its mean Distance is 16 Minutes.

3. Hence we can find the Height of the shadowy Cone CB; for in the Right-angled Triangle CBD there is given the Side CD = 1, and the Angle CBD = 16 Minutes; therefore to find the Side CB, say,

As the Tangent of	CBD = 00° 16′	=	7.667849
Is to the Radius	90 00	=	10.000000
So is Unity	1	=	0.
To the Length of the Side	CB = 214,8	=	2.332151

4. The Height of the Earth's Shadow being at the mean Distance of the Sun 214,8 Semidiameters, when the Sun is at its greatest Distance it will make CB = 217 Semidiameters of the Earth, which is its greatest Height. Hence we see the Height of the Shadow is near *three times* as great as the

THE

THE COMETARIUM is a very curious Machine, which exhibits an Idea of the Motion or Revolution of a Comet about the Sun; and as this Sort

mean Distance of the Moon, or 60 Semidiameters: But the Height of the terrestrial Shadow falls far short of the Distance of *Mars*; and therefore can involve no one of the heavenly Bodies but the Moon.

5. After the same manner it may be shewn that the Angle of the Moon's Shadow (and indeed of all Spheres whose Semidiameters bear no sensible Proportion to their Distance from the Sun) is of the same Dimensions with that of the Earth; whence those Cones are similar Figures, and so have their Heights proportional to the Diameters of the Bases. Therefore say, As the Diameter of the Earth 100 is to the Diameter of the Moon 28, so is the Altitude of the Earth's Shadow 214,8 to the Altitude of the Moon's Shadow $60\frac{144}{1000}$ of the Earth's Semidiameters. The Shadow of the Moon therefore will just reach the Earth in her mean Distance, which it cannot do in her Apogee; but in her Perigee it will involve a small Part of the Earth's Surface.

6. Besides the dark Shadow of the Moon, there is another Pl. LXII.
call'd the *Penumbra*, or Partial Shadow; to represent which Fig. 3.
let S be the Sun, T the Earth, D the Moon; and let KCF and ABE be two Lines touching the opposite Limbs of the Sun and Moon; then 'tis evident that CFEB will be the dark or absolute Shadow of the Moon, in which a Person on the Earth's Surface between F and E is wholly deprived of the Sun's Light. Moreover, let KBG and ACH be two other Lines touching the Sides of the Sun and Moon alternately, and intersecting each other in the Point I above the Moon. Then will HCBG be the *Penumbra* above-mention'd, and is the *Frustum* of the Cone GIH; for 'tis evident that a Part of the Sun will be seen and Part thereof hid to a Spectator on the Earth's Surface between F and H, and E and G; or, in other Words, the Sun in those Parts of the Earth will appear only *partially eclipsed*.

7. To calculate the Angle of the Cone HIG, draw SB, then in the oblique Triangle BIS, the external Angle BID is equal to both the inward and opposite Angles IBS and ISB; but ISB is that under which the Semidiameter of the Moon appears at the Sun, and is therefore insensibly small; whence the Angle BID = IBS or KBS = the apparent Semidiameter of the Sun. Therefore the Part of the Penum-

of Motion is not perform'd in *circular*, but very *elliptic Orbits*, so in this Instrument, a peculiar Contrivance by *elliptical Wheels* is necessary to effect it; which as a great Curiosity

bral Cone CIB is equal and similar to the dark Shadow of the Moon.

8. Let us now see how much of the Earth's Surface can be at any time involved in the Moon's dark Shadow, or the Quantity of the Arch EF. In order to this, let us suppose the Sun to be in Apogee, and the Moon in Perigee; and in that Case the Height of the conical Shadow will be about 61 Semidiameters, and the Distance of the Moon about 56;
Plate LXII. that is, (in *Fig*. 4.) DK = 61, DT = 56, and TE = 1. In this Case also the Half-Angle of the Shadow TKE = 15′ 50″, as being least of all. Therefore say,

As Unity, or the Side	TE = 1 =	0.
Is to the Side	TK = 5 =	0.698970
So is Sine of the Semi-angle TKE = 15′ 50″ =		7.663238
To Sine of the Angle TEK = 1° 19′ 10″ =		8.362208

Wherefore TEK + TKE = ATE = AE = 1° 35′, and so FE = 3° 10′ = 190′ = 220 Miles Statute-Measure; which is therefore the Diameter of the dark Shadow on the Earth's Surface when greatest.

9. After a like manner you find the Diameter of the Penumbral Shadow at the Earth, as GEFH, when greatest of all, that is, when the Earth is *in Perihelio*, and the Moon in her *Apogee*; for then will the Sun's apparent Diameter be equal to 16′ 23″ = TIG, the greatest Semi-angle of the Cone; and thence we shall find ID = 58½ Semidiameters of the Earth. In this Case also the Distance of the Moon from the Earth is DT = 64 Semidiameters. Therefore, As TG = 1 : TI = 122½ :: Sine of the Angle TIG = 16′ 23″ : Sine of the Angle IGN = 35° 42′. But IGN = TIG + ITG, and so ITG = IGN — TIG = 35° 25′; the double of which is 70° 50′ = GEFH = 4900 *English* Miles nearly.

10. Since an Eclipse of the Sun proceeds from an Interposition of the Moon, 'tis evident, if the Sun and Moon were always in the same Plane, there would necessarily be an Eclipse of the Sun every time the Moon came between the Sun and Earth, that is, at every *New Moon*. For let X be
Fig. 5. the Sun, T the Earth, and FBGH the Moon's Orbit in the Plane of the Ecliptic; then when the Moon comes to be at B

will

will be shewn, together with all Parts of the Machine, in my new Construction thereof. The Comet here represented is that which appear'd in the Year 1682, whose Period

in the Right Line TX, which joins the Centers of the Sun and Earth, it will be exactly interposed between the Sun and a Spectator on the Earth at V; and since the apparent Magnitude or Disk of the Sun is the same nearly with that of the Moon, it must necessarily be hid behind the Sun's Disk at that Time, and so eclipsed from the Sight of the Spectator; and this must be the Case whenever the Moon comes into the said Line or Point B, *viz.* every New Moon.

11. But if (as the Case really is) the Orbit of the Moon be not in the Plane of the Ecliptic, but inclined thereto under a certain Angle, there may be a New Moon, and yet no Eclipse of the Sun at the same Time. To illustrate this, let ABCDE be a Circle in the Plane of the Ecliptic, described at the Distance of the Moon's Orbit AGH, and intersecting the same in the Points B and D, making an Angle therewith as ABF, whose Measure is the Arch GC, as being 90 Degrees distant from the angular Points or *Nodes* B and D.

12. Now 'tis evident, if the Arch GC be somewhat greater than the Sum of the apparent Semidiameters of the Sun and Moon, then at G, and some Distance from G towards B, there may be a New Moon, and yet no Eclipse of the Sun, because in this Case the Disk of the Moon G is too much elevated or depressed above or below the apparent Disk or Face of the Sun C to touch it, much less to hide or eclipse any Part thereof; as is evident from the Figure.

13. At a certain Point M in the Moon's Orbit, the Moon will have a Latitude equal to the Sum of the Semidiameters of the Sun and Moon; and therefore when the Moon is New in that Point, she will appear to a Spectator in the Point Z to touch the Sun only; from whence this Point is call'd the *Ecliptic Limit*, inasmuch as it is impossible there should happen a New Moon in any Part between this and the Node D (on each Side) without eclipsing the Sun less or more; as you see the *Partial Eclipse* at K, and the *Total Eclipse* in the Node itself B.

14. What we have hitherto said has been with regard to the *Phænomena* of an Eclipse of the Sun as they appear to a Spectator on the Earth's Surface, in whose Zenith the Moon then is, and where there is no Refraction to alter the true Latitude of the Moon: But where the Moon has any Lati-

is 75 Years and a half, and therefore will again appear in 1758. By this Piece of Machinery is shewn the *unequal Motion* of a Comet in every

tude, there the Process of calculating the Appearances of a Solar Eclipse will be somewhat more complex, on account of the Variation of the Moon's Latitude and Longitude for every different Altitude, and consequently every Moment of the Eclipse.

Pl. LXIII. Fig. 1. 15. But that I may give a clear Idea of this Affair of Refractions, let ABD be the Surface of the Earth, M the Moon, S the Sun, seen from the Center of the Earth T in the same Point of the Heavens with the Moon, and consequently *centrally eclipsed* to a Spectator at C, in whose Zenith the Moon is: But to a Spectator any where else situated, the same *Phænomenon* will not happen in the same Circumstances, if at all. Thus a Spectator at B will view the Moon in the Direction of the Right Line BMN, and so her apparent Place in the Heavens will be at N, where it is evident her upper Limb will but just touch the lower Limb of the Sun, and so will not eclipse it at all: But to a Spectator any where between B and C the Sun will appear to be *partially eclipsed* less or more, as you go from B towards C.

16. This Arch SN in the Heavens is call'd the *Parallax*, or Difference between the *true and apparent Place* of the Body at M, and is equal to the Angle SMN or BMT. Now this Angle or Parallax is constantly diminishing, as the *Phænomenon* at M approaches towards the Zenith at E, where it entirely vanishes; but increases as it approaches the Horizon at G, where it is greatest of all, and is there call'd the *Horizontal Parallax*, which in the Moon amounts to a *whole Degree*, as was shewn *Annot.* CXXXV.

17. It is here observable, that the Parallax always depresses the Object, and therefore when the Moon has North Latitude it is diminished, but the South Latitude is increased, with respect to us; and so the Ecliptic Limits are variable in every particular Latitude. But a *Solar Eclipse* may in an absolute Manner be best represented by a Projection of the Earth's Disk, and of the Section of the dark and penumbral Shadow of the Moon, as they appear (or would appear) to a Spectator at the Distance of the Moon in a Right Line joining the Centers of the Sun and Earth.

18. In order to this, we are to find the Dimensions of the apparent Semidiameters of the Earth, dark Shadow, and *Pe-*

Part

Part of its Orbit, and how from thence it moves with a retarded Velocity till it arrives at the *Aphelion* Point, where it moves slowest of all; and from thence it is seen continually accelerating its Motion to-

numbra, at the Distance of the Moon. As to the first, *viz.* the *Earth's Semidiameter*, it is equal to the Moon's *horizontal Parallax*, as we have shewn. That of the *dark Shadow* is thus estimated: Let C be the Center of the Moon, DB its Diameter, DHB its dark Shadow, and KAL the Penumbral Cone. Then let EF be the Diameter of the *Penumbra* at the Earth, and IG that of the dark Shadow, and draw CG and CE; then is the Angle CGB = BHC + HCG, and so GCH = BGC — BHC; that is, the apparent Semidiameter of the dark Shadow is equal to the Difference between the apparent Semidiameters of the Moon and Sun. (See *Art.* 2 and 5.) Pl.LXIII, Fig. 2.

19. In like manner the Angle ECH = DEC + DAC, that is, the apparent Semidiameter of the *Penumbra* at the Earth is equal to the Sum of the apparent Semidiameters of the Moon and Sun. (See *Art.* 7.) Now the Semidiameters of the Sun and Moon, and also the Moon's Horizontal Parallax, are all ready calculated for the various Distances of the Sun and Moon from the Earth, and for least, mean, and greatest Eccentricities of the Lunar Orbit, in the *Astronomical Tables*.

20. Therefore let AE represent a small Portion of the annual Orbit, and FH the visible Path of the Center of the Lunar Shadows, which will exactly correspond to the Position of the Moon's Orbit with respect to the Ecliptic in the Heavens; and therefore the Point of Intersection ☊ will be the Node, and the Angle H ☊ E the Angle of Inclination of the Lunar Orbit to the Plane of the Ecliptic, which is about 5 Degrees. Fig. 3.

21. Hence if ÆPQS represent the Disk of the Earth (according to the *Orthographic Projection*) in the several Places ☊, B, C, D, whose Semidiameter is made equal to the Number of Minutes in the Moon's Horizontal Parallax at the Time of the Eclipse; and if in the Path of the Shadows in the Points ☊, R, N, G, we describe a small Circle whose Semidiameter is equal to the Difference between the Semidiameters of the Sun and Moon, that shall be the circular Section of the Moon's dark Shadow at the Distance of the Earth: (by *Article* 18.) Lastly, if on the same Center we describe a larger Circle, whose Semidiameter is equal to the Sum of the Semidiameters of the Sun and Moon, that shall represent the Section of

wards the *Perihelium*, in such manner as the Laws of Attraction require. The Comet is represented by a small Brass Ball, carried by a *Radius Vector*,

the Penumbral Shadow, (by *Art.* 19.) and is here shewn by the dotted Area.

22. Here then it is evident, if the Moon, when New, be at the Distance ☊G from the Node, the Penumbral Shadow will not fall near the Earth's Disk, and so there cannot possibly happen any Eclipse. If the Moon's Distance from the Node be equal to ☊N, then the Penumbral Shadow will just touch the Disk, and consequently ☊C the *Ecliptic Limit*; which may be found as follows. The Line NC, as being the nearest Distance of the Centers of the Shadows and Disk, is perpendicular to the Path FH, and is equal to $TC + NT = 62' \, 10'' + 16' \, 52'' + 16' \, 23''$, *viz.* the Sum of the Moon's Horizontal Parallax, and of the Semidiameters of the Sun and Moon, all of them when greatest: Also the Angle N☊C, when least, is $5^\circ \, 30'$. Therefore in the Right-angled Triangled N☊C, to find the Side ☊C, we have the following Analogy.

As the Sine of the Angle N☊C =	$5^\circ \, 30'$ =	8.981573
Is to Radius	$90^\circ \, 00'$ =	10,000000
So is the Logarithm of the Side NC =	$95',5$ =	1,980003
To the Logarithm of the Side ☊C =	$996',4$ =	2,998430

24. The Ecliptic Limit, therefore, is $996',4 = 16^\circ \, 36'$, beyond which Distance from the Node ☊ there can be no Eclipse; and within that Distance, if the Moon be New, the Shadow will fall on some Part of the Disk, as at B; where all those Places over which the Shadows pass will see the Sun eclipsed, *in part only* by the dotted Penumbral Shadow, but *totally* by the dark Shadow; and the Sun will be *centrally* eclipsed to all those Places over which the Center of the Shadows passeth.

24. If the Moon be new in the Node itself, then will the Center of the Shadows pass over the Center of the Disk, as represented at ☊. In this Case if the apparent Diameter of the Moon be greater than that of the Sun, the Face of the Sun will be *wholly obscured* to all Parts over which the Center passes; but if not, the Sun will only be *centrally eclipsed*, but his Circumference will appear a *bright Annulus*, or luminous

or Wire, in an *elliptic Groove*, about the Sun in one of its *Foci*; and the Years of its Period are shewn by an Index moving with an equable Mo-

Ring, whose Width will be equal to the Difference of the Diameters of the Luminaries.

25. As the Disk of the Earth is here projected, it represents the Case of an Eclipse on an *Equinoctial Day*, so that AK is the Ecliptic, ÆQ the Equator, XY the Axis of the Ecliptic, PS the Axis of the Equator or of the Earth, P and S the North and South Poles; besides the Tropics and Polar Circles, here represented by Right Lines, as in the common *Analemma*. And by those who understand this Projection, the Disk of the Earth and the Passage of the Shadows over it may be exhibited for any Place of the Sun, or Declination of the Moon; for which see my *Young Trigonometer's Guide*, Vol. II.

26. LUNAR ECLIPSES are not quite so complicated in Theory, nor near so tedious and difficult in Calculation, as Solar ones. The latter are only *apparent*, the former *really such*; that is, the Moon is really deprived of its Light, and therefore must appear obscured to all the Inhabitants of the Earth equally, by whom she can be seen; whereas the Sun, not being deficient in Light, will ever appear resplendent to those who do not happen to live on that Part of the Earth where the Lunar Shadows pass.

27. As a *Lunar Eclipse* is occasion'd by the Immersion of the Moon into the Earth's Shadow, we have only to calculate the apparent Semidiameter of the Earth's Shadow at the Moon, in order to delineate an Eclipse of this Sort. Thus let AB be the Earth, T its Center, AEB its Conical Shadow, DC the Diameter of a Section thereof at the Moon; and drawing TD, we have the outward Angle ADT = DTE + DET; therefore DTE = ADT − DET; that is, the Angle DTE, under which the Semidiameter of the Earth's Shadow at the Distance of the Moon appears, is equal to the Difference between the Moon's Horizontal Parallax ADT, and the Semidiameter of the Sun DEF. Plate LXIII. Fig. 4.

28. If therefore AE represent the Path of the Earth's Shadow at the Distance of the Moon near the Node ☋, and FH a Part of the Lunar Orbit, and the Section of the Earth's Shadow be delineated at ☋, B, C, D, and the Full Moon at ☋, I, N, G; then 'tis evident, where the least Distance of the Centers of the Moon and Shadow exceeds the Sum of their Semidiameters, there can be no Eclipse of the Moon, Fig. 5.

tion

tion over a graduated ſilver'd Circle: The Whole being a juſt Repreſentation of the preſent Theory of thoſe prodigious and wonderful *Phænomena* of the Planetary Syſtem (CXLIV).

as at D. But where that Diſtance is leſs, the Moon muſt be partly or wholly involved in the Shadow, and ſo ſuffer an Eclipſe, as at B and ☋; in which latter Caſe the Moon paſſes over the Diameter of the Shadow.

29. But in a certain Poſition of the Shadow, as at C, the leaſt Diſtance of the Centers N C is equal to the Sum of the Semidiameters; and therefore ☋ C is the *Ecliptic Limit* for Lunar Eclipſes: To find which, we have N C = 63′ 12″ nearly when greateſt, and the Angle N ☋ C = 5° 00′. Therefore ſay,

As the Sine of the Angle N ☋ C =	5° 00′ =	8.940296
Is to Radius	90° 00′ =	10,000000
So is the Logarithm of the Side N C =	63′,2 =	1.800717
To the Logarithm of the Side ☋ C =	725′,2 =	2,860421

Hence, if the Moon be at a leſs Diſtance from the Node ☋ than 725′ = 12° 5′, there will be an Eclipſe; otherwiſe none can happen.

30. If the Earth had no Atmoſphere, the Shadow would be abſolutely dark, and the Moon involved in it quite inviſible; but by means of the Atmoſphere many of the Solar Rays are refracted into and mix'd with the Shadow, by which the Moon is render'd viſible in the midſt of it, and of a duſky red Colour.

(CXLIV) 1. I ſhall here preſent the Reader with as large a Compendium of the NEWTONIAN COMETOGRAPHY as the Limits of this Work will permit, or, perhaps, as he may have an Inclination to read. Sir *Iſaac* has made the Doctrine or *Aſtronomy* of *Comets* the laſt Part of his immortal *Principia*, and declares it to be by far the moſt difficult and intricate Part of Philoſophy.

2. A COMET is a Sort of Planet revolving about the Sun, in a very eccentric Orbit or Ellipſis, and which conſequently approaches very near the Sun in one Part of its Orbit, and recedes to a very remote Diſtance from it in another. Hence 'tis evident, they muſt undergo extreme Degrees of Heat and Cold. Hence it appears that the Comets are ſolid, compact, fix'd, and

and durable Bodies, and not a Vapour or Exhalation of the Earth, Sun, or Planets, as has been usually supposed; because if it were such, it must inevitably be dissipated and dispersed in passing so near the Sun: For the Distance of the Comet of 1680 *in Perihelio* was so small, that it conceived a Degree of Heat above 2000 times greater than that of red-hot Iron.

3. Yet are they not so fix'd, but that they emit a fine, thin, lucid Vapour; which at first, while the Comet is yet a great way from the Sun, surrounds the Body in Form of an Atmosphere, and begins to render the Comet visible. As the Comet approaches nearer the Sun, this Vapour begins to ascend from the Head or *Nucleus*, to Heights greater and greater, as the Comet gets nearer and nearer to the Sun, and makes those amazing Streams of Light we usually call their *Tails*. All which is easy to conceive from a View of the Figure.

4. These Tails also are so fine and translucid, that the Stars are distinctly visible through them. As they rise from the Head and ascend, they become rarified, and grow broader towards the upper End. The Form of the Tail is well known to all now living who saw the late Comet; in which we observed the Tail had a small Flexure or Curvature, as they all have, being convex on the anterior Part, and concave behind, which arises from the twofold Motion of the Particles of the Tail, the one of the Ascent from the Head, the other being the progressive Motion in common with the *Nucleus* itself. But as the former is much the greatest, so its Direction is but little alter'd by the latter, and so the Position of the Tail but a little oblique and incurvated.

5. As to the Cause of the Ascent of the Cometary Vapour or Tail towards the Parts opposite to the Sun, there have been various Surmises and Conjectures, for so I call them, as not being attended with Certainty and Demonstration. *Kepler* ascribes it to the Action of the Sun's Rays rapidly carrying the Matter of the Tail away with them. And Sir *Isaac* does not think it dissonant to Reason, to suppose the subtil *Æther* in those free Spaces may yield to the Action and Direction of the Sun-Beams. It is certain from Experiments, that the Solar Rays collected by a Burning-Glass to a Focus, impel light and pendulous Bodies very notably, even so as to make them vibrate backwards and forwards: And though this Impulsion of the Rays of Light with us, in our gross Mediums, and on our sluggish Matter, be inconsiderable; yet in those free Spaces, and on the subtil Effluvia or fine Particles of the Cometary Atmosphere, it may be very great. I know there are other and later Hypotheses to account for the Motion and Form of a Comet's Tail; but on Examination they appear

to

to be insufficient, improbable, and unphilosophical, and therefore shall not trouble the Reader with them.

6. The Bodies of Comets are very small, and above the Orbit of the Moon, as is evident from hence, that they have no perceptible *horizontal* or *diurnal Parallax*, and when view'd with a Telescope at their nearest Distances appear less than to the naked Eye, by having the Splendor of their Tails taken off, and that of the Atmosphere abated by being magnified. The *Nucleus* of the last Comet measured but a few Seconds, as I found by measuring the Atmosphere by a Micrometer, and taking a proportional Part.

7. On the other hand, by their *annual Parallax* they are proved to descend within the Regions of the Planets; they also appear sometimes *direct* and slower than they really move, sometimes *retrograde* and swifter than the true Motion, and lastly they are sometimes *stationary*; all which Phænomena arise from the same Causes as were before explain'd of the Planets. (See *Annot.* CXXXIX.)

8. Since the Comets by Observation are found to describe curve Lines about the Sun, they must be drawn by some Force from a rectilineal Course by the first Law of Motion. And since this Force in all the Planets tends to the Sun, as being the largest Body in the System, therefore also this Force in the Comets respects the Sun in a more immediate Manner, as being so much less than it than most of the Planets are. And lastly, as this Force in the Planets is inversely in the duplicate Ratio of the Distance from the Sun, the same Law is undoubtedly observed by the Comets, which are in other Respects Bodies similar to the Planets. The Comets therefore move in Conic Sections about the Sun, having their Foci in the Sun's Center. (See *Annot.* CXL.)

9. Hence, if Comets return in an Orbit, those Orbits must be *Ellipses*; and their Periodical Times will be to the Periodical Times of the Planets in the sesquiplicate Ratio of the principal Axes: And therefore the Comets being for the most part beyond the Planetary Regions, and on that account describing Orbits with much larger Axes than the Planets, revolve more slowly. Thus if the Axis of a Comet's Orbit be 4 times as long as that of *Saturn*'s Orbit, then would the Time of the Period of the Comet be to that of the Planet as $4\sqrt{4}$ to 1, or as 8 to 1; *viz.* $8 \times 30 = 240$ Years.

10. Since it is found by Observations that the Cometary Orbits are extremely eccentric, and that the Portion which a Comet describes during the whole Time of its Appearance is but a very small Part of the Whole, the Center of such an Ellipsis being removed to so vast a Distance must occasion the

Curvature

Curvature at each End to be vastly near that of a Parabola having the same focal Distance; and consequently the Motion of a Comet may be calculated in a Parabolic Orbit without any sensible Error.

11. Therefore the Velocity of a Comet *in Perihelio* (*viz.* in the Vertex of the Parabola P) is to the mean Velocity of a Planet describing a Circle about the Sun, at the same focal Distance SP, as $\sqrt{2}$ to 1. And supposing the Earth to be that Planet, let us put the Radius of its Orbit SP = 100000, and then say, As the whole Periodical Time of the Earth $365\frac{1}{4}$ is to the whole Periphery 628318, so is 1 Day to 1720,2 Parts described in one Day; and in one Hour it will describe 71,67 Parts. But as $1 : \sqrt{2} :: 1720,2 : 2432,747$, the Parts described by the Comet in one Day; and so the Parts described by the Comet in one Hour will be 101,364. Plate LXIV. Fig. 1.

12. Whence if the *Latus Rectum* LR of the Parabola be equal to 4 times the Radius SP of the Earth's Orbit, and we put $SP^2 = 100000000$, the Area which the Comet will describe each Day, by a Ray drawn to the Sun, will be $1216373\frac{1}{2}$ of those Parts, and each Hour an Area of $50682\frac{1}{4}$ of those Parts. To demonstrate this we must consider, that the Square of the Diameter of any Circle is to its Area as $1 : 0,7854 :: 4 : 3,14159$; therefore the Square of Radius or PM = 1. Whence the Area of the Circle is to the said Square PM as 3,14159 to 1. And the Rectangle PL = 2. But the Parabolic Area $PLS = \frac{2}{3} PL = \frac{2}{3} \times 2 = \frac{4}{3}$. Hence this Area PLS is to the Area of the Circle as $\frac{4}{3}$ to 3,14159. And if the Velocity of the Comet and Planet at P were the same, the Time in which the Comet would describe the Arch of the Parabola PL would be to the Time in which the Planet describes its Orbit in the same Ratio of $\frac{4}{3}$ to 3,14159. But these Velocities are as $\sqrt{2}$ to 1; therefore the said Times will be $\frac{4}{3} \times \frac{1}{\sqrt{2}}$ to $\frac{3,14159}{1}$, that is, as $\sqrt{\frac{16}{18}} = \sqrt{\frac{8}{9}}$ to 3,14159. Wherefore say, As $3,14159 : \sqrt{\frac{8}{9}} :: 365\text{D. } 6\text{H. } 9' : 109\text{D. } 14\text{H. } 46'$, the Time in which the Comet will describe the Arch PL. If then $PS^2 = PM = 100000000$, we have the Parabolic Area PLS = 133333333

133333333 Parts described in 109 D. 14 H. 46′; and therefore the proportional Parts for a Day and Hour as above.

13. What those diurnal and horary Areas are in different Parabolas may be thus shewn. Let prq be a Parabola similar to the former PRQ; then will the Time T of describing the Arch PR be to the Time t of describing the similar Arch pr, as the Periodical Time P of describing a Circle on PS to the Periodical Time p of describing a Circle on ps, by the last Article. But $P : p :: PS^{\frac{3}{2}} : ps^{\frac{3}{2}} :: R^{\frac{3}{2}} : r^{\frac{3}{2}} :: T : t$, also the similar Areas $PRS = A$, and $prS = a$, are as the Squares of their like Sides PS and pS; that is, $A : a :: R^2 : r^2$. Now since in the same Figure equal Spaces are described in equal Times, whatever Number of Days or Hours are contain'd in T and t, the Areas A and a will consist of as many equal Parts respectively; and which therefore we may call the $\frac{1}{T}$ Part of A, and $\frac{1}{t}$ Part of a, or x and y; so that $x : y :: \frac{A}{T} : \frac{a}{t} :: \frac{R^2}{R\sqrt{R}} : \frac{r^2}{r\sqrt{r}} :: \sqrt{R} : \sqrt{r}$.

14. Let the Quadrantal Area PSR of the Parabola PRQ be divided into 100 equal Parts, that is, let $A = 100$; then $\frac{A}{100} = 1$ of those Parts, and so $\frac{A}{100} : R^2$. Again, let N be the Number of those Parts described in 1 Day; then will this diurnal Area be $N \times \frac{A}{100} : \frac{A}{T} : N \times R^2 : \sqrt{R}$, (by *Art.* 13.) therefore $N : \frac{1}{R^{\frac{3}{2}}}$.

15. In like manner it is shewn, that if the Quadrantal Area prS of the Parabola prq be divided into an 100 equal Parts, and $pS = r$, and $n =$ Number of those Parts in the diurnal Area; then $n : \frac{1}{r^{\frac{3}{2}}}$. And so $N : n :: \frac{1}{R^{\frac{3}{2}}} : \frac{1}{r^{\frac{3}{2}}}$; $n = N \times \frac{1}{r^{\frac{3}{2}}}$, if $R = SP = 1$, or the Radius of the Earth's Orbit.

16. On these Principles the *Cometary Calculus* depends; for in any Parabolic Orbit the Quantity $n = N \times \frac{1}{r^{\frac{3}{2}}}$ is the diurnal Area, and may therefore be esteemed the *mean Motion* or *Anomaly* of the Comet for a Day; which multiplied by the Time (express'd in Days) before or after the Comet is *in Perihelio*

helio at P, will give the whole mean Motion or Area PRQS for any Place of the Comet Q in its Orbit. In order to this we must have the Time ascertain'd from Observation when the Planet was *in Perihelio* at P, and also the Perihelian Distance SP from the Sun; as also the Place in the Ecliptic at the same Time, the Position of its Nodes, and Inclination of its Orbit: All which Particulars for 24 Comets the Industry of the great Astronomer of this Age has supplied, *viz.* Dr. *Halley*, in his *Cometagraphia*; which I have transcribed, and added thereto the same Things for the last Comet, as they were determined by the Reverend Mr. *Betts*, from the Observations of Mr. Professor *Bliss* of *Oxford*, at the Observatory of the Right Hon. the Earl of *Macclesfield*, at *Sherbourn* in *Oxfordshire.*

17. From the Place of the Comet Q draw QA perpendicular to the Axis; and let ab be a Tangent to the Curve in the Point Q, and BQ drawn perpendicular thereto; then by the Nature of the Parabola we have AB = SR, the *Semi-Latus Rectum*. And putting the given Area PQS = a, and AQ = x, we have $\frac{1}{12}x^3 + \frac{1}{4}x = a$, or $x^3 + 3x = 12a$; which Cubic Equation resolv'd gives the Ordinate AQ, and thence we have PA; but PA + PS = SQ = Distance of the Comet from the Sun, which therefore is given. Therefore in the Triangle SAQ, right-angled at A, we have SQ and AQ to find the Angle QSA; and then PSQ the Angle from the *Perihelium* is known. When this is done, all the other Particulars are the same as in the *Planetary Calculus.* Pl. LXIV. Fig. 2.

18. These are the Principles or Elements of Calculation; which we shall now proceed to illustrate by Example, that so the *Praxis* may not remain so difficult and obscure as it has hitherto been; and we shall make choice of the last Comet for this Purpose, whose *mean Anomaly* or *diurnal Area* is in the first Place to be determined.

19. In order to this, we have the constant mean Motion of a Comet moving in a Parabola, whose Perihelion-Distance PS = R = 1 = Semidiameter of the Earth's Orbit, *viz.* N =

$$\frac{100}{109\text{D. } 14\text{H. } 46'} = 0{,}91228, \text{ whose Logarithm } 9.960128$$

is therefore always at hand for constant Use.

20. The Perihelion-Distance PS = r = 0,22206, and its Logarithm 9,346472, as in the Table, for the Comet of $174\frac{3}{4}$. But we have its mean Anomaly $n = \text{N} \times \frac{1}{r^{\frac{3}{2}}}$ (by *Article* 15.); therefore to find n by Logarithms the Process is as follows:

The

The Logarithm of Perihelion-Distance	$r =$	9.346472
Which multiply by		3
The Product is the Logarithm of	$r^3 =$	8.039416
Divide by 2, the Quotient is Log. of	$r^{\frac{3}{2}} =$	9.019708
Arithmetical Complement is the Log. of	$\frac{1}{r^{\frac{3}{2}}} =$	0.980292
To which add the Logarithm of	$N =$	9.960128
The Log. of Mean Anomaly	$n = 8,718 =$	0.940420

21. Having thus obtain'd the *diurnal Area*, if we multiply this by any Number of Days and Decimal Parts of a Day, it will give the Area PRQS, or *mean Anomaly*, for the given Time. Thus let it be required for *January* 23 D. 6 H. 11'.

		D.	H.	M.
Then from the Time of Perihelion,	*Feb.*	19	8	12
Subduct the given Time,	*Jan.*	23	6	11
The Difference will be		27	2	1

Wherefore to Log. of diurnal Area	$8,718 =$	0.940420
Add the Log. of the given Time	$27,0833 =$	1.432702
The mean Anomaly required	$= 236,1 =$	2.373122

22. Having therefore the Area PRQS $= 236,1$, we can find AQ $= x$, from the Equation $x^3 + 3x = 12a$; for if when the Quadrantal Area PSR is 100, we put SR $= x = 1$, then 'tis plain, $x^3 + 3x = 1 + 3 = 4 = 12a$ in that Case. Therefore when the *mean Anomaly* is but $\frac{1}{100}$ Part of this, we have $x^3 + 3x = \frac{4}{100} = 0,04$; which will be a constant Multiplier for reducing any given Anomaly to fit it for the Equation. Thus $0,04 \times 236,1 = 9,444 = x^3 + 3x$ in the present Case, which resolved according to the usual Methods gives $x = 1,65$ nearly.

23. Then by the Nature of the Parabola $\frac{\overline{AQ}^2}{4PS} = AP = \frac{1,65 \times 165}{2} = 1,3612$. Also AP + PS = SQ = 1,8612, the Distance of the Comet from the Sun for the given Time. But to express this Distance in the same Parts as the Sun's mean Distance from the Earth contains 1,00000, we must consider that the Perihelion-Distance PS $= 0,22206$; whence

SR

SR = 0,44412. Wherefore say, As 1 : 0,44412 :: 1,8612 : 0,82650, the Expression required.

24. In the Right-angled Triangle QAS, having all the Sides, we find the Angle QSA = 62° 36½′; whence the obtuse Angle PSQ = 117° 33½′, which is the Heliocentric Distance of the Comet from the Perihelion. Now since the Perihelion is in ♎ 17° 12′ 55″, if we subduct 117° 33′ 30″, we have the Heliocentric Longitude in ♉ 19° 39′ 25″.

25. Also the *Descending Node* is in ♏ 15° 45′ 20″, from which subtract the Comet's Place now found, the Difference 147° 05′ 55″ is the Distance of the Comet from the Node. Let the Line of the Nodes be ☋S☊; then, since the Perihelion P is 151° 27′ 35″ distant from the Node ☊, it will be but 28° 38′ 25″ distant from the Node ☋. If then from the Angle QSA = 62° 36½′ we deduct PS☋ = 28° 38′ 25″ = AS☊, we shall have QS☊ = 30° 58′ 5″. Pl. LXIV. Fig. 3.

26. From Q let fall the Perpendicular QN on the Line of Nodes; then in the Right-angled Triangle QSN, having the Angle at S and the Side SQ, we can find QN as follows.

As Radius	90° =	10.000000
To the Sine of the Angle QSN = 33° 58′ =		9.747374
So is the Side	SQ = 0,82650 =	9.917227
To the Length of the Side QN = 0,46200 =		9.664601

27. Again: In the Right-angled Triangle QND we have the Side now found QN, and the Angle of the Inclination of the Comet's Orbit QND = 47° 9′, to find the Side or Perpendicular QD. Thus say,

As Radius	90° =	10.000000
Is to the Sine of Inclination	QND = 47° 9′ =	9.865138
So is the Side	QN = 0,46200 =	9.664601
To the Perpendicular	QD = 0,33860 =	9.529739

28. We can now find the Heliocentric Latitude of the Comet, or the Angle QSD; for

As the Side	QS = 0,82650 =	9.917227
Is to the Side	QD = 0,33860 =	9.529739
So is Radius	90° =	10.000000
To Sine of Helioc. Lat.	QSD = 24° 12′ =	9.612512

29. To find the Comet's Curtate Distance from the Sun, viz. SD, we have this Analogy from the Right-angled Triangle QSD.

As Radius	90° =	10.000000
To the Sine of the Angle	SQD = 65° 48′ =	9.960052
So is the Side	SQ = 0,82650 =	9.917227
To the Curtate Distance	SD = 0,75380 =	9.877279

30. To find the Side N D in the Right-angled Triangle QND, say,

As Radius	90° =	10.000000
To Co-sine of Inclination	DQN = 42° 51′ =	9.832616
So is the Side	QN = 0,46200 =	9.664601
To the Side	DN = 0,31420 =	9.497217

31. Then in the Right-angled Triangle NSD we can find the *Heliocentric Place of the Comet in the Ecliptic*, or Angle DSN, thus:

As the Curtate Distance	SD = 0,75380 =	9.877279
To the Side	ND = 0,31420 =	9.497217
So is Radius	90° =	10.000000
To the Sine of the Angle	DSN = 24° 38′ =	9.619938

Therefore to the Place of the Node ☊,	♉ 15° 45′ 20″
Add the Angle now found	24 38 00
The Sum is the Helioc. Place in the Ecliptic,	♊ 10 23 20

32. The next thing to be done is to find the Place of the Sun, and consequently of the Earth in her Orbit for the given Time; which is calculated from the Tables in the usual Method as follows:

	Mot. of the Sun. S.	°	′	″		*Mot. of Perihelion.* S.	°	′	″
1741.	9	21	1	58	——	3	8	13	30
3.	11	29	17	00				2	30
Jan. 23.	00	22	40	12					3
Hours 6			14	47					
Min. 11				27					
Mean Mot.	10	13	14	24	——	3	8	16	3
Equat. add.		1	7	39		10	13	14	24
True Place	10	14	22	3		7	4	58	23 M. Anom.

33. The Sun's Place being found in ♒ 14° 22′ 03″, the Earth's Place will be in the opposite Part of the Ecliptic, *viz.* in ☊ 14° 22′ 03″ at T. If therefore from this we subtract the Comet's Heliocentric Place at H, we shall have the Arch HT = 63° 49′ 43″ = DST, the *Angle of Commutation*. And as the Earth's mean Anomaly is 7 S. 4° 58′ 23″, the Logarithm of the Earth's Distance ST will be 9,993947. But SD is also known; therefore we can find the Angle DTS, or Elongation of the Comet from the Sun, thus:

As

As the Sum of the Sides	$ST + SD$	$= 1,74000 =$	0.240549
Is to their Difference	$ST - SD$	$= 0,23240 =$	9.366236
So the Tang. of $\frac{1}{2}$ the Ang.	$\frac{D+T}{2}$	$= 58^\circ\ 00' =$	10.204211
To Tang. of $\frac{1}{2}$ their Diff.	$\frac{D-T}{2}$	$= 12^\circ\ 03' =$	9.329898

34. Hence $58^\circ + 12^\circ\ 03' = 70^\circ\ 03' =$ TDS, and $58^\circ - 12^\circ\ 03' = 45^\circ\ 57' =$ STD, or Longitude of the Comet from the Sun; which added to the Sun's Place at I gives the *Geocentric Longitude*, of the Comet at L, in ♈ 00° 19′. And to find the *Geocentric Latitude*, or Angle DTQ, we have this Analogy:

As the Sine of Commutation	TSD	$= 64^\circ\ 00' =$	9.953650
Is to the Sine of Elongation	STD	$= 45^\circ\ 57' =$	9.856568
So is Tang. of Helio. Lat.	DSQ	$= 24^\circ\ 12' =$	9.652650
To the Tang. of Geo. Lat.	DTQ	$= 19^\circ\ 46' =$	9.555568

35. Thus you have the whole Process of Calculation, as it relates to the Phænomena of a Comet moving in a Parabola near the Vertex, and is the same with that used for the Planets (from the 25th *Article* inclusive). And though it is certain (from what will be shewn by and by) that this Comet does not describe a *Parabola*, but an *Ellipsis*, yet the computed Longitude and Latitude are the same which the Comet was observed to have at that very Time; whence the Accuracy of this Method sufficiently appears: But as it is thus limited to a Parabola, and only one small Part of that, and cannot be extended to determine the Axis of the Orbit, or the Time of its Revolution, I shall here supply this great Deficiency by shewing a direct and geometrical Method of Computation of all the Phænomena of a Comet moving in any Conic Section, which was first invented by M. *Bonguer* in *Mon Paris.* An, 1733; which Method I shall explain, illustrate, and exemplify in the following Articles.

36. Let AKBI be the Trajectory of a Comet, AB its longest Axis, IK the shortest; S, F, the two *Foci*, in one of which the Sun is at S; C the Place of the Comet, CS its Distance from the Sun; DCE a Tangent to the Curve in the Point C; C*c* the Space pass'd over by the Comet in a small Particle of Time; SD, FE, Perpendiculars from the Foci to the Tangent: And draw SG parallel to DE, and join FC. Also let ANO be the elliptic Orbit of any Planet; S, *f*, its Foci. Lastly, let ALB be a Circle described on the longer Axis, AB; APTB a Rectangle about the Ellipsis AIB; and AQRB as the Square about the Circle ALB; and put SC Pl. LXV. Fig. 1.

$= a$, $SD = b$, $Cc = e$, the Time in which it is described $= f$. The longer Axis of the Cometary Orbit $AB = x$, of the Planetary Orbit $AO = q$, the Circle described on the same Axis $AVO = p$; the Periodical Time of the Comet $= t$, and that of the Planet $= n$.

37. The Space Cc described, the Distance SC, and the Angle SCD, are all known by Observation, and therefore given Quantities. The mean Distance of the Comet is $AH = \frac{1}{2}x$, and of the Planet is $Ag = \frac{1}{2}q$. And because the *Squares of the Periodical Times are as the Cubes of the mean Distances*, we have $\frac{1}{8}q^3 : \frac{1}{8}x^3 :: n^2 : t^2$; and therefore $t = \frac{nx}{q}\sqrt{\frac{x}{q}}$. (*Annot.* XXXIV. 11.)

38. It is necessary now to find another Expression of the Periodical Time t, thus: Because Cc is a very small Portion of the Orbit, it may be esteem'd a Right Line, and the Sector CSc as an evanescent Triangle, whose Area $\frac{1}{2}SD \times Cc = \frac{1}{2}be$ is given; but as the Area $\frac{1}{2}be$ is to the Time f, so is the whole Area of the Ellipsis $AKBI = A$ to the whole Periodical Time t; that is, $t = \frac{f}{\frac{1}{2}be} \times A$.

39. Now in order to determine the Area A, we must find the Semi-conjugate HK, thus: Because $AB = SC + FC$, therefore $FC = x - a$; and by similar Triangles SDC and FEC we have $SC : SD :: FC : FE$, that is, $a : b :: x - a : \frac{bx - ab}{a} = FE$; and therefore $FG = (FE - GE)$ $\frac{bx - 2ab}{a}$. Again, $SC : CD :: FC : CE$; or $a : \sqrt{a^2 - b^2} :: x - a : \frac{x - a}{a}\sqrt{a^2 - b^2}$. Hence DE or $SG = CE + CD = \frac{x - a}{a}\sqrt{a^2 - b^2} + \sqrt{a^2 - b^2} = \frac{x}{a}\sqrt{a^2 - b^2}$. But $FG = \frac{bx - 2ab}{a}$; therefore $FS = \sqrt{SG^2 - FG^2} = \sqrt{\frac{b^2x^2 - 4ab^2x + 4a^2b^2 + a^2x - b^2x^2}{a^2}}$ $= \sqrt{\frac{a^2x^2 - 4ab^2x + 4a^2b^2}{a^2}}$. And therefore $SH = \frac{1}{2}SF$ $= \frac{1}{2}\sqrt{\frac{a^2x - 4ab^2x + 4a^2b^2}{a^2}}$.

40. Moreover, by the Nature of an Ellipsis, $SK =$

AH

$AH = \frac{1}{2}x$, and therefore $\sqrt{SK^2 - SH} = HK = \sqrt{\frac{1}{4}x^2 - \frac{a^2x^2 + 4ab^2x - 4a^2b^2}{4a^2}} = \frac{b}{a}\sqrt{ax - a^2}$; therefore $IK = 2HK = \frac{2b}{a}\sqrt{ax - a^2}$. Consequently, $\frac{xb}{a}\sqrt{ax - aa} = APTB$, the Semi-Area of the Ellipse. Let $Q =$ Diameter of the Circle ALB, and P its Periphery; then since $\frac{1}{2}LH \times P = \frac{1}{4}QP$ is the Area of the Circle, we shall have $Q^2 : \frac{1}{4}QP$ (:: $\frac{1}{2}Q^2 : \frac{1}{8}QR$) :: $AQRB : ALB$ :: $APTB : AIB$ :: $q^2 : \frac{1}{4}qp$. That is, $q^2 : \frac{1}{4}qp$ :: $\frac{xb}{a}\sqrt{ax - aa} : \frac{bpx}{4aq}\sqrt{ax - a^2} = AIB$. But $2AIB = AIKB = A = \frac{bpx}{2aq}\sqrt{ax - aa}$; therefore the above Expression $t = \frac{f}{\frac{1}{2}be}A = \frac{fpx}{aeq}\sqrt{ax - aa}$. Then $t = \frac{nx}{q}\sqrt{\frac{x}{q}} = \frac{fpx}{aeq}\sqrt{ax - aa}$. And, reducing the Equation, we get $x = \frac{af^2p^2q}{f^2p^2q - ae^2n^2} = AB$, the principal Axis of the Section, or Trajectory of the Comet.

41. If we substitute this Value of x in the Equation above for t, we shall have $t = \frac{p^3f^3n^2a^{\frac{3}{2}}}{\overline{qf^2p^2 - ae^2n^2}|^{\frac{3}{2}}} =$ the Periodical Time. Also because the Conjugate $IK = \frac{2b}{a}\sqrt{ax - aa} = c$, therefore $x = \frac{c^2a^2 + 4b^2a^2}{4b^2a} = \frac{af^2p^2q}{f^2p^2q - ae^2n^2}$; whence $c = IK = 2ben\sqrt{\frac{q}{f^2p^2q - ae^2n^2}}$.

42. From these Equations it plainly appears, that when the Velocity of the Comet is such that $f^2p^2q = ae^2n^2$, the Axis x is *infinite*, and consequently the Trajectory will be a *Parabola*; but if ea^2n^2 be greater than f^2p^2q, it will be an *Hyperbola*; in both which Cases the Comet can never return: But in all Cases where f^2p^2q is greater than ea^2n^2, the Comet will describe *Ellipses*; among which we reckon that of

 the

the *Circle*, where $x = 2a = \frac{af^2p^2q}{f^2p^2q - ea^2n^2}$, and hence $e = Cc = \frac{fp}{n}\sqrt{\frac{q}{2a}}$, the Arch of the Circle described in one Day.

43. Let the Planet we supposed to describe the Ellipsis ANO be the Earth; then will its mean Distance $\frac{1}{2}q =$ 100000 equal Parts; and so $q = 200000$, and $p = 628318$. Also the Periodical Time $n = 1$ Year; and then if Cc be the Space described in one Day, we have $f = \frac{1}{365,2565} =$ 0.0027378. Then also the other Expressions will become for the principal Axis $x = \frac{591826599535 \times a}{591826599535 - ae^2}$, and for the Periodical Time $t = \frac{4750560000 \times a^{\frac{3}{2}}}{591826599535 - ae^{2^{\frac{3}{2}}}}$.

44. Hence it appears, that if Observations could be made sufficiently exact to determine the Distance of the Comet, and the Space it moved over in its Orbit in one Day, then the Axes of the Orbit and the Periodical Time of the Comet may as well be computed as those of a Planet; but this is a Matter of the greatest Nicety, and of course the greatest Difficulty, because the *elliptic Orbit* of a Comet, if it be such, can scarcely be distinguish'd by Observation (however well made) from a Parabolical Orbit, in all that Part of the Orbit which the Comet describes during its Appearance. Hence the Quantity ae^2 will generally come out either equal to, or greater than the Number 591826599535, and so gives the Axis x infinite or negative: And if it chance that ae^2 be less than the said Number, then if a or e be not defined to the last Degree of Exactness, the Axis x, and Periodical Time t, will be very different from the Truth. But more of this in another Place.

45. A *Parabola* therefore is fully sufficient to account for all the Circumstances and Phænomena of a Comet's Motion during the Time of its Appearance; as Sir *Isaac* has shewn with respect to the Comets of 166$\frac{4}{5}$, 1680, 1682, 1683, 1723, and Mr. *Betts* for the last Comet of 174$\frac{3}{4}$. And that the Reader may see the wonderful Agreement between the Theory (though grounded on the *Parabolical Hypothesis*) and the Phænomena of Longitude and Latitude of the Comet by Observation, I shall here subjoin a Table exhibiting the same both by Computation and Observation, and the Differences between them severally for each respective Time of Observation.

	Equal Time at Oxford.			Longit. Comet observed.				North Latit. observed.			Longit. Comet computed.				North Latit. computed.			Diff. in Long.	Diff. in Latit.
	D.	H.	′		°	′	″	°	′	″		°	′	″	°	′	″	″	″
Dec. 1743.	23	5	32	♈.	14	10	2	17	33	11	♈.	14	10	3	17	33	37	1 −	26 −
	27	5	7½	♈.	12	2	25	17	51	29	♈.	12	2	26	17	51	47	1 −	18 −
	28	5	1½	♈.	11	32	11	17	55	54	♈.	11	32	14	17	56	8	3 −	14 −
	31	4	44	♈.	10	4	57	18	9	3	♈.	10	5	16	18	8	53	19 −	10 +
		5	53	♈.	10	4	11	18	9	37	♈.	10	3	55	18	9	6	16 +	31 +
Jan. 1743.	12	9	10	♈.	4	52	5	18	59	37	♈.	4	52	24	18	59	13	19 −	24 +
	13	6	20	♈.	4	31	40	19	2	31	♈.	4	31	13	19	2	49	27 +	18 −
		8	20	♈.	4	29	27	19	3	32	♈.	4	26	6	19	3	12	21 +	20 +
	16	6	33	♈.	3	18	43	19	15	47	♈.	3	18	27	19	15	13	16 +	34 +
		8	00	♈.	3	17	31	19	16	7	♈.	3	17	00	19	15	30	31 +	37 +
	23	6	11	♈.	0	19	45	19	42	30	♈.	0	19	16	19	42	1	29 +	29 +
	23	7	29	♈.	0	17	58	19	42	47	♈.	0	17	45	19	42	12	13 +	35 +
Feb. 1743.	5	7	31½	♓.	21	52	37	19	35	00	♓.	21	52	56	19	34	42	19 −	18 +
	11	6	37½	♓.	14	42	45	17	23	30	♓.	14	42	58	17	24	5	13 −	35 −
	12	6	33	♓.	13	10	36	16	38	40	♓.	13	10	52	16	39	17	16 −	37 −
	13	6	25	♓.	11	32	50	15	43	45	♓.	11	33	16	15	44	16	26 −	31 −
	16	23	41½	♓.	5	9	14	10	17	40	♓.	5	9	1	10	18	8	13 +	28 −
	17	23	35	♓.	3	37	37	8	15	39	♓.	3	37	11	8	16	3	26 +	24 −

Plate LXV. Fig. 2. 46. Having thus shewn the several Affections of a Comet's Motion, I shall conclude with a Word or two in relation to their *Tails*. The Atmosphere of Comets consisting of a very fine Vapour, will, when the Comet is in its *Aphelion*, be nearly spherical, and its Density greatest. As the Comet approaches the Sun, the Sun's Heat enters the Atmosphere, and rarifies it by degrees, causing at the same time the finest Part to rise from the Comet, like the Flame from a Candle, towards the Parts averse from the Sun; and as the Comet comes nearer and nearer the Sun, this Fume will rise and extend itself to greater and greater Lengths, and make what is call'd the *Tail* of the Comet; so that when they are view'd with a Telescope, the *Nucleus*, Atmosphere, and Tail of a Comet appear much like what is represented in the Figure.

Fig. 3. 47. The Length of the Tail is thus found by Observation. Let S be the Sun, C the Comet, T the Earth, Ce the Comet's Tail; draw TS, TC, SC, and Te touching the End of the Tail, and meeting the Line SC produced in E. The Place of the Sun and Comet being known, the Angle TCE is known (for $TCE = STC + CST$). Also the Angle of Deviation ECe is known from Observation; whence TCe is known. Moreover the Angle CTe is known also by Observation. Therefore in the Triangle TCe, having the two Angles TCe and CTe, and the Side TC, (from the Theory) we can find the Side Ce, which is the Length of the Tail. And thus they have been found to be 40, 60, and 80 Millions of Miles.

48. Draw Se cutting the Comet's Orbit in d; then because the whole Motion of a Particle from C to e may be resolved into two Motions Cd and ed, 'tis plain, since de is that directly averse to the Sun, the Comet would have possess'd the Point d when the Particle at e first rose from the *Nucleus*, if the Motion had been every where in the Direction of Se, as the Line Se kept moving from Se to SE.

49. But since this is not the Case, but the Particles move in the oblique Direction Ce, therefore parallel to Ce draw SF cutting the Orbit in D, and join De; then will the compound Motion Ce, arising from the progressive Motion of the Comet in the Direction CD, and its Motion of Ascent in the Direction Ce, give the Point D for the Comet's Place when first the Particle at e began to ascend from the *Nucleus*.

50. Now the Time in which the Comet describes any given Part of its Orbit DC may be found from the Theory, and consequently the Time of the Ascent of the Tail of a Comet from the *Nucleus* to the Extremity e. Thus I have finish'd

finish'd a *compleat Compendium of the* NEWTONIAN *Philosophy of* COMETS.

Jam patet horrificis quæ sit via flexa Cometis;
Jam non miramur barbati Phænomena Astri.

Dr. HALLEY.

APPENDIX

APPENDIX
TO
LECTURE XI.

Of TIME, *and its* MEASURE *by the* Celeſtial *Motions. Of the* YEAR Tropical *and* Sydereal, *and the Quantity of each. The Time of the* EQUINOXES *and* SOLSTICES *determined by* Calculation. *Of* DAYS, Natural *and* Artificial. *The* EQUATION *of* Time *explain'd. Of* WEEKS. *Of* MONTHS, Periodical *and* Synodical. *Of* Old *and* New Style. *Of* CYCLES; *the* CYCLE *of the* SUN, *and* Dominical LETTERS; *the* METONIC Cycle, *or* Cycle *of the* MOON, *and* GOLDEN NUMBERS. *The* Cycle *of* INDICTION. *The* Dionyſian PERIOD, *or* Paſchal Cycle. *The* Julian PERIOD. *The* Aſtronomical Principles *of* CHRONOLOGY, *by Sir* ISAAC NEWTON, *explain'd and exemplified.*

1. I SHALL here give the Reader an Idea of of the YEAR, as the grand and original Meaſure of Time, and derived from the Aſtronomical Principles of the Earth's Motion; and then afterwards conſider its Subdiviſions and Diſtributions into leſſer Parts, as *Months*, *Days*, *Hours*,

Hours, *Minutes*, *Seconds*, *Thirds*, &c. for the Purposes of common Life, and the Uses of Chronology, History, and other Sciences.

2. TIME is in itself a flowing Quantity, measuring the Duration of Things; and its Flux is always equable and uniform; and therefore to estimate the Quantity of Time, we should measure it by something that is in its own Nature always of one and the same Tenor. For this Purpose we have no Expedient so convenient as that of Motion; and because the Measure of Time ought to be permanent, we can find no other Motion fit for this Purpose but that of the Heavenly Bodies.

3. AMONG these, none of the Motions are so obvious to every Body, and plain to common Sense, as that of the *Sun* and *Moon*; which therefore have been agreed upon by the Consent of all Nations for this End, and indeed this seems to have been a principal Part of the Design of their Creation. For we are told they were appointed for *Times and Seasons, for Days and for Years*, Gen. i. That is, the Sun by his Diurnal Motion affords the Measure for DAYS, and by his Annual Motion the Measure for YEARS; and the Moon, by her Revolutions, gives the Measure of another Part of Time we call MONTHS.

4. FOR it is a compleat Revolution of those Luminaries that constitutes a Year, a Month, and a Day in the Abstract, or absolutely consider'd. Hence it is necessary to consider the Point which is to be esteem'd the *Exordium* or Beginning of these

these Revolutions. And this, with respect to the Annual Revolution of the Sun, is fix'd in that Point of the Ecliptic which is the Beginning of *Aries*; and the Time which the Sun takes in going from, and returning to this Point again, is call'd a YEAR.

5. ALSO the Space of Time which the Sun takes to compleat one Revolution about the Earth, is call'd a *Natural Day*, or the *Nychthemeron*, including a Common Day and Night; which Space of Time is subdivided into 24 equal Parts, we call HOURS; and each of these are again subdivided into 60 equal Parts or Minutes; each of these again into 60 other equal Parts call'd Second Minutes, or Seconds; each of these into Thirds, and so on in a Sexagesimal Subdivision for any lesser Parts of Time.

6. Now, if this first Point or Beginning of *Aries* were fix'd, each Annual Revolution of the Sun would be constantly the same, and therefore a just and equal Measure of the Year, which is call'd the *Periodical Year*, as being the Time of the Earth's Period about the Sun; and which consists of 365 D. 6 H. 9′ 14″. For so long is the Earth in departing from any fixed Point in the Heavens, and returning to the same again.

7. BUT since, as we have shewn (*Annot.* CXLI.) the several Points of the Ecliptic have a retrograde Motion, 'tis easy to understand, that by this Recession of the Equinox it will, as it were, meet the Sun, and cause that the Sun shall arrive to the Equinox, or first Point of *Aries*, before his Revolution

volution is compleated. And therefore this Space of Time (which is call'd the *Tropical Year*) is not so long as the former; for by Observations made at the Distance of many Years of the Time of two Equinoxes, and dividing the Time elapsed between by the Number of Revolutions, the Quotient will shew the Quantity of this *Tropical Year* to be 365 D. 5 H. 48′ 57″; which is 20′ 17″ less than the Periodical Year.

8. THE Beginning of the Year or Time when the Sun enters the Equinox is thus determin'd by Observation. Let ABC be a Portion of the Equinoctial, and DBE an Arch of the Ecliptic; Pl. LXVI. Fig. 2. then with a very nice Instrument take the Meridian Altitude of the Sun, the Day before and after the Equinox; the Difference between these Altitudes and that of the Equator will be the Sun's Declination on those two Days, which suppose to be AD and EC; which being thus known, and the Angle of Obliquity ABD = EBC = 23° 29′, we find the Arch DB and EB; and therefore we say, As DB + EB is to DB, so is 24 Hours to the Time between the first Observation and Moment of the Sun's Ingress to the Equinoctial Point B.

9. BUT the Quantity of the Tropical Year is better defined from a Calculation of the Moments of the Solstices. The Invention of which curious and most certain Method was owing to our late celebrated Dr. *Halley*; and is founded on an easy Observation, and therefore practicable by any Person but moderately skill'd in the Conic Geometry. The Method is as follows: Let AVO represent Fig. 3. a small

a small Portion of the Tropic, which the Ecliptic K V M touches in the Solstitial Point V. Suppose the Sun at several Times near the Solstice be in the Points K, I, L, V, C, M, N, then will the Right Lines T L, I D; B C, E M, &c. (perpendicular to the Tropic A O) be the Deficiencies of the Sun's Declination at those Times from his greatest Declination in V.

10. AND from the Elements of Geometry, the Subtenses T L, D I, &c. of the Angle of Contact A V K, are as the Squares of the Conterminal Arches V L, V I, &c. that is, of the Lines V T, V D, &c. which are nearly equal to those Arches. Now when the Sun is in L, part of its Path that Day will be the Line L C; and when in M, the Line I M, drawn parallel to A O. Let V Q be part of the Solstitial Colure; then we have V T = L F, and V D = G I, &c. also V F = T L, V G = D I, &c. whence $LF^2 : IG^2 :: VF : VG$, &c. so that the Figure K V N has really the Property of a Parabola, and may be taken for such, without any sensible Error.

11. THEREFORE let three Points F, G, H, in the Axis V Q be determined by Observation, thus; let *ab* be an upright Object, *ac* the Ground or Horizon, and *cd* a Plane set nearly perpendicular to the Sun's Rays at Noon. Then let the Points *b*, *f*, *g*, on the Plane mark the Shadow of the Apex *b*, on three several Days at Noon; suppose two before, and one after the Solstice. By this means we have the Proportion of Distance between the Points F H and F G, for as *fb* : *fg* ::

F H

FH : FG. By the first Observation from the Point H the Sun's Place at K is given; by the second, from the Point F, we have the Place at L; and by the third, having G, we have the Point M in the Curve.

12. Now let the Time between the first and second Observation AT (= KL) = a; the Time between the second and third Observation TE (= LVM) = b, FH = c, FG = d, and TV = x = the Time between the second Observation and the Moment of the Solstice, to be found. Then AV = $a + x$, and VE = $b - x$; and let the *Latus rectum* of the Parabola be p. Then (*per Conics*) we have $x^2 = \text{VF} \times p$, and therefore VF = $\frac{x^2}{p}$. In like Manner VH = $\frac{a^2 + 2ax + x^2}{p}$, and VG = $\frac{b^2 - 2bx + x^2}{p}$, therefore FH (= VH − VF) = $\frac{a^2 + 2ax}{p} = c$, and FG (= VG − VF) = $\frac{b^2 - 2bx}{p} = d$; wherefore $p = \frac{a^2 + 2ax}{c} = \frac{b^2 - 2bx}{d}$; and reducing the Equation, we have $x = \frac{b^2 c - a^2 d}{2ad + 2bc}$ = TV, the Time required.

13. BUT if the Order of the Observations be such, as that the Observation of the Shadow of the *Gnomon* in f is exactly in the Middle between those of the Shadow in h and g; then will AT = TE, and so $a = b$; and the Equation will become $x = \frac{ac - ad}{2d + 2c}$ = TV; which gives this Analogy,

2d

$2d+2c:c-d::a:x$, that is, 2 FG + 2 FH : GH :: AT : TV.

14. I shall illustrate this Calculation by an Example of each Case. In the Year 1500, *Bernard Walker*, in the Month of *June*, at *Nuremberg*, observed the Chord of the Sun's Distance from the Zenith by a large Instrument, as follows;

June 2, 45467		and	*June* 8, 44975
June 9, 44934			*June* 12, 44883
June 16, 44990			*June* 16, 44990

The Differences of these Chords are equal very near to the small Distances FG and FH; therefore $c=533$, and $d=56$, and $c-d=477$; and since the Time was 7 Days between Observations, therefore $a=7$. Whence we have 1178 : 477 :: 7 : 2 D. 20 H. 2′, which added to the Time of the middle Observation, gives *June* 11 D. 20 H. 2′ for the Time of the Solstice.

15. Again, by the other three Observations, we have $c=107$, $d=92$, $c-d=15$, and $a=4$; wherefore say, As 398 : 15 :: 4 D. = 96 H. : 3 H. 37′, the Difference between the 2ᵈ Observation, *June* 12, and the Moment of the Solstice, which therefore must be *June* 11 D. 20 H. 23′, which is but 21′ different from the former. The Time of the Tropic therefore, in *Anno* 1500, we may conclude was *June* 11 D. 20 H. 12′.

16. We will now give an Example of the former Method by the Shadow of a Gnomon 55 Feet high, which *Gassendus* at *Marseilles* made use of for determining the Proportion of the Gnomon to its Solstitial Shade. This he did in the

the Year 1636; and the Experiments were as follow.

June 19	New Style, The Shadow	31766	Parts, of which the *Gnomon* was 89428.
20		31753	
21		31751	
22		31759	

Here indeed the End of the Shadow, instead of being receiv'd on the Plane $c\,d$, perpendicular to the Rays, was taken on the horizontal Line, where the Points f, g, h, are referr'd to F, G, H, in three of the Observations; yet is the Ratio between FH and FG the same nearly as the Ratio between fh and fg, because the Rays at that Distance from h, in so small an Angle, differ little from parallel Rays.

17. HENCE the Case of the Problem is still the same. Therefore, let the Shadow, on *June* 19, be $aH=31766$; on the 21st, $aF=31751$; and on the 22d, $aG=31759$; then $2\,c=30$, $2\,d=16$, $c-d=7$, and $a=2$, $b=1$; then the Theorem $\frac{c\,b^2-d\,a^2}{2a\,d+2\,b\,c}=0{,}274=00$ D. 17 H. 25′, which is the Time by which the Solstice preceded the second Observation. The Solstice therefore was on *June* 20 D. 17 H. 25′ *N.S.* or *June* 10 D. 17 H. 25′ *O.S.*

18. THE Difference between the Time of this and the other Solstice is 1 D. 2 H. 47′; of which 1 D. 1 H. 12′ arises from the Deficiency of the Length of the Tropical Year from that of the *Julian* Year, (as will by and by appear) and the other Part 1 H. 45′ from the Progression of the

Sun's Apogæum during that Space of Time, *viz.* 136 Years.

19. THE DAYS are the next Part of Time we shall consider. These may be divided into *Solar* and *Sidereal* Days. The *Solar Day* is that Space of Time which intervenes between the Sun's departing from any one Meridian, and its Return to the same again. But a *Sidereal Day* is the Space of Time which happens between the Departure of a Star from, and its Return to the same Meridian again. And each of these are divided into 24 equal Parts, or *Hours*.

20. BECAUSE the Diurnal Motion of the Earth about its Axis is equable, every Revolution will be perform'd in the same Time; and therefore all the *Sidereal Days*, and the Hours of those Days, will be equal. And on the other hand, the *Solar Days* are all unequal, and that on two Accounts, *viz.* because of the Elliptic Figure of the Earth's Orbit, and because of the Obliquity of the Ecliptic to the Equator.

Plate LXVI. Fig. 4.

21. THIS will appear as follows. Let S be the Sun, AB a Part of the Ecliptic, A the Centre of the Earth, and MD a Meridian whose Plane passes through the Sun. Now in the Time of one Revolution about its Axis, let the Earth be carried about the Sun from A to B, and then the Meridian will be in the Position *m d*, parallel to the former MD. But 'tis plain, the Meridian *m d* is not yet directed to the Sun, nor will not, till by its angular Motion it has attain'd the Situation *e f*, describing the Angle $eBm = BSA$; whence

whence it appears that all the *Solar Days* are longer than the Time of one Revolution, or *Sidereal Day*.

22. If the Earth revolved in the Plane of the Equator, and in a Circle about the Sun, then would the Angle A S B, and consequently the Angle *e* B *m* be always of the same Quantity, and therefore the Time of describing the said Angle *e* B *m* would always be equal, and so all the Solar Days would be equal among themselves. But neither of these two Cases have Place in Nature.

23. For by the Earth's Theory, founded on the nicest Observations, the Orbit is an *Ellipsis*, and therefore (as we have shewn) her Annual Motion cannot be equable, or the Angle A S B described in the same Space of Time will not be equal; for in the Aphelion, the Velocity of the Earth will be less than in the Perihelion, therefore also the Arch A B will be less, and consequently the similar Arch *em*, and therefore also the Time of describing it; whence it appears, the Part of Time to be added to the Sidereal Day, to compleat the Solar Day, is always variable.

24. The other Part of the Equation of Time (and most considerable) is that which arises from the Plane of the Earth's Orbit or Ecliptic being inclined to that of the Equator or Plane of the Diurnal Motion; to explain which, let ♈ ♑ ♎ be a Semicircle of the Ecliptic, and ♈ H ♎ of the Equinoctial, S the Centre of the Sun, and A that of the Earth in the third Quarter of the Ecliptic; *bf* the Meridian passing through the true Sun S, and Plate LXVI. Fig. 5.

and its apparent Place at I in the first Quarter of the Ecliptic ♈ ♋.

25. SUPPOSE, now, the Motion of the Earth in every Respect equable, and first that it sat out from ♎, and proceeded in the Equator in a given Time to D, the Sun would apparently describe in the same Time the Arch of the Equator ♈ I. Again, suppose it sat out from the same Point ♎, and spent the same Time with the same equable Velocity in the Ecliptic, it would arrive to the Point A, so that the Arch ♎ A = ♎ D, and ♈ I = ♈ C. Then 'tis evident, as the Earth revolves about its Axis from West to East, the Meridian of any Place will first arrive at the Sun I in the Ecliptic, and afterwards at the Sun C in the Equinoctial; that is, the Time of Noon by the Sun in the Ecliptic will be sooner than that Noon which would happen by the Sun in the Equinoctial; and that by the Quantity of the Arch b D turn'd into Time.

26. Now the Arch b D = BC is the Difference of the Sun's Longitude ♈ I or ♈ C, and his Right Ascension ♈ B. Draw ge parallel to DC, and the Angle e A f will be equal to the Angle D S b, and the Arch ef similar to the Arch D b; therefore the Time in which the Meridian bf revolves into the Situation ef, is that which is to be added to the Ecliptic Noon to equate it with the Time of the Equinoctial Noon, in the first and third Quarters of the Ecliptic. In the second and fourth Quarter, the said Equation is to be sub-

ſubtracted; as would eaſily appear by making the ſame Conſtruction there.

27. Now becauſe in different Parts of the Quadrant this Arch D *b* or BC is of a different Length, the *Equation of Time* will be a variable Quantity; and therefore ſince the Motion and Time meaſured by the Sun in the Equinoctial is always equal, (there being nothing to make it otherwiſe) it follows, that the Times (*i. e.* the Days) meaſured by the Sun in the Ecliptic muſt be always unequal; or, in other Words, the *Solar Days* are ſometimes ſhorter, ſometimes longer, than the *equal Time* meaſured out in the Equinoctial.

28. It has been ſhewn already, that the True Motion of the Earth precedes the Mean in the firſt Semicircle of Anomaly, and is preceded by the Mean in the ſecond. Therefore while the Earth is going from the Aphelion to the Perihelion, or while the Sun apparently moves from the Apogæum to the Perigæum, the Apparent Time will be before the Mean, and in the other Semicircle of Anomaly it will be after it. The Difference of theſe Motions converted into Time is the *Equation of Time* in this Reſpect, and is to be ſubtracted from the Apparent Time to gain the Mean, or added to the Mean to gain the Apparent, in the firſt Semicircle of Anomaly, and *Vice verſa* in the latter.

29. Now both theſe Parts of the Equation of Time are calculated by Aſtronomers for every Degree of Anomaly, and for every Degree of the

Sun's Longitude in the Ecliptic, and disposed in two several Tables, with Directions for *adding and subtracting*, as the Case requires; so that at all times the true or equal Time may be had. And from thence it appears that the apparent Time, or that shewn by the Sun, *viz.* by a *Sun-dial*, is but four Days in the whole Year the same with the mean or equal Time shewn by a good *Clock* or *Watch*, *viz.* about *April* the 4th, *June* the 6th, *August* the 20th, and *December* the 13th. Also about the 23d of *October* the Equation is greatest of all in the Year, being then about 16' 11'', Clocks being then so much slower than *Sun-dials*.

30. As the Solar Days are unequal, the Hours must be so of course; and hence it appears, that there is no natural Body which can by its Motion measure Time truly or equally; and the only way to do this is, by the artificial Contrivance of Clocks, Watches, Clepsydræ, Hour-Glasses, *&c.*

31. In different Parts of the World, the natural Day has a different Beginning. The ancient *Egyptians* began their Day at Midnight, as do also the modern Nations of *France*, *Spain*, *Great-Britain*, and most Parts of *Europe*. The *Jews*, with the *Germans*, begin their Day at Sun-setting. The *Babylonians* began theirs at Sun-rising. And the Astronomers begin the Day at Noon, and reckon on to twenty-four Hours, and not twice twelve, as we do by our Clocks in civil Life.

32. A Week is another common Measure of Time consisting of seven Days; and because the Ancients supposed the seven Planets had an In-

fluence

fluence upon the Earth and all terrestrial Things, they allotted the first Hour of each Day to the Planet they supposed then to preside; from whence the several Days of the Week received their Names. Thus *Sunday* was *Dies Solis*, i. e. the Day of the *Sun*; *Monday* was *Dies Lunæ*, i. e. the Day of the *Moon*; *Tuesday* was *Dies Martis*, i. e. the Day of *Tuisco* or *Mars*; *Wednesday* was *Dies Mercurii*, i. e. the Day of *Wooden* or *Mercury*; *Thursday* was *Dies Jovis*, i. e. the Day of *Thor* or *Jupiter*; *Friday* was *Dies Veneris*, i. e. the Day of *Friga* or *Venus*; and *Saturday* was *Dies Saturni*, i. e. the Day of *Saturn*.

33. A MONTH is another Part of Time, so call'd from the Moon, because it is the Time of her Revolution about the Earth, and is therefore also call'd a *Lunation*. If we respect the Revolution of the Moon from any fixed Point in the Heavens (as a Star) to the same again, it is call'd a *Periodical Month*, and consists of 27 D. 7 H. 43′. But if we regard the Time that passes between one Conjunction or New-Moon, and the next following, it is call'd a *Synodical Month*, and is equal to 29 D. 12 H. 44′ 3″.

34. THESE now mention'd are the *Astronomical Years*, *Months*, and *Days*; But those used in common Life are somewhat different. Thus the Civil Month is a Space of 28, 29, 30, or 31 Days, and 12 Synodic Months make 354 Days, which is call'd a *Civil Lunar Year*; and a Civil Solar Year is the Space of 365 Days. Therefore to equate the *Civil Lunar* to the *Solar Year*, 11

Days are to be added, which were call'd by the *Greeks Epagomenæ*, and by us the *Epacts*.

35. THE *Civil Soli-Lunar* Year of 365 Days, being short of the true by 5 H. 48′ 57″, occasion'd the Beginning of the Year to run forwards thro' the Seasons one Day nearly in four Years, and in 1460 Years through all the Months of the Year. On this Account *Julius Cæsar* ordain'd that every 4th Year *one Day* should be added to *February*, by causing the 24th Day to be reckon'd twice; and because this 24th of *February* was the Sixth (*Sextilis*) before the Kalends of *March*, there were in this Year two of those *Sextiles*, which gave the Name of *Bissextile* to this Year. The Year, thus corrected, was from thence call'd the *Julian Year*.

36. BUT the six Hours, added by *Julius Cæsar*, is too much, that is, exceeds 5 H. 48′ 57″ by 11′ 3″, and therefore the Sun each Year begins his Course 11′ 3″ before the *Julian* Year is ended, which in 131 Years amounts to a whole Day. Hence at the Council of *Nice*, A. D. 325, (at which the Time of *Easter* was fix'd) the Vernal Equinox being upon the 21st Day of *March*, it was found in the Year 1582 to happen on the 11th of *March*, 10 Days sooner than before.

37. POPE *Gregory* XIII. thought the Kalendar too erroneous, and resolved to reform it, by restoring the Equinox to its former Place in the Year, *viz.* to the 21st of *March*. To do this, he took 10 Days out of the Kalendar, by ordering the 5th of *October* 1582 to be call'd the 15th; and to prevent the Regress of the Equinox for the fu-

ture,

ture, order'd every 100^{dth} Year to consist of only 365 Days, whereas in the *Julian* it has 366, as being *Bissextile*. This Reformation is therefore call'd the *Gregorian* Account, or New-Stile, and is used by *Papists* in *Italy*, *Spain*, *France*, *Germany*, and by some *Protestants* abroad; but we still retain the *Julian* Year, and call the Reckoning by it *Old-Stile*.

38. SINCE the Council of *Nice*, to the present Year 1746, there have elapsed upwards of 1421 Years, by which means the Equinox does in the Old-Stile, at this time, fall on the 10^{th} of *March*, and the *Julian* Account is 11 Days later than the *Gregorian*. But even the *Gregorian* Emendation is not sufficient; for whereas by that four Days in 400 Years are rejected, a considerable Error is committed; for the odd $11' 3''$, by which the *Julian* Year exceeds the Truth, will not amount to more than three Days in 391 Years. If therefore at the End of every 391 Years we expunge three Days, the Equinox will very nearly always keep to the same Day of the Month.

39. IN Computations of Time, we find it necessary to fix upon some remarkable Transaction, or memorable Event, for the *Exordium* or Beginning of the Reckoning; these are call'd EPOCHA's or ÆRA's, Thus some compute from the *Creation of the World:* The ancient *Greeks* from the *Institution of the Olympiads*, beginning 776 Years before CHRIST: The *Romans* from the *Building of Rome*, about 750 Years before CHRIST. The

Chaldeans

Chaldeans and *Egyptians* used the *Æra of Nabonassar*, beginning *A. ante C.* 752. The *Turkish* Epocha is the *Hegira* or *Flight of Mahomet*, A. C. 622. The *Persian* Æra is call'd *Yesdegird*, A. C. 632. And that of the *Christians*, the *Birth of Christ*, since which Time we reckon 1746 Years.

40. BESIDES the Measure of Time by Common Years, we find it became necessary to introduce the Use of CYCLES (*i. e.* Circles) *of Years*; as the *Metonic Cycle*, the *Cycle of the Sun*, the *Cycle of Indiction*, and the *Julian Period* compounded of all the rest. Of each of these I shall give the following short Account.

41. THE CYCLE of the SUN arises hence: If the Number 365 be divided by 7, it will have a Remainder of 1, which shews the last Day of the Year is the same Day of the Week with the first. Now it was always customary to place against the seven Days in the Week, the seven first Letters of the Alphabet, A, B, C, D, E, F, G, and therefore, as they were continued thro' the Year, it is evident the same Letter must stand against the first and last Day of the Year, *viz.* the Letter A.

42. HENCE, if the 1st of *January* be a *Sunday*, the Letter A will point out all the *Sundays* in that Year; and since the 1st of *January* in the next Year is *Monday*, the first *Sunday* will be on the 7th, against which stands the Letter G, which therefore will be the *Sunday* Letter for all that Year. Again, the first Day of the following Year being *Tuesday*, the first *Sunday* will be on the 6th, against which

which ſtands the Letter F, which therefore indicates the *Sundays* thro' that Year, and ſo on; whence 'tis eaſy to obſerve, that the Letters which point out the *Sundays* in every Year will be in a retrograde Order, *viz.* A, G, F, E, *&c.* And becauſe theſe Letters ſhew the *Dies Domini*, or *Lord's-Days*, they have been call'd DOMINICAL LETTERS.

43. Now, if all the Years were common ones, the ſame Letter would not be the *Dominical*, or the *Sundays* would not be upon the ſame Days of the Week, till after a Cycle or Revolution of ſeven Years; and ſince every 4th Year has a Day extraordinary, this Day will interrupt the Succeſſion of the *Dominical Letters*, and cauſe that the ſame Days will not be ſhewn again by the ſame Letters after a *Cycle* of ſeven Years, but of $4 \times 7 = 28$ Years, which is call'd the *Cycle* of the *Sun*.

44. BECAUSE in every *Biſſextile* Year the 24th or 25th of *February* is reckon'd twice, and both thoſe Days have the ſame Letter, it follows, that that Letter which ſhew'd the *Sundays* before the 24th of *February* will not ſhew it afterwards, and therefore in every ſuch Year there will be two *Dominical Letters*. For Example, the Year 1744 was *Biſſextile*, *January* 1. *Sunday*, the *Dominical Letter* A; but the 24th of *February* being *Friday* had the Letter F, and alſo *Saturday* the 25th; therefore *Sunday* the 26th muſt have G, which for that reaſon was the *Sunday* Letter the remaining Part of the Year.

45. To

45. To find what Year of the *Cycle* the present or any Year of CHRIST is, add 9 to the given Year, (because the first Year of CHRIST was the 9th of the *Cycle*) and divide by 28, the Remainder is the Year of the *Cycle* required. Example: The Year 1746 + 9 = 1755, then 1755 divided by 28 leaves 19, the Year of the *Cycle* required, whose *Dominical Letter* is E, according to the following Table:

Cycle	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Dom. Let.	G F.	E.	D.	C.	B A.	G.	F.	E.	D C.	B.	A.	G.

Cycle	13.	14.	15.	16.	17.	18.	19.	20.	21.
Dom. Let.	F E.	D.	C.	B.	A G.	F.	E.	D.	C B.

Cycle	22.	23.	24.	25.	26.	27.	28.
Dom. Let.	A.	G.	F.	E D.	C.	B.	A.

46. THE METONIC CYCLE, (so call'd from the Inventor *Meton*) otherwise call'd the *Cycle of the Moon*, is a Period of nineteen Years, after which the *New* and *Full Moons* were supposed to return on the same Days of the Month, and Hours, as before; because if the Solar and Lunar Year began together at any Time, these Years being to each other as 365 to 354, could not coincide again at their Beginning till after a certain Time, *viz.* 235 Lunations, which make 6939 D. 16 H. 31′ 45″, and in nineteen Solar Years are 6939 D. 18 H.; the Difference being only 1 H. 28′ 15″ shews the two Years will then begin again very nearly at the same Time, and the *New* and *Full*

Moons

Moons come round again upon the same Days of the Month.

47. YET this Deficiency of an *Hour and half* will cause the *New* and *Full Moons* to happen so much sooner each *Cycle* in the Heavens than by this Reckoning, and this in 304 Years amounts to a whole Day; and therefore, at this Time, they happen almost five Days sooner than they should do, by the Rule settled by the *Nicene* Council for finding the same by the *Golden Numbers*; the Nature and Use of which are to be understood as follows.

48. TAKING any Year for the First of the *Cycle*, the Ancients observ'd all the Days on which the *New Moons* happen'd thro' the Year, and against each such Day they placed the Number 1; in the 2d Year of the *Cycle* they did the like, and to each Day of the *New Moon* annexed the Number 2. In like Manner to every *New Moon* Day in the 3d Year of the *Cycle* they subjoin'd the Number 3; and so on, thro' all the Years of the *Cycle*. This being done for one *Cycle*, the same Numbers were fitted to the Kalendar to shew the *New Moons* in each Year of any future *Cycle*; and, upon Account of this their excellent Use, they were in *Gold*, and were therefore call'd the *Golden Numbers* for those Years respectively.

49. BUT because these Numbers for the observed *New Moons* are not of lasting Use (as above shewn) the best way of disposing these Numbers is by the *Mean Lunations*, as they may be found

found from Astronomical Tables for each Year of the *Cycle*, which are the ſame in every *Cycle*; and do not vary greatly from the true. But, however advantageous this may be in civil Life, we are not to expect this Innovation ſhould take Place in the Liturgy of the Church of *England*, which ſtill continues to compute the Moons, as it does the Equinoxes, by the old erroneous Rule eſtabliſhed by the Council of *Nice*, which are call'd *Eccleſiaſtical New Moons*, in Contradiſtinction to the true ones in the Heavens.

50. Besides theſe, there was another Period call'd the Cycle of Indiction, conſiſting of 15 Years; it was ſo call'd, becauſe the Numbers of this Cycle *indicated* the Time of *Eaſter*. But as this *Cycle* has no Connection with the Motions of the Heavenly Bodies, I ſhall ſay no more of it here, but refer the Reader for a farther Account of this and other Matters purely *Chronological*, to the Authors who have wrote on *Chronology*, or, if they pleaſe, to an Epitome of that Science in my *Philological Library of Literary Arts and Sciences*.

51. The Dionysian Period is one that is made by multiplying together the *Cycles* of the *Sun* and *Moon*, and therefore conſiſts of 532 Years, for $28 \times 19 = 532$. After the Completion of this Period, not only the New and Full Moons return to the ſame Days of the Month, but alſo the Days of the Month return to the ſame Days of the Week; and therefore the *Dominical Letters* and the *Moveable Feaſts*

Feasts all return again in the same Order. Hence this *Cycle* was call'd the GREAT PASCHAL CYCLE.

52. THE JULIAN PERIOD is the last I shall mention, and the largest of all, consisting of 7980 Years, being composed of the *Cycles of the Sun, Moon, and Indiction*; thus 28 x 19 x 15 = 7980. The Beginning of this Period was 764 Years before the Creation, and is not yet compleated; and therefore comprehends all other *Periods, Cycles,* and *Epochas*, and the Times of all memorable Actions and Histories. It had its Name from its Inventor *Julius Scaliger*, who has eternized himself thereby.

53. I CAN'T conclude this Essay, without laying before the Reader the *Astronomical Principles* of CHRONOLOGY, which Sir *Isaac Newton* makes use of for settling the Grand EPOCHA of the *Argonautic Expedition*, and which he makes the Basis of his Chronology. He observes, that *Eudoxus*, in his Description of the Sphere of the Ancients, placed the Solstices and Equinoxes in the Middles of the Constellations *Aries, Cancer, Chelæ*, and *Capricorn:* And also that this Sphere or Globe was first made by *Musæus*, and the *Asterisms* delineated upon it by *Chiron*, two of the *Argonauts*.

54. Now it has been shewn, that by the Precession of the Equinoxes the Stars go back 50″ *per Annum*. And since at the End of the Year 1689, the *Equinoctial Colure* passing thro' the middle Point, between the first and last Star of *Aries*, did then cut the Ecliptic in ♉ 6° 44″, it

is

is evident; that the Equinox had then gone back 36° 44′; therefore, as 50″ is to one Year, so is 36° 44′ to 2645 Years, which is the Time since the *Argonautic* Expedition to the Beginning of the Year 1690; that is, 955 Years before CHRIST is the Æra of the *Argonautic Expedition*.

55. BUT our great Author is more particular and subtile in this Affair. He finds the Mean Place of the *Colure of the Equinoxes and Solstices*, by considering the several Stars they pass'd thro' among the other Constellations, as follows, according to *Eudoxus*.

56. IN the Back of *Aries* is a Star of the 6[th] Magnitude, mark'd ν by *Bayer*; in the End of of the Year 1689, its Longitude was ♉ 9° 38′ 45″; and the Equinoctial Colure passing thro', according to *Eudoxus*, cuts the Ecliptic in ♉ 6° 58′ 57″.

57. IN the Head of *Cetus* are two Stars of the 4[th] Magnitude, call'd ν and ξ by *Bayer*. *Eudoxus*'s Colure passing in the Middle between them, cuts the Ecliptic in ♉ 6° 58′ 51″, at the End of the Year 1689.

58. IN the extreme Flexure of *Eridanus* there was formerly a Star of the 4[th] Magnitude (of late it is referr'd to the Breast of *Cetus*). It is the only Star in *Eridanus*, thro' which this Colure can pass; its Longitude was at the End of the Year 1689 ♈ 25° 22′ 10″, and the Colure of the Equinox passing thro' it cuts the Ecliptic in ♉ 7° 12′ 40″.

59. IN the Head of *Perseus*, rightly delineated, is a Star of the 4[th] Magnitude, call'd τ by *Bayer*; its

its Longitude was ♉ 23° 25′ 30″ at the End of the Year 1689; and the Colure of the Equinox passing through it cuts the Ecliptic in ♉ 6° 18′ 57″.

60. IN the Right Hand of *Perseus*, rightly delineated, is a Star of the 4th Magnitude, whose Longitude at the End of the Year 1689 was ♉ 24° 25′ 27″, and the Equinoctial Colure passing through it cuts the Ecliptic in ♉ 4° 56′ 40″.

		°	′	″
61. Now the Sum of all these five Places of the Colure, *viz.*	♉	6	58	57
	♉	6	58	51
	♉	7	12	40
	♉	6	18	57
	♉	4	56	40
Is =	1	2	26	05
The 5th Part of which is =	♉	6	29	13

which is therefore the Mean Place, in which the Colure in the End of the Year 1689 did cut the Ecliptic.

62. AFTER a like Manner he determines the Mean Place of the *Solstitial Summer Colure* to be ♌ 6° 28′ 46″, which as it is just 90 Degrees from the other, shews it to be rightly deduced. The *Equinoxes* having then departed 1s 6° 29′ from the Cardinal Points of *Chiron*, shews that 2628 Years have elapsed since that Time, which is more correct than the former Number (*Article* 55.) tho' less by only seventeen Years.

63. BY some other Methods, of a like Nature, he also shews the *Æra of the Argonauts* ought to be placed in that Age of the World; and having

fix'd this moſt antient *Epocha*, he makes his Computation with Reference thereto in the future Part of his Book.

64. AND thus our great Author has with his uſual Sagacity, ſo conducted his Deſign, as to make his Chronology ſuit with the *Courſe of Nature*, with the *Principles of Aſtronomy*, with *Sacred Hiſtory*, with *Herodotus*, the Father of Profane Hiſtory, and with itſelf. And tho' many have thought fit to cavil, and find great fault with his Chronology, yet, how little Regard ought to be paid to them may from hence appear, that Sir *Iſaac Newton* was undoubtedly equal to any Man *in all the common Qualifications of a Chronologiſt*, and *vaſtly ſuperior to all* in thoſe which were eſſential. Gentlemen ſhould have the Modeſty not to criticiſe on the greateſt Man that ever lived, till they have convinced the World, at leaſt, *that they underſtand him.*

LEC-

LECTURE XII.

The Use of the GLOBES.

Of the GLOBES *in general. The* CIRCLES *of the* SPHERE *described. The* POSITIONS *of the* SPHERE. *The* SOLUTION *of* PROBLEMS *on the* CELESTIAL GLOBE. *The* TERRESTRIAL GLOBE *described.* PROBLEMS *on the same. Of the* CONSTELLATIONS *of the* Northern *and* Southern HEMISPHERE. Flamsted's CATALOGUE *of the* STARS. *Of the* DISTANCE *and other* Phænomena *of the* STARS. *A* Calculation *of the surprizing* VELOCITY *of* LIGHT. *Of the* ABBERRATION *of* Light, *and the* Telescopic MOTION *of the Stars by Dr.* BRADLEY. *The* PRINCIPLES *of* GNOMONICS, *or* Art *of* DIALLING *demonstrated, by a* DIALLING-SPHERE. Astronomical Doctrine *of the* SPHERE, *and Method of calculating* SPHERICAL TRIANGLES. *The* HARVEST-MOON *explain'd. How to find a* MERIDIAN LINE. *The* FIGURE *and* DIMENSIONS *of the Earth determined by* actual Mensuration *of a* DEGREE *under the* ARCTIC CIRCLE *and at* PARIS. *A new* CALCULATION *on that Head. Of the* ORTHOGRAPHICAL PROJECTION. *Of the* STEREOGRAPHICAL PROJECTION. *The* Globular Projection.

jection. *Of* MERCATOR'S CHART, *and a new Method of Constructing the Table of* MERIDIONAL *Parts by Fluxions. The Nature of the* RHUMB-LINE *investigated, and applied in* Sailing. *A new* MAP *of the* WORLD *on the* Globular Projection. *A* MAP *of the* Country *in* Lapland *where the Arch of the* Meridian *was measured by the French King's* Mathematicians.

IN this Lecture I shall explain the *Nature and Use of both the Globes*, by giving you a succinct Account of the Nature and Design of each, and a Solution of the *principal Problems* that are usually perform'd thereby.

Plate LXVIII. Fig. 4. EACH Globe is suspended in a General Meridian, and moveable (within an Horizon) about its Axis, in the same manner as the *Armillary Sphere* of the Orrery; and the Circles of that Sphere, already described, are laid on the corresponding Parts of the Surface of each Globe; and are therefore supposed to be known.

THE Surface of the CELESTIAL GLOBE is a Representation of the Concave Surface of the *Starry Firmament*, there being depicted all the Stars of the first and second Magnitude, and the most noted of all the rest that are visible. So that by this Globe we may shew the *Face of the Heavens* for any required Time, by Day or Night, throughout the Year, in general; or in regard to any particular Body, as the *Sun*, *Moon*, *Planet*, or *Fix'd Star*.

THE

THE Stars are all diſpoſed into Conſtellations, under the Forms of various Animals, whoſe Names and Figures are printed on the Paper which covers the Globe; which were invented by the ancient Aſtronomers and Poets, and are ſtill retain'd for the ſake of Diſtinction and better Arrangement of thoſe Luminaries, which would be otherwiſe too confuſed and promiſcuous for eaſy Conception, and a regular Method of treating on them (CXLV).

(CXLV) 1. The Surface of the CELESTIAL GLOBE may be eſteem'd a juſt and adequate Repreſentation of the concave Expanſe of the Heavens, notwithſtanding its Convexity; for 'tis eaſy to conceive the Eye placed in the Center of the Globe, and viewing the Stars on its Surface, ſuppoſing it made of Glaſs, as ſome of them are; and alſo, that if Holes were made in the Center of each Star, the Eye in the Center of the Globe, properly poſited, would view through each of thoſe Holes the very Stars in the Heavens repreſented by them.

2. Becauſe it would be impoſſible to have any diſtinct or regular Ideas or Notions of the Stars in reſpect of their Number, Magnitude, Order, Diſtances, *&c.* without firſt reducing them to proper Claſſes, and arranging them in certain Forms, which therefore are call'd ASTERISMS or CONSTELLATIONS; this was done in the early Ages of the World by the firſt Obſervers of the Heavens, and thoſe who made Spheres or Delineations; of whom Sir *Iſaac Newton* reckons *Chiron* the Centaur the firſt who form'd the Stars into Conſtellations, about the Time of the *Argonautic Expedition*, or ſoon after; and that the ſeveral Forms or *Aſteriſms* were, as it were, ſo many ſymbolical Hiſtories, or Memorials of Perſons and Things remarkable in that Affair. Thus *Aries*, the Ram, is commemorated for his *Golden Fleece*, and was made the firſt of the Signs, being the Enſign of the Ship in which *Phryxus* fled to *Colchis*. *Taurus*, the Bull, with brazen Hoofs, tamed by *Jaſon*; *Gemini*, the Twins, *viz. Caſtor* and *Pollux*, two of the *Argonauts*; the Ship *Argo*, and *Hydrus* the Dragon, *&c.* which all manifeſtly relate to the Affairs of that Expedition, which happen'd about forty or fifty Years after *Solomon*'s Death.

In order to underſtand the following Problems, it will be neceſſary to premiſe the following Definitions in relation thereto, *viz.*

I. The Declination of the Sun and Stars is their Diſtance from the *Equinoctial* in Degrees of the general Meridian, towards either Pole, *North* or *South*.

II. Right Ascension is that Degree of the Equinoctial reckon'd from the Beginning of *Aries*, which comes to the Meridian with the Sun or Star.

III. Oblique Ascension is that Degree of the Equinoctial which comes to the Horizon when the Sun or Star is riſing: And *Oblique Deſcenſion* is that Point which comes to the Horizon on the Weſt Part, when the Sun or Star

3. By this means they could make Catalogues of the Stars, record their Places in the Heavens, and call them all by their Names. *Hipparchus* is ſaid to be the firſt who framed a Catalogue of the Stars, which was afterwards copied by *Ptolomy*, and adjuſted to his own Time, *A. D.* 140. The Number in this was 1026. After this *Ulug Beigh* made a Catalogue of 1022, reduced to the Year 1437. *Tycho Brahe* rectified the Places of 1000 Stars; but his Catalogue, publiſh'd by *Longomontanus*, contains but 777, for the Year 1600. *Bayer* publiſh'd a Catalogue of 1160. *Hevelius* compoſed a Catalogue of 1888 Stars, adjuſted to the Year 1660. But the largeſt and moſt compleat of all is the *Britiſh* Catalogue by Mr. *Flamſted*, containing about 3000, of which ſcarce 1000 can be ſeen by the naked Eye in the cleareſt and darkeſt Night. They are rectified for the Year 1689. They are diſtinguiſh'd into ſeven Degrees of Magnitude, in their proper Conſtellations; whoſe Names, Latitudes, and Longitudes here follow, together with the Number of Stars in each, and of each particular Magnitude, as I have taken them from the third Volume of the *Hiſtoria Cœleſtis*. Note, The firſt Latitude is South, the other North, in the Twelve Signs, unleſs mark'd to the contrary.

is deſcending or ſetting in an oblique Sphere.

IV. ASCENSIONAL DIFFERENCE is the Difference between the *Right* and *Oblique Aſcenſion.*

V. THE LONGITUDE of the Sun or Star is an Arch of the Ecliptic, between the firſt Point of *Aries*, and that Point of the Ecliptic to which the Luminary is referr'd by the Meridian paſſing through it; and is therefore reckon'd in Signs and Degrees of the Ecliptic.

4. The *Conſtellations* of the TWELVE SIGNS.

Names.	*Long.* ° ′	*Lat.* ° ′	*Num.*	1	2	3	4	5	6	7
Aries.	♈ 26 48 ♉ 21 06	00 01 12 31	65	0	1	2	5	6	28	23
Taurus.	♉ 16 49 ♊ 26 36	18 27 09 46	135	1	1	4	13	21	44	51
Gemini.	♊ 14 11 ♋ 12 33	10 07 13 18	79	1	2	4	6	12	32	22
Cancer.	♋ 22 49 ♌ 12 19	10 19 14 59	71	0	0	0	6	7	39	19
Leo.	♌ 10 57 20 42	07 39 17 38	95	2	2	6	15	10	50	10
Virgo.	♍ 00 10 ♎ 29 23	00 24 21 24	89	1	0	5	10	19	45	9
Libra.	♏ 04 11 28 35	18 34 *N* 11 27 *S.*	49	1	2	7	5	11	21	2
Scorpio.	♏ 26 48 ♐ 20 46	12 46 *N.* 13 57 *S.*	51	0	2	2	12	5	25	5
Sagittarius.	♐ 22 55 ♑ 26 29	10 59 07 31	50	0	1	5	6	11	23	4
Capricornus.	♑ 27 26 ♒ 21 29	08 53 07 27	51	0	0	3	3	9	34	2
Aquarius.	♒ 07 24 ♓ 21 57	21 04 23 02	99	1	0	4	7	31	50	6
Piſces.	♓ 11 06 ♈ 26 47	23 06 09 05	109	0	0	1	6	27	54	21

VI. THE LATITUDE of a Star is its Diſtance from the Ecliptic towards the North or South Pole.

VII. AMPLITUDE is the Diſtance at which the Sun or Star riſes or ſets, from the Eaſt or Weſt Point of the Horizon, towards the North or South.

VIII. AZIMUTH is the Diſtance between the

5. The *Conſtellations* of the NORTHERN HEMISPHERE.

Andromeda.	♈ 3 29 ♉ 18 4	15 55 49 53	66	0	3	2	12	13	34	2 *neb.*
Aquila cum Antinoo.	♑ 29 46 ♒ 8 6	10 5 43 27	70	1	0	10	7	15	32	5
Anſer cum Vulpe.	♑ 20 20 ♓ 1 14	37 39 47 46	34	0	0	0	4	12	18	0
Auriga.	♊ 11 22 ♋ 12 30	2 29 32 13	68	1	2	1	10	18	31	5
Bootes.	♍ 22 34 ♏ 00 54	25 15 60 33	55	1	0	8	10	11	17	8
Caſſiopeia.	♈ 21 6 ♊ 8 4	38 18 59 53	56	0	0	5	7	9	30	5
Camelopardus.	♊ 10 40 ♋ 10 39	29 24 45 43	58	0	0	0	4	18	27	9
Cepheus.	♈ 00 39 ♉ 26 37	59 32 75 27	35	0	0	3	7	8	14	3
Coma Berenices.	♍ 16 53 ♎ 4 38	15 14 33 56	40	0	0	0	8	14	14	4
Corona Septen.	♏ 00 58 20 54	44 21 56 25	21	0	1	0	6	8	6	0
Cygnus.	♑ 20 55 ♓ 23 17	37 39 74 10	107	0	1	6	21	31	48	0
Delphinus.	♒ 8 49 16 31	23 00 33 44	18	0	0	6	0	2	9	1
Draco.	*Per totam Circumf.*	57 13 87 25	49	0	1	7	8	13	19	1
Equuleus.	♒ 14 12 21 7	20 9 25 13	10	0	0	0	4	1	5	0

North Point of the Horizon, and the Point where the Vertical Circle, passing through the Body of the Sun or Star, cuts the Horizon.

IX. THE ALTITUDE of the Sun or Star is its Height above the Horizon, measured in the Degrees of the *Quadrant of Altitude*, or moveable *Azimuth Circle*.

Hercules.	♎ 28 7 ♐ 28 29	25 15 69 33	95	0	0	11	15	31	38	0
Leo Minor.	♋ 29 8 ♍ 4 4	9 21 30 50	53	0	0	1	5	11	33	3
Lacerta.	♓ 19 49 6 27	43 12 55 34	16	0	0	0	3	6	7	0
Lynx.	♊ 28 24 ♌ 9 45	17 3 40 39	44	0	0	0	3	12	21	8
Lyra.	♑ 3 35 26 14	54 28 66 13	21	1	0	2	2	5	11	0
Perseus. C. M.	♉ 8 8 ♊ 10 48	11 17 41 13	67	0	2	5	11	15	28	6
Pegasus.	♒ 23 37 ♈ 7 17	9 13 44 24	93	0	4	3	10	13	58	5
Sagitta.	♑ 20 00 ♒ 8 37	35 35 43 15	23	0	0	0	4	1	18	0
Serpens Ophiuchi.	♏ 7 38 ♑ 11 31	7 59 42 28	59	0	1	7	6	3	32	10
Scutum.	♑ 0 23 10 8	4 59 18 17	7	0	0	0	2	4	1	0
Serpentarius, or *Ophiuchus.*	♏ 27 58 ♑ 1 29	6 54 37 18	69	0	1	7	15	13	26	7
Triangulum.	♉ 0 5 13 15	13 55 20 34	15	0	0	0	3	1	7	4
Ursa Major.	♊ 10 41 ♎ 6 58	17 6 61 3	215	0	6	5	35	58	91	20
Ursa Minor.	♊ 21 43 ♌ 17 19	65 42 77 50	24	0	0	4	3	5	6	6
Canes Venatici.	♍ 0 5 25 43	52 52 33 56	25	0	1	0	2	5	14	3

X. A Star is ſaid to riſe or ſet *Coſmically*, when it riſes or ſets when the Sun riſes.

XI. A Star riſes *Acronically*, if it riſes when the Sun ſets.

XII. A Star riſes *Heliacally*, when it emerges out of the Sun-beams, and is ſeen in the Morn-

6. *Conſtellations in the* SOUTHERN HEMISPHERE.

Ara cum Thuribulo.	♐ 15 6 27 18	23 5 37 15	9	0	0	1	6	2	0	0
Argo, or *Navis.*	♋ 24 57 ♌ 14 2	22 24 49 14	25	0	0	4	6	6	9	0
Apus.	♐ 9 45 21 24	44 32 62 4	11	0	0	0	4	3	4	0
Canis major.	♊ 3 7 ♋ 25 12	34 44 59 14	32	1	7	1	4	11	5	3
Canis minor.	♋ 16 48 ♌ 0 49	9 45 23 47	15	1	0	1	0	3	9	1
Cetus.	♓ 18 36 ♉ 14 30	2 42 34 14	78	0	2	6	13	9	44	4
Centaurus, cum Lupo.	♎ 25 42 ♏ 28 30	11 28 21 59	13	0	1	0	5	6	1	0
Cameliontis.	♏ 26 30 ♐ 3 39	63 35 75 24	10	0	0	0	0	9	1	0
Columba Noahi.	♊ 14 54 ♋ 6 46	55 42 60 41	10	0	2	0	1	6	1	0
Corona Auſtr.	♑ 1 18 10 14	12 28 22 36	12	0	0	0	1	3	8	0
Corvus.	♎ 6 26 13 3	10 21 21 44	10	0	0	3	2	2	3	0
Crater.	♍ 19 26 ♎ 3 58	11 18 22 42	11	0	0	0	8	2	2	0
Eridanus.	♈ 16 38 ♊ 11 15	18 26 54 33	68	0	0	12	15	20	20	1
Grus.	♒ 14 54 18 2	39 43 41 55	3	0	0	0	2	1	0	0
Hydrus.	♑ 26 59 ♈ 3 37	64 10 78 5	10	0	0	4	2	3	1	0

ing

ing before Sun-rising: And it sets *Heliacally*, when it is so near the Sun that it cannot be seen.

XIII. A *Right Sphere* is that whose Poles are in the Horizon and the Equinoctial, and all its Parallels cut the Horizon at Right Angles. Plate LXVII, Fig. 1, 2, 3.

XIV. A *Parallel Sphere* is that whose Poles co-incide with the Poles of the Horizon, or *Ze-*

Lepus.	♊ 6 44 28 9	34 45 45 46	19	0	0	3	7	3	6	0
Musca.	♏ 16 20 22 22	55 11 58 47	4	0	0	0	2	2	0	0
Monosceros.	♊ 29 34 ♌ 10 50	13 13 31 11	19	0	0	0	10	7	2	9
Orion.	♊ 7 32 ♋ 15 11	3 11 34 4	80	2	4	4	25	20	25	0
Pavo.	♐ 24 7 ♑ 24 41	36 11 50 49	14	0	1	3	5	4	1	0
Phœnix.	♒ 29 47 ♓ 24 14	31 39 55 5	13	0	1	5	6	1	0	0
Piscis Volans.	♎ 11 19 ♏ 14 11	67 52 82 35	8	0	0	0	0	7	1	0
Robur Carolinæ.	♎ 3 34 ♏ 7 6	51 2 72 12	12	0	1	2	7	2	0	0
Sextans.	♌ 19 59 ♍ 13 5	1 21 19 43	41	0	0	0	1	7	32	1
Toucan.	♒ 3 16 22 43	45 27 59 46	9	0	0	4	2	3	0	0
Triangulum.	♐ 5 35 17 2	41 32 48 1	5	0	1	2	0	2	0	0
Xiphias.	♐ 7 36 ♉ 18 32	70 12 88 14	6	0	0	1	2	1	2	0

	Num.	1	2	3	4	5	6	7
7. In the *Zodiac*,	943	7	11	43	94	169	445	174
In *Northern Hemisphere*,	1511	4	23	93	227	356	695	113
In *Southern Hemisphere*,	547	4	20	56	136	145	176	10
Sum of all the STARS.	3001	15	54	192	457	670	1316	297

with

nith and *Nadir*; and the Equinoctial with the

8. The Use of such a Catalogue of Stars is very great; for from hence we learn, (1.) If any *new Stars* at any time appear, which have never been observed before. (2.) If any Star, which now appears, shall in Time to come disappear. (3.) If the *new Star* which shall appear be the same with a Star that has disappear'd formerly; and therefore, (4.) If the Stars have any periodical Times of Apparition. Hence (5.) The Means or Method of predicting the Appearing or Disappearing of Stars. (6.) By a Catalogue of the Stars we compare their respective Places, Situations, and Distances with Ease. (7.) By this means we also compare and determine the true Places and Motions of the heavenly Bodies in general, and of the Sun, Moon, Planets, and Comets in particular, with many other useful Purposes it serves besides.

9. Now it is actually Fact, that some new Stars appear, and that others disappear; yea, that they change their apparent Magnitude, and disappear by degrees. *Hipparchus* the first of Men observed a new Star, (120 Years before *Christ*) which occasion'd his making a Catalogue of the Stars. Another is said to have appear'd *A. D.* 130; another *A. D.* 389; one exceeding bright in the 9th Century, and another in the Year 1264.

10. But the first *new Star*, of which we have any good Account, is that in the Chair of *Cassiopeia*, first observed by *Cornelius Gemma* on the 9th of *November* 1572, and by *Tycho Brahe* on the 11th. Sir *Isaac Newton* says it equall'd *Venus* in Brightness at its first Appearance, and gradually declined in its Lustre, till it totally disappear'd in the *March* following. This Star is supposed to be the same that appear'd in the Years 945 and 1264, having its Period about 310 or 320 Years.

11. In *Aug.* 13, 1596, *D. Fabricius* observed another *new Star* in the *Neck of the Whale*; and through the 17th Century this Star was observed to appear and disappear periodically, its Period being equal to 333 Days. The Phænomena of this and the like Stars are supposed to be owing to the Spots on their Surface, which sometimes increase and sometimes decrease, in the manner as we have observed they do on the Surface of our Sun.

12. For that the Stars are really *Suns*, and have each a System of Planets, &c. about them, like ours, can be no Doubt to those who understand the Rules of Reasoning rightly, as I have before observed, *Annot.* CXXXI. And there-

Horizon

Horizon; and all the Parallels parallel thereto.

fore as they revolve about their Axis, those Spots may cause a great Alteration of Lustre, and sometimes wholly obscure them for a time. But it is no Wonder if Bodies at such a Distance should have Appearances produced by Causes quite unknown to us. See more on this Head in Dr. *Long*'s *Astronomy*.

13. As to the Distance of the Fix'd Stars, we had but small Hopes of any Estimation of it, till Dr. *Bradley* began his Observations on them with an Instrument so very exact, as that he is of Opinion, if the Parallax of a Star amounted to but one single Second, he must have observed it; and therefore that such a Star must be above 400000 times farther from us than the Sun.

14. For if S represent the Sun, T the Earth, ATB its Orbit, and R a Star at such a Distance SR or TR, that the Semidiameter of the Orbit ST shall subtend an Angle TRS $= 30'''$, or half a Second, then we find the Distance SR by this Analogy: Pl. LXV. Fig. 4.

As the Tangent of the Angle TRS	$= 30''' =$	4.371914
Is to Radius	$90° =$	10.000000
So is the Sun's Distance	$ST = 1 =$	0.000000
To the Distance of the Star SR	$= 424700 =$	5.628086

15. But the Distance of the Sun ST = 20000 Semidiameters of the Earth (see *Annot*. CXXXIV.); and supposing SR = (TR =) 400000 ST, then is the Distance of the Star from the Earth TR = 400000 x 20000 = 8000000000 Semidiameters of the Earth, or 8000000000 x 4000 = 32000000000000 Miles of *English* Measure. Hence it appears, that though the Velocity of Sound be so very great as at the Rate of 1142 Feet *per* Second, or 7000000 Miles *per Annum*, yet it would take up 4571430 Years to pass from the nearest Star to us. A Cannon-Ball would take up twice that Time to pass from us to the Star; (see *Annot*. XXV. 4.) yea, Light itself, with the inconceivable Velocity of 10000000 Miles *per* Minute, takes up more than 6 Years in coming from the Star to us. Therefore how immensely great must those Luminaries be, which appear so bright, and of such different Magnitudes, at such immense Distances!

16. The different apparent Magnitudes of the Stars are owing to their different Distances from us. Had we Telescopic Eyes, we should see many more. *Seventy Stars*, and more, have been discover'd in the *Pleiades* (commonly call'd

XV. An

XV. AN *Oblique Sphere* is that, one of whose

the *Seven Stars*; and all that Tract of the Heavens call'd the *Milky Way* (or *Galaxy*) is well known to be owing to the Refulgence of a prodigious Multitude of Stars disseminated thro' those Parts of the Universe, though at so great a Distance as to be invisible to the naked Eye; yet are they discernible in great Numbers through a Telescope, and more in Proportion as the Instrument is better.

17. Hence likewise we account for that particular Phænomenon we call a *nebulous Star*, or cloudy faintish bright Spots that appear like Stars in an indirect View; for in order to this you have no more to do than only to direct a good Telescope to any one of them, and you will be agreeably surprized with a View of a great Multitude of very small Stars, which were the Cause of the *luminous Spot* to the naked Eye.

18. To the very small apparent Magnitude of the Stars we owe their constant *Twinkling*; for being but lucid Points, every opake Corpuscle or Atom floating in the Air will be big enough to cover and eclipse them, when they get in the Right Line between the Star and the Eye; which Alternations of momentary Occultations and Apparitions make the Twinkling of the Stars we now speak of.

19. I shall here give a fuller Account of the small elliptic apparent Motion of each Star about its true Place, which I have already begun in a former *Annotation*. And in order to understand the Force of the Argument, the following Representations are necessary, *viz*. Let S be the Sun, ABCD the Earth's Orbit; and from S suppose a Perpendicular erected, as SP, passing through a Star at P. Now if the Spectator were at S, he would view the Star in the same Perpendicular, and in its true Place P, projected in the Point *p* in the visible Surface of the Heavens. But if the Spectator be carried about the Sun in the Circle ABCD, whose Diameter is sensible at the Distance P, or subtends a sensible Angle APC, then in the Position A he will see the *Phænomenon* P in the Right Line AP*a*, projected in the Point *a*. For the same Reason, in the Points B, C, D, the Star will appear in *b*, *c*, *d*; so that it will seem to have described the little Circle *abcd*.

Plate LXV. Fig. 5, 6, 7.

20. If the Distance of the Star SP be so great, that the Diameter of the Earth subtends no sensible Angle, but appears as a Point, then will also the small Circle *abcd* become insensible; and all the Lines AP, BP, &c. may be esteem'd perpendicular to the Plane of the Ecliptic, and be directed to the same Point in the Heavens with the Perpendicular SP,

Poles

Poles is above the Horizon, and the other below as to Senſe. So that in this Caſe the Star P would ever appear in the ſame Point *p*, if Light were propagated in an Inſtant.

21. But if in this very Caſe, in which the Star is ſo remote, Light be propagated in Time, or with a certain Velocity, then as the Earth deſcribes its Orbit a Spectator will ſee the Star in an oblique Direction, and not in the Perpendicular, as we have formerly ſhewn: That is, if GF be a Tangent to the Earth's Orbit in B, and BE perpendicular to the Plane of the Ecliptic in the Point B, then while the Earth moves through the indefinitely ſmall Arch GB, a Star at E will appear to move from E to *e*, or to be in *e* when the Earth arrives at B.

22. Now ſince the Diſtance SB is but a Point with reſpect to the great Diſtance SP of the Star, it follows, that we may refer the Spectator from the ſeveral Points A, B, C, D, to the central Point S, for obſerving the *Phænomena* of the Star at P, which will not be alter'd thereby. Therefore if *ca* be parallel to AC, and you make the Angle PS*a* equal to the Angle EB*e*, 'tis plain the Star P muſt appear in *a*, in the Direction S*a*. Alſo when the Earth is at D, the Star will be ſeen in the oblique Direction S*c* at *c*, the Spectator being referr'd to S.

23. For the like Reaſon, *viz.* becauſe *bd* is parallel or alike ſituated in reſpect of DB, and to the Tangents in D and B, therefore the Star at P will appear in *d* and *b* when the Earth is at C and A; and ſo during the Space of one Year the Star P will appear to deſcribe a ſmall Circle *adcb*, ſuppoſing the Star in the Zenith E of the Spectator; but if the Star be at any Diſtance from the Zenith, the ſaid ſmall Circle will become an Ellipſe, as in *Fig.* 7.

24. Theſe ſmall elliptic Motions of the Stars occaſion'd their Declinations to vary, and alſo their Diſtances from the Poles of the World, and that by the Space of $20''\frac{1}{4}$ on one Side and on the other. Now this could not happen on any account of Refraction, becauſe the ſame thing was as well obſerved of Stars near the Zenith, where there is no Refraction, as elſewhere ſituated. Nor could it reſult from any *Nutation* of the Earth's Axis; for that would have made the equal Diſtances of the Stars on oppoſite Sides of the Pole unequal, which never happen'd.

25. Neither can this be a *Parallactic Motion* of the Stars; for then while the Earth deſcribed the Half of its Orbit ABC,

it;

it; and the Equinoctial and its Parallels obliquely cutting the same (CXLVI).

the Star would describe the Semicircle *abc*; whereas it is found by Observation, that the Star describes the said Semicircle *abc* while the Earth describes its Semi-Orbit BCD. (See *Art.* 22, 23.) Therefore it must arise solely from *the Velocity of Light bearing a sensible Proportion to the annual Motion of the Earth*; which accounts for all the *Phænomena* to the greatest Exactness, without any the least Difficulty or Intricacy; as they may see who will consult the *Professor*'s own Account in the *Transactions*, and what Mr. *Symson* and Mr. *Mac Laurin* have wrote on this Subject.

Plate LXVII. Fig. 1. (CXLVI) 1. The three Positions of the Sphere here described are represented in so many Figures; the first of which is the *Direct* or *Right Sphere*, which is proper to those People only who live under the Equinoctial Circle ÆQ, because to them the Poles of the World P and S will both be in the Horizon HO.

Fig. 2. 2. The second Figure represents the *Parallel Sphere*, where the Axis of the Earth PS is perpendicular to the Horizon, or the Poles P, S, are in the *Zenith* and *Nadir*. This Position of the Sphere is peculiar to the Parts of the Earth under each Pole; whose Inhabitants, if any there were, would perceive no circular Motion of the Sun, Moon, or Planets, nor any Motion of the Stars at all. But this must be understood of a Person standing precisely on the Ends of the Earth's Axis, which are the only Points on the Earth's Surface which have no real Motion, and consequently which can produce no apparent Motion.

Fig. 3. 3. The *Oblique Sphere* is represented in the third Figure. In this the Axis of the World PS makes an Angle PEO with the Horizon HO, of a greater or lesser Number of Degrees according to the Latitude of the Place. Hence it appears, that all the Inhabitants of the Earth have such a Position of the Sphere, except those under the *Equinoctial* and the *Poles*.

4. The Arch PO measures the *Altitude* or Height of the Pole, or what is commonly call'd the *Pole's Elevation*; and this Arch PO is ever equal to the Latitude of the Place ÆZ; as will easily appear thus: It is ÆZ + ZP = (ÆP = Quadrant =) ZP + PO = ZO; if therefore from the two equal Quadrants ÆP = ZO you subduct the common Part

THE

THE *Problems* on the *Celestial Globe* are the following.

PROB. I. *To rectify the Globe:*

ELEVATE the Pole to the Latitude of the Place, and every thing as directed under PROB. II. of the *Terrestrial Globe*, which see.

PROB. II. *To find the* SUN'S PLACE *in the Ecliptic:*

FIND the Day of the Month in the Calendar on the Horizon, and right against it is the Degree of the Ecliptic which the Sun is in for that Day.

PROB. III. *To find the Sun's* DECLINATION:

RECTIFY the Globe, bring the Sun's Place in the Ecliptic to the Meridian, and that Degree which it cuts in the Meridian is the Declination required.

or Arch ZP, the remaining Arches ÆZ = PO; which was to be shewn.

5. Hence appears also the Reason of the Method of rectifying the Sphere or Globe for any given Place Z, or Latitude ÆZ, *viz.* because if the Pole P be elevated so high above the Horizon as the Place is distant from the Equator, the said Place will then be the highest Point of the Globe, and consequently that to which alone all the *Phænomena* of the Heavens and the Earth, in such a Position of the Globe, can agree.

6. Hence also we observe, that the Complement of the Latitude ZP is equal to the Elevation of the Equator ÆH above the Plane of the Horizon. For ÆZ + ZP = (ÆP = ZH =) ZÆ + ÆH; therefore subduct the common Part ÆZ, and there remains on each Side ZP = ÆH; which was to be shewn. Whence the Angle ZEP = ÆEH.

7. Any Great Circle of the Sphere passing through the Zenith and Nadir Z and N, as ZEN, ZAN, are call'd *Azimuths* or *Vertical Circles*; of which that which passes through the East and West Points of the Horizon, as ZEN, is call'd

PROB. IV. *To find the Sun's* RIGHT ASCENSION:

BRING the Sun's Place to the Meridian, and the Degree in which the Meridian cuts the Equinoctial is the Right Afcenfion required.

PROB. V. *To find the Sun's* AMPLITUDE:

BRING the Sun's Place to the Horizon, and the Arch of the Horizon between it and the Eaft or Weft Point is the Amplitude, North or South.

PROB. VI. *To find the Sun's* ALTITUDE *for any given Day and Hour:*

BRING the Sun's Place to the Meridian; fet the Hour-Index to the upper XII; then turn the Globe till the Index points to the given Hour, where let it ftand; then fcrewing the Quadrant of Altitude in the *Zenith*, lay it over the Sun's Place, and the Arch contained between it and the Hori-

the *Prime Vertical*. The Arch of the Horizon AE is the *Amplitude* of a Phænomenon emerging above the Horizon at the Point A; this is call'd the *Ortive Amplitude*, becaufe it is *rifing*; as on the Weftern Side it is call'd the *Occafive Amplitude*, becaufe it is there *fetting*. The Arch AB meafured by a Quadrant of Altitude ZA is the *Altitude* of any Celeftial Body at B, above the Horizon.

8. As I judge this a proper Place, I fhall here explain the *Philofophical Principles* of GNOMONICS, or the *Art of* DIALLING. In order to this we are to confider, that as the Time which paffes between any Meridian's leaving the Sun and returning to it again is divided into 24 Hours, fo if we conceive a Sphere to be conftructed with 24 of thefe Meridians, the Sun will orderly come upon or be in one of them at the Beginning of every Hour. Such a Sphere may be reprefented by the Figure PDSB, where the feveral Meridians are reprefented by P1S, P2S, P3S, and fo on to twice 12, or 24 in all. Plate LXVII. Fig. 4.

9. Since thefe Meridians divide the Equinoctial into 24 equal Parts, each Part will contain juft 15°, becaufe 15 x 24

zon

zon will give the Degrees of Altitude required.

PROB. VII. *To find the Sun's* AZIMUTH *for any Hour of the Day:*

EVERY thing being done as in the laſt Problem, the Arch of the Horizon contain'd between the North Point and that where the Quadrant of Altitude cuts it, is the *Azimuth* Eaſt or Weſt, as required.

PROB. VIII. *To find the Time when the Sun riſes or ſets:*

FIND the Sun's Place for the given Day; bring it to the Meridian, and ſet the Hour-Hand to XII; then turn the Globe till the Sun's Place touches the Eaſt Part of the Horizon, the Index will ſhew the Hour of its Riſing: After that, turn the Globe to the Weſt Part of the Horizon,

= 360° = the whole Circle; and ſince all the Meridians paſs through the Poles of the World, the Planes of thoſe Meridians all interſect each other in one common Line PS, which is the Axis of the Sphere, therefore the ſaid Axis PS is in the Plane of each of the 12 Meridians.

10. Suppoſe Z to be the Zenith of any Place, as *London*, and DWBE the Plane of the Horizon fix'd within the Sphere, conſtructed with the ſaid 12 *Meridians* or *Hour-Circles*, 1,1; 2,2, 3,3, 4,4, &c. then will the Axis of the Sphere PS paſs through the Center of the Plane at N, ſo that one Half NP will be above the Plane, and the other Half NS below it.

11. Suppoſe now this *Dialling-Sphere* to be ſuſpended by the Point Z, and moved about ſo as to have the Points D and B exactly in the *South* and *North* Points of the Horizon, and E and W in the *Eaſt* and *Weſt* Points; then will the Sphere have a Situation every way ſimilar to that of the Earth and Heavens with reſpect to the given Place *London*, and the Axis of the Sphere to that of the Earth.

12. Therefore the Sun ſhining on ſuch a Sphere will be attended with all the ſame Incidents, and produce all the ſame Effects, as would happen if the ſaid Sphere were at the Cen-

and the Index will shew the Time of its Setting for the given Day.

PROB. IX. *To find the Length of any given Day or Night:*

THIS is easily known by taking the Number of Hours between the Rising and Setting of the Sun for the Length of the Day; and the Residue, to 24, for the Length of the Night.

PROB. X. *To find the Hour of the Day, having the Sun's Altitude given:*

BRING the Sun's Place to the Meridian, and set the Hour-Hand to XII; then turn the Globe in such manner, that the Sun's Place may move along by the Quadrant of Altitude, (fix'd in the *Zenith*) till it touches the Degree of the given Altitude; where stop it, and the Index will shew

ter of the Earth, or the Center N of the Sphere coincided with the Center of the Earth; because the Distance betwixt the Surface and Center of the Earth is insensible at the Distance of the Sun.

13. Now 'tis evident, as the Sun revolves about such a Sphere, it will every Hour be upon one Half or other of the 12 Hour-Circles; *viz.* from Midnight to Noon it will be on those Parts of the Circles which are in the *Eastern Hemisphere*, and from Noon to Midnight it will pass over all those in the *Western*. It is also farther evident, that while the Sun is in the Eastern Hemisphere it will be first below and then above the Plane of the Horizon, and *vice versa* on the other Side.

14. Again: When the Sun is upon any one of these Hour-Circles, by shining upon the Axis it causes it to cast a Shadow on the contrary Side, on the Plane of the Horizon, on the nether or upper Surface, as it is below or above the said Plane. This Shadow of the Axis will be precisely in the Line in which the Plane of the Hour-Circle would intersect the Plane of the Horizon: If therefore Lines were drawn through the Center N, joining the Points on each Side the Plane where the Hour-Circles touch it, as 4N4, 5N5, 6N6, &c. the

on

on the *Horary Circle* the Hour required.

PROB. XI. *To find the Place of the* MOON *or any* PLANET, *for any given Day:*

TAKE *Parker*'s or *Weaver*'s *Ephemeris*, and against the given Day of the Month you will find the Degree and Minute of the Sign which the Moon or Planet possesses at *Noon*, under the Title of *Geocentric Motions*. The Degree thus found being mark'd in the Ecliptic on the Globe by a small Patch, or otherwise, you may then proceed to find the *Declination*, *Right Ascension*, *Latitude*, *Longitude*, *Altitude*, *Azimuth*, *Rising*, *Southing*, *Setting*, &c. in the same manner as has been shewn for the Sun.

PROB. XII. *To explain the Phænomena of the* HARVEST-MOON.

IN order to this we need only consider, that

Shadow of the Axis will fall on those Lines at the Beginning of each respective Hour, and thereby indicate the Hour-Circle the Sun is in for every Hour of the Day.

15. These Lines are therefore properly call'd *Hour-Lines*; and among the rest, that which represents the Hour of 12 at Noon is NB, half the Meridian-Line DB; whence it appears, that the Hour-Lines N 1, N 2, N 3, &c. which serve for the Afternoon, lie on the East Side of the Plane, and are number'd from the North to the East; and on the contrary.

16. It also appears, that as the Sun's Altitude above the Plane is greater or less, the Number of Hour-Circles the Sun will possess above the Horizontal Plane will be also greater or less. Thus when the Sun is at S in the Equinoctial, its *diurnal Path* for that Day being the Equinoctial Circle itself ÆEQW, 'tis plain, since the Arch ÆE = EQ, the Sun will apply to six Hour-Circles below the Horizon, and to six above it, in each Half of the Day; and consequently, that on that Day the Shadow will occupy but 12 of the Hour-Lines on each Surface of the Plane, beginning and ending at 6.

when the Sun is in the Beginning of *Aries*, the Full Moon on that Day must be in the Biginning of *Libra:* And since when the Sun sets, or Moon rises, on that Day, those Equinoctial Points will be in the Horizon, and the Ecliptic will then be least of all inclined thereto, the Part or Arch which the Moon describes in one Day, *viz.* 13 Degrees, will take up about an Hour and a Quarter ascending above the Horizon; and therefore so long will be the Time after Sun-set, the next Night, before the Moon will rise. But at the opposite Time of the Year, when the Sun is in the *Autumnal*, and Full Moon in the *Vernal Equinox*, the Ecliptic will, when the Sun is setting, have the greatest Inclination to the Horizon; and therefore 13 Degrees will in this Case soon ascend, *viz.* in about a Quarter of an Hour; and

17. But when the Sun is in the Tropic of *Cancer*, its diurnal Path for that Day being the Tropic itself TCRF, 'tis manifest the Sun in the Forenoon ascends above the Plane in passing between the Hour-Circles of 3 and 4 in the Morning, and descends below it in the Afternoon between the Hours of 8 and 9: Therefore on the Summer-Tropic the Shadow will pass over 16 of those Hour-Lines. And *vice versa*, when the Sun is in the Winter-Tropic at O, its Path being then OGIH, it rises above the Plane between 8 and 9, and leaves it between 3 and 4.

Plate LXVII, Fig. 5. 18. From what has been said 'tis evident, that if the Circles be supposed removed, and only the horizontal Plane remain, with the Half of the Axis NP above it, in the same Position as before, then should we have constituted an HORIZONTAL DIAL, every way the same with those in common Use, as represented in the next Figure, with only the Addition of a Substyle PO, to render the Style NP very firm.

19. Hence appears the Reason why the *Gnomon* or Style NP in those Dials is always directed to the North Pole, and always contains such an Angle PNO with the Hour of 12 NB as is equal to the Latitude of the Place: Lastly, the Rea-

so

ſo long after Sun-ſet will the Moon riſe the next Day after the Full: Whence, at this Time of the Year, there is much more Moon-Light than in the Spring; and hence this Autumnal Full Moon came to be call'd the *Harveſt Moon*, the *Hunter's* or *Shepherd's Moon*: All which will clearly be ſhewn on the Globe.

PROB. XIII. *To repreſent the Face of the Starry Firmament for any given Hour of the Night:*

RECTIFY the Globe; and turn it about, till the Index points to the given Hour; then will all the upper Hemiſphere of the Globe repreſent the viſible Half of the Heavens, and all the Stars on the Globe will be in ſuch Situations as exactly correſpond to thoſe in the Heavens; which may therefore be eaſily found, as will be ſhewn.

ſon why the Number of Hour-Lines on theſe Dials exceeds not 16, and are all drawn from 6 to 12 and 6 again on the Northern Part, the reſt on the Southern; and why the Hour-Line of 6 lies directly *Eaſt* and *Weſt*, as that of 12 does *North* and *South*.

20. If a Plane be fix'd with the ſame Sphere in a Vertical Poſition, or perpendicular to the Horizon, and coinciding with the Plane of the *Prime Vertical*, i. e. facing full South and North; then will the Axis PS ſtill paſs through the Center of the Plane N, and the lower Semiaxis NS will by its Shadow mark out the Hour-Lines on the Southern Surface, and the upper Semiaxis NP will do the ſame on the Northern. Theſe Hour-Lines are determined in the ſame Manner as thoſe on the Horizontal Dial; and it is plain, the Sun cannot come on the Southern Face of this Plane before Six in the Morning, nor ſhine on it after Six in the Evening. Plate LXVII. Fig. 6.

21. Alſo it is evident, that all the Hours before Six in the Morning, and after Six at Night, will be ſhewn on the Northern Face or Side of this Plane, for the Time of the Sun's being above the Horizon in any Place. Hence the Reaſon of a *Direct South and North Vertical Dial* eaſily appears; the lat-

PROB. XIV. *To find the Hour when any known Star will rise, or come upon the Meridian:*

RECTIFY the Globe, and ſet the Index to XII; then turn the Globe till the Star comes to the Horizon or Meridian, and the Index will ſhew the Hour required.

PROB. XV. *To find at what Time of the Year any given Star will be on the Meridian at* XII *at Night:*

BRING the Star to the Meridian, and obſerve what Degree of the Ecliptic is on the North Meridian under the Horizon; then find in the Calendar on the Horizon the Day of the Year againſt that Degree, and it will be the Day required. (CXLVII).

Plate LXVII. Fig. 7. ter of which is here repreſented apart from the Sphere, with its Style NS, Subſtyle, and Hour-Lines: And the ſame may be conceived for a *North Erect Dial.*

22. The *Gnomon* NS contains an Angle SND = ZNP with the Meridian or Hour-Line of 12, *viz.* ZD, which is exactly the Complement of the former PNB to 90 Degrees; or the Elevation of the *Gnomon* is in theſe equal to the Complement of the Latitude of the Place: And what has been ſaid about the Reaſon of the Hour-Lines is the ſame for the Half-Hours, Quarters, &c. Likewiſe if the *Rationale* of a *Direct South Dial* be underſtood, nothing can be difficult to underſtand of a Dial which does not face the South or North directly, but declines therefrom any Number of Degrees towards the Eaſt or Weſt. But they who would know more of the Mathematical Structure and Calculations for all Sorts of Dials may have Recourſe to the Second Volume of my *Young Trigonometer's Guide*, or other Books on that Subject.

(CXLVII) 1. I ſhall here repreſent the Caſes of theſe Aſtronomical Problems, as they are performable by the Circles of the Celeſtial Globe, or by the *Stereographical Projection* of the Sphere *in Plano.* Thus

THESE

THESE are the chief *Problems* on the *Celestial Globe*: We now proceed to those on the *Terrestrial*; but shall first premise the following Definitions relating thereto.

I. THE LATITUDE of any Place is its Distance from the Equator towards either Pole; and is reckon'd in Degrees of the General Meridian, beginning at the Equator.

II. LONGITUDE is the Distance between the Meridian of any Place, and the first or standing Meridian, reckon'd in the Degrees of the Equator towards the East or West.

III. A CLIMATE is a Space of the Earth's Surface, parallel to the Equator, where the Length of the Day is *half an Hour* longer in the Parallel which bounds it on the North, than in that which terminates it on the South.

IV. A ZONE is also a Division of the Earth's Surface parallel to the Equator, in regard of the different Degrees of *Heat and Cold*, which we have described in the preceding Lecture.

V. THE ANTOECI are those Inhabitants of the Earth, who live under the same Meridian, but on opposite Parallels, and are therefore equally di-

Let ÆNQS be the General Meridian. Plate LXVIII. Fig. 1.
NS the Axis of the Sphere.
ÆQ the Equinoctial Line.
HO the Horizon of *London*.
♋C♑ the Ecliptic, or Sun's Path.
ZD the *Prime Vertical*, or *Azimuth*.
EP the Axis of the Ecliptic.
NAS an Hour-Circle or Meridian.
ZAD an Azimuth Circle.

stant

ſtant from the Equator. Their Noon and Midnight are at the ſame Time; the Days of one are equal to the Nights of the other; and their Seaſons of the Year are contrary.

VI. THE PERIOECI are thoſe People who live under the ſame Parallel, but oppoſite Meridians. The ſame Pole is elevated and depreſs'd to both; are equally diſtant from the Equator, and on the ſame Side; when Noon to one, it is Midnight to the other; the Length of Days to one is the Complement of Night to the other, and the contrary, and the Seaſons of the Year are the ſame to both, at the ſame Time.

VII. THE ANTIPODES are thoſe who live *Feet to Feet*, or under *oppoſite Parallels and Meridians.* They are equally diſtant from the Equator on different Sides; have the contrary Poles equally elevated; the Noon of one is Midnight to the other; the longeſt Day or Night to one is ſhorteſt to the other; and the Seaſons of the Year are contrary, *&c.*

VIII. ALSO the Inhabitants of the Torrid Zone are call'd AMPHISCII, becauſe their *Shadows fall on both Sides* of them.

IX. THOSE of the Frigid Zone are called

EYP a Circle of Longitude.
♋ I ♋ the Tropic of *Cancer.*
♑ ♑ the Tropic of *Capricorn.*

2. By means of theſe Circles various Spherical Triangles are form'd for Calculation. Thus let A be the Place of the Sun in the Ecliptic; then in the Right-angled Triangle AXÇ we have

CA the Sun's *Place*, or Longitude from the Equinox Ç,
AX the Sun's *Declination* North.

PERISCII,

PERISCII, because their *Shadows fall all around* them.

X. AND the Inhabitants of the Temperate Zones are call'd HETEROSCII, because they cast *their Shadows only one way.*

XI. A CONTINENT is the largest Division or Space of Land, comprehending divers Countries and Kingdoms, not separated by Water.

XII. AN ISLAND is any small Tract of Land surrounded by Water.

XIII. A PENINSULA is a Part of Land encompass'd with Water all around, except on one Part, which is call'd

XIV. AN ISTHMUS, being that narrow Neck of Land which joins it to the Continent.

XV. A PROMONTORY is a mountainous Part of Land standing far out in the Sea; whose Forepart is call'd a *Cape*, or *Head-Land.*

XVI. THE OCEAN is the largest Collection of Waters, which lies between, and environs the Continents.

XVII. THE SEA is a smaller Part of the aqueous Surface of the Earth, interceding the Islands, Promontories, *&c.*

XVIII. A GULF is a Part of the Sea every

CX the Sun's *Right Ascension.*

ACX the Angle of Obliquity of the *Ecliptic.*

3. And supposing the Sun rising in the Horizon at M on the Day of the Summer Tropic, and NMS an Hour-Circle; then there is form'd the Right-angled Triangle NOM, in which we have

NO = ÆZ = the *Latitude* of the Place Z.

MO the *Amplitude* from the North.

NM the Complement of the Sun's *Declination* RM.

where

where environ'd with Land, except on one small Part call'd

XIX. A STRAIT, which is that narrow Passage joining it to the adjacent Sea.

XX. A LAKE is any large Quantity of stagnant Water entirely surrounded by Land.

THE other Parts of Land or Water need no Explanation.

I SHALL now proceed to the Solution of the most useful *Problems* on the *Terrestrial Globe*, first premising that *the Latitude of a Place is equal to the Elevation of the Pole at that Place*; for if the Arch of the Meridian between the Place and the Pole be added to the Latitude of the Place, it makes 90 Degrees; also if it be added to the Pole's Elevation, or Arch between the Pole and Horizon, the Sum is 90 Degrees: Whence the Proposition is evident.

PROB. I. *To find the Latitude of any Place:*

BRING the given Place to the Brazen Meridian, and observe what Degree it is under, for that is the Latitude required.

PROB. II. *To rectify the Globe for any given Place:*

ONM the Angle of the *Hour* from Midnight.
OMN the Angle of the Sun's *Position*.

4. On the same Tropical Day the Sun is at I at Six o'Clock, because the Hour-Circle of Six is projected upon the Axis NCS; therefore in the Right-angled Triangle ICK we have
IK the Sun's *Altitude* at Six.
CK the *Azimuth* from the East at Six.
CI the *Declination* North.
ICK the *Latitude* of the Place.

RAISE

RAISE the Pole so high above the Horizon, as is equal to the Latitude of the Place; screw the Quadrant of Altitude in the *Zenith*; find the Sun's Place, and bring it to the Meridian; set the Hour Hand to the upper XII; and place the Globe North and South by a Needle; then is it a just Representation of the Globe of the Earth, in regard of that Place, for the given Day at Noon.

PROB. III. *To find the Longitude of a given Place:*

BRING the Place to the Brazen Meridian, and observe the Degree of the Equator under the same, for that expresses the Longitude required.

PROB. IV. *To find any Place by the Latitude and Longitude given:*

BRING the given Degree of Longitude to the Meridian, and under the given Degree of Latitude you will see the Place required.

PROB. V. *To find all those Places which have the same Latitude and Longitude with those of any given Place:*

BRING the given Place to the Meridian, then all those Places which lie under the Meridian have the same Longitude: Again, turn the Globe round on its Axis; then all those Places, which

5. Again; when the Sun on the same Day comes to the *Prime Vertical* ZCD, his Place when due *East* and *West* is at G; therefore in the Right-angled Triangle GBC we have

GB the Sun's *Declination* North.
GC the Sun's *Altitude* when East or West.
BC the *Hour* of his being due East or West.
BCG the *Latitude* of the Place.

6. Suppose the Sun in the Horizon at M once more; then in the Right-angled Triangle MCR we have

pass

paſs under the ſame Degree of the Meridian with any given Place, have the ſame Latitude with it.

PROB. VI. *To find all thoſe Places where it is Noon at any given Hour of the Day, in any Place:*

BRING the given Place to the Meridian; ſet the Index to the given Hour; then turn the Globe, till the ſaid Index points to the upper XII; and obſerve what Places lie under the Braſs Meridian, for to them it is Noon at that Time.

PROB. VII. *When it is Noon at any one Place, to find what Hour it is at any other given Place:*

BRING the firſt given Place to the Meridian, and ſet the Index to the upper XII; then turn the Globe till the other given Place comes to the Meridian, and the Index will point to the Hour required.

PROB. VIII. *For any given Hour of the Day in the Place where you are, to find the Hour of the Day in any other Place:*

BRING the Place where you are to the Meridian;

CM the *Amplitude* from Eaſt or Weſt.
MR the *Declination* North.
CR the *Aſcenſional Difference*.
RCM the *Co-Latitude* of the Place.
RMC the Angle of *Poſition*.

7. In the oblique Triangle AZN we have
ZN the *Co-Latitude* of the Place Z.
AN the *Co-Declination*.
AZ the Complement of the *Altitude* AF.
ANZ the *Hour* from Noon, equal to ÆX.
AZN the *Azimuth* from the North.

And the ſame may be done for any Star at A, or any other Place.

dian, set the Index to the given Hour; then turn the Globe about, and when the other Place comes to the Meridian, the Index will shew the Hour of the Day there, as required.

PROB. IX. *To find the Distance between any two Places on the Globe in* English *Miles:*

BRING one Place to the Meridian, over which fix the Quadrant of Altitude; and then laying it over the other Place, count the Number of Degrees thereon contain'd between them; which Number multiply by 69 and a half, (the Number of Miles in one Degree) and the Product is the Number of *English* Miles required.

PROB. X. *To find how any one Place bears from another:*

BRING one Place to the Brass Meridian, and lay the Quadrant of Altitude over the other; and it will shew on the Horizon the Point of the Compass on which the latter bears from the former.

8. Lastly, let Y be any Star, then in the oblique Triangle YNE we have

YE the *Co-Latitude* of the Star, *viz.* YK.
YE the *Co-Declination* of the Star.
NE = Æ♋ = the *Obliquity* of the Ecliptic.
NEY the Star's *Longitude* in the Ecliptic.
ENY the *Hour* for Midnight.

9. For the Canons and Method of Calculation I shall refer the Reader to the Second Volume of my *Young Trigonometer's Guide*, what I have done being as much as the Nature of the Subject at present requires: And those who have no Globes may solve most of these (and many other) Problems by my *Synopsis Scientiæ Cœlestis*, at a very small Expence, and with the greatest Exactness.

10. The Reason of the *Phænomenon* we call the HARVEST-MOON is extremely easy by the Globe, and may also be re-

PROB.

PROB. XI. *To find those Places to which the Sun is vertical in the Torrid Zone, for any given Day:*

FIND the Sun's Place in the Ecliptic for the given Time, and bring it to the Meridian, and observe what Degree thereof it cuts; then turn the Globe about, and all those Places which pass under that Degree of the Meridian are those required.

PROB. XII. *To find what Day of the Year the Sun will be vertical to any given Place in the Torrid Zone:*

BRING the given Place to the Meridian, and mark the Degree exactly over it; then turn the Globe round, and observe the two Points of the Ecliptic which pass under that Degree of the Meridian: Lastly, see on the Wooden Horizon, on what Days of the Year the Sun is in those Points of the Ecliptic; for those are the Days required.

Plate LXVIII. Fig. 2.

presented in a Diagram thus. Let HO be the Horizon, ÆQ the Equinoctial; then will T*r* be the Ecliptic, when the Beginning of *Aries* is in the Western Horizon; but when the other Equinox is there, *t*R will be the Position of the Ecliptic. On the *Vernal Equinox* if a Full-Moon happens, it will be at C in the Eastern Horizon at Rising; in one Day the Moon will describe the Arch C*c*; wherefore the following Night so much Time will intervene between Six o'Clock and the Hour of the Moon's rising, as is spent in the Motion of the Globe while the Arch C*c* is ascending above the Horizon.

11. Whereas at the opposite Time of the Year, *viz.* at the *Autumnal Equinox*, if a Full-Moon happen, then the next Night the Moon's diurnal Arch to be elevated above the Horizon is C*b* = C*c*; but since the Position of C*b* is so much nearer to the Horizon than C*c*, it will ascend much sooner above it, *viz.* in about one fifth Part of the Time, and some-

PROB.

PROB. XIII. *To find those Places in the North Frigid Zone, where the Sun begins to shine constantly without setting, on any given Day between the* 10th *of* March *and the* 10th *of* June.

FIND the Sun's Place in the Ecliptic for the given Day; bring it to the General Meridian; and observe the Degrees of Declination; then all those Places which are the same Number of Degrees distant from the Pole, are the Places required to be found.

PROB. XIV. *To find on what Day the Sun begins to shine constantly without setting, on any given Place in the North Frigid Zone, and how long:*

RECTIFY the Globe to the Latitude of the Place; and, turning it about, observe what Point of the Ecliptic between *Aries* and *Cancer*, and also between *Cancer* and *Libra*, co-incides with the North Point of the Horizon; then find, by

times in less, because the Moon's Orbit sometimes makes a greater Angle with the Horizon than TCH $=$ *a*C*c*, and sometimes a less Angle than *t*CH $=$ *a*C*b*. But for more on this Subject see my *Philosophical Grammar*.

12. Because in many Cases it is absolutely necessary to have a MERIDIAN-LINE at hand, I shall here shew the best Way of making or drawing such a one on any Plane where the Sun can shine, thus. Let a strait Brass-Pin or Steel-Wire AB be fix'd upright in the Point A, on which Point as a Center you had before described several concentric Circles, as CDE, FGH, &c. Now to make the Pin AB exactly perpendicular, let three Points be chosen in the outmost Circle, as F, G, H, in which place one Foot of the Compasses, and extend the other to the Top of the Pin B. The Pin is to be bent one way and the other, till the said Point of the Compasses Plate LXVIII. Fig. 3.

 the

the Calendar on the Horizon, what Days the Sun will enter those Degrees of the Ecliptic, and they will satisfy the Problem.

PROB. XV. *To find the Place over which the Sun is vertical, on any given Day and Hour:*

FIND the Sun's Place, and bring it to the Meridian, and mark the Degree of Declination for the given Hour; then find those Places which have the Sun in the Meridian at that Moment; and among them, that which passes under the Degree of Declination is the Place desired.

PROB. XVI. *To find, for any given Day and Hour, those Places wherein the Sun is then rising, or setting, or on the Meridian; also those Places which are enlighten'd, and those which are not:*

FIND the Place to which the Sun is vertical at the given Time, and bring the same to the Meridian, and elevate the Pole to the Latitude of the

will fall nicely on the Middle of the Top B from each Point of the Circle F, G, H, and then is the Pin well adjusted.

13. Then observing in the Forenoon where the Top of the Shadow AC touches any one Circle, there make a Mark, as at C; and then in the Afternoon make a Mark at E, where the Shadow's Point is in the same Circle again. Then bisect the Arch CE in the Point D, through which and the central Point A draw the Line AD, and it will be the *Meridian Line* required. If this be done in several Circles, the Operation will be the more exact and certain.

Plate LXVIII. Fig. 4, 5. 14. I have here added the Figures of the Celestial and Terrestrial Globe, with all the principal Circles and their Names, as they are rectified for the Latitude of *London*. *Note*, These Globes are made and sold by Mr. *Cushee*, at the *Globe and Sun* in *Fleet-street*.

Place;

Place; then all those Places which are in the Western Semicircle of the Horizon have the Sun *rising*, and those in the Eastern Semicircle see it *setting*; and to those under the Meridian it is *Noon*. Lastly, all Places above the Horizon are enlighten'd, and all below it are in Darkness or Night.

PROB. XVII. *The Day and Hour of a Solar or Lunar Eclipse being given, to find all those Places in which the same will be visible:*

FIND the Place to which the Sun is vertical at the given Instant, and elevate the Globe to the Latitude of the Place; then in most of those Places above the Horizon will the Sun be visible during his Eclipse; and all those Places below the Horizon will see the Moon pass through the Shadow of the Earth in her Eclipse.

PROB. XVIII. *The Length of a Degree being given, to find the Number of Miles in a great Circle of the Earth, and thence the Diameter of the Earth:*

ADMIT that one Degree contains $69\frac{1}{2}$ *English* Statute Miles; then multiply 360 (the Number of Degrees in a Great Circle) by $69\frac{1}{2}$, and the Product will be 25020, the Miles which measure the Circumference of the Earth. If this Number be divided by 3.1416, the Quotient will be $7963\frac{86}{100}$ Miles, for the Diameter of the Earth.

PROB XIX. *The Diameter of the Earth being known, to find the Surface in Square Miles, and its Solidity in Cubic Miles:*

ADMIT the Diameter be 7964 Miles; then multiply the Square of the Diameter by 3.1416, and the Product will be 199250205 very near, which are the Square Miles in the Surface of the Earth. Again, multiply the Cube of the Diameter by 0.5236, and the Product 264466789170 will be the Number of Cubic Miles in the whole Globe of the Earth.

PROB. XX. *To express the Velocity of the diurnal Motion of the Earth:*

SINCE a Place in the Equator describes a Circle of 25020 Miles in 24 Hours, 'tis evident the Velocity with which it moves is at the Rate of $1042\frac{1}{2}$ in one Hour, or $17\frac{3}{10}$ Miles *per* Minute. The Velocity in any Parallel of Latitude decreases in the Proportion of the *Co-Sine of the Latitude to the Radius*. Thus, for the Latitude of *London*, 51 deg. 30 min. say,

As Radius — — — —	10.000000
To the Co-sine of Lat. 51 deg. 30 m.	9.794149
So is the Velocity in the Equator, $17\frac{3}{10}$ M. — —	2.232046
To the Velocity of the City of *London*, $10\frac{8}{10}$ M. — —	2.032195

That is, the City of *London* moves about the Axis of the Earth at the Rate of $10\frac{8}{10}$ Miles every Minute of Time. But this is far short of the Velocity of the annual Motion about the Sun; for that

that is at the Rate of 60000 Miles *per* Hour, or about 1000 Miles each Minute, ſuppoſing the Diameter of the annual Orbit to be 82 Millions of Miles (CXLVIII.)

(CXLVIII) 1. I might here ſhew how the ſeveral Spherical Triangles are form'd for the Solution of moſt of theſe *Geographical Problems*, as I did before for the *Aſtronomical* ones; but as the Method is the ſame, I need not again repeat it. However, to facilitate the Ideas of the above Definitions, &c. I have added (as before mention'd) a Print of each Globe, as they are made with new Improvements by Mr. *R. Cuſhee* in *Fleetſtreet*. The *Rationale* of the ſeveral Methods of ſolving Problems of this Sort cannot be well ſhewn without an Eye upon the Globe, and a Praxis *cum vivâ voce* of a Demonſtrator.

2. I ſhall here ſubjoin a few Things relating to the Magnitude of the Earth, and the Dimenſions of the ſeveral Parts, together with the Manner of acquiring the Knowledge thereof. Firſt then, the moſt natural, eaſy, and certain Method of doing this is, by firſt *meaſuring the Length of a Degree of Latitude* in the Meridian of any Place; becauſe if the Meaſure of *one Degree* be once found, the Earth being ſuppoſed round, 'tis plain all the other Meaſures may eaſily be deduced from this.

3. Thus if I take the Height of the North-Pole-Star in this Place with a very good Quadrant or Sextant, and then proceed directly Northward or Southward, till by the ſame Inſtrument I find the ſaid Star raiſed or depreſſed juſt *one* Degree; then 'tis evident I muſt have paſs'd over juſt one Degree on the Earth's Surface, which therefore might be known by actual Menſuration, were it poſſible to find ſuch a Part of the Earth's Surface as is exactly even and ſpherical, and truly in the ſame Meridian.

4. Now this is ſcarcely to be expected any where, except in ſuch a Country as *Holland*, which is level, and when over-flow'd with Water, and that frozen into Ice, the icy Surface may be near the Truth; and a Degree meaſured in the Meridian upon this Ice muſt of courſe be pretty exact, if due Regard be had to Refractions in taking the Height of the Pole. Thus *Snellius* actually meaſured the Diſtance between a Tower at *Leyden* and another at *Souterwode* three times over, and then a ſtrait Line in the Meridian on the Ice, whence by a Trigonometrical Proceſs he meaſured a Degree;

THERE is a geometrical Method of describing the Superficies of the Celestial and Terrestrial Globe on a Plane; and this is call'd the *Projection*

but as some Mistakes had been made in the Calculations, the indefatigable Mr. *Muschenbroeck* attempted the Thing anew, and form'd Triangles upon the fundamental Base of *Snellius* in the Year 1700, and found 57033 *Toises* to a Degree.

5. Now this was but 27 Toises less than had been found by the Royal Academy of *Paris*; and this was but little different from the Measure of a Degree some time before by our Countryman *Norwood*, which resulted from his measuring the Distance between *London* and *York*, which he did in the Year 1635; and according to him the Length of a Degree is $69\frac{1}{2}$ of *English* Miles.

6. Mr. *Greaves* compared the *English* Foot taken from the Iron-Standard in *Guild-Hall*, *London*, with the Standards of divers Nations. The Proportion between some of them is as follows:

The *English* Foot,	1.000
The present *Roman* Foot,	0.967
The *Grecian* Foot,	1.007
The *Paris* Foot,	1.068
The *Leyden* or *Rhinland* Foot,	1.033
The *Bologna* Foot,	1.250

7. If the *French* Measure of a Degree, *viz.* 57060 Toises, be corrected by making proper Allowances for the *Precession of the Equinoxes*, the *Aberration of Light* in the Stars he made use of, and the *Refraction of Light* through the Air, (all which were neglected by *Picard*) the true Measure of a Degree at *Paris* will be 56925,7 Toises.

8. Now since the famous *Cassini* and Sir *Isaac Newton* had both of them shewn the Earth could not naturally have a *spherical Form*, but must be a *Spheroid*; and since these great Men differ'd in their Accounts of what Sort this Spheroid was, Sir *Isaac* shewing it to be an *Oblate Spheroid*, and *Cassini* strongly contending for the *Oblong Spheroid*; the King of *France* was nobly inclined to have this important Affair decided, and accordingly order'd the Length of a Degree to be measured at the Equator, and at the Polar Circle; that by comparing them with the Length of a Degree near *Paris*, it might be known whether the Earth were oblong or flat towards the Poles.

9. Upon this Business he order'd two Voyages, one to *Peru*, the other to the *Arctic Circle*. The Success of the former is not yet known, those who made it not being heard of

of the Sphere in Plano: Thus, one Half of the Globe is projected on one Side of the Plane, and the other Half on the other; and if the Plane be

till lately; and returning in Time of War they were disperfed, and their Papers fuppofed to be loft or conceal'd as yet, for none are to be found in the Ships that fell into *English* Hands. But thofe *Mathematicians* who went Northwards finifh'd their Defign with great Accuracy, and have fince publifh'd an Account of the fame.

10. And fince a Determination of the Figure of the Earth, and its Dimenfions by actual Menfuration, is a Problem of the higheft Concern in *Navigation*, *Aftronomy*, *Geography*, *Levelling*, *Hydraulics*, &c. I think it quite neceffary the Reader fhould have an Idea of the Manner in which this was effected by the *French* Mathematicians, and which therefore I fhall give from the Book entitled *The Figure of the Earth determin'd*, &c. by *Maupertuis*.

11. The arduous Tafk was perform'd in *Lapland* by Meffieurs *Clairaut*, *Camus*, *Le Monnier*, *Maupertuis*, the *Abbé Outhier*, and M. *Celfius* of *Upfal*. They fat out for *Stockholm*, and from thence for the Bottom of the Gulph of *Bothnia*. Being arrived at *Tornea* they began their Work; for from thence they fat out, *July* 6, 1736, to reconnoitre the Country, of which I have here added their Map, by which the Affair is made eafy to underftand. Plate LXIX.

12. After twelve Hours Voyage up the River, they came to the Hamlet *Korpikyla*, and from thence through the Foreft they went on Foot to the fteep Mountain *Niwa*, whofe Summit, (a bare Rock) they made their firft Station. Farther up the River they met with another high Mountain call'd *Avafaxa*, on the Top of which they built a *Signal*. They then went up the River *Tenglio*, and crofs'd a Morafs to the great Mountain *Horrilakero*, where they built another *Signal*. From hence they return'd back again, and in their Way crofs'd the Foreft to another very fteep Mountain call'd *Cuitaperi*, which afforded a very fair Profpect to all the reft.

13. After this they went fome to one Part, and fome to another, and built Signals on the Summits of other Mountains, *viz. Kakama*, *Pullingi*, *Niemi*, and *Kittis*, near the Village *Pello*. Then taking the Angles which the Vifual Rays made connecting the feveral Signals by a Quadrant of two Feet Radius, furnifh'd with a Micrometer, they conftituted a *Heptagonal* Figure TCAPQNK, extending from the Tower of the Church of *Tornea* at T, to *Kittis* at Q. Fig. 2.

that of the Ecliptic or Equinoctial, as in the Case of the Celestial Globe, these Projections are then call'd the *Celestial Hemispheres*. But with

14. And because the Truth of their Work may the better appear, I shall here set before the Reader the Sum of all the Angles, of which the several Angles of the Heptagon did consist, *viz.*

		°	′	″
1. The Angle	CTK =	24	22	54,5
2. The Angle TCA =	KCT =	37	9	12
	KCH =	100	9	56,8
	HCA =	30	56	53,4
3. The Angle CAP =	CAH =	112	21	48,6
	HAP =	53	45	56,7
4. The Angle APQ =	APH =	31	19	55,5
	HPN =	37	22	2,1
	NPQ =	87	52	24,3
5. The Angle	PQN =	40	14	52,7
6. The Angle QNK =	QNP =	51	53	4,3
	PNH =	93	25	7,5
	HNK =	27	11	53,3
7. The Angle NKT =	NKH =	9	41	47,7
	HKC =	43	45	35,6
	CKT =	118	28	12
	The Sum of all,	900	1	37

15. But since the Angles of any Polygon are equal to twice the Number of Right Angles that the Figure has Sides, abating 4, therefore the Sum of the Angles of a Heptagon is $14 - 4 = 10 \times 90° = 900°$. Hence if their Heptagon had been taken on a Plane, it would have exceeded the Truth but by 1′ 37″; but since the Figure lay on a convex Surface, the Sum ought to be a little more than 900°. And thence it appears to what a surprising Degree of Exactness they attain'd in this Undertaking.

16. Now in order to measure the *Meridian-Line* QM, which lay through the Middle of the Heptagon, or rather the Line q m, which was the correct Distance between the two Parallels where they made their *Astronomical Observations* with a Sector, (whose Accuracy is incredible, and of a Structure not here to be described) I say, in order to measure this Line q m, it was necessary to begin with some Base Line to be first

regard

regard to the Terrestrial Globe, they are generally made on the Plane of the *General Meridian or Horizon*, and then they are commonly call'd

of all measured, and then to compute a fundamental Triangle or two, for the Grounds of their future Work.

17. Thus they pitch'd on the Distance between *Niemisby* and the Village *Poiki*, for the *Base Line* B*b*, because it lay along the River, and could be most accurately measured on the Ice. It was measured twice over, and

	Toises.	*Feet.*	*In.*
The first Mensuration gave ——	7406	5	0
The second —— —— ——	7406	5	4
The mean Length therefore is ——	7406	5	2

18. Having this Base Line known, they calculated the two Triangles AB*b* and ABC, from which they found the Distance between *Avasaxa* and *Cuitaperi* to be 8659,94 = AC; from whence they proceeded to find the Sides and Angles of all the other Triangles round the Figure, as AHC, AHP, PQN, CTK, *&c.* and from thence having found the Sides AP, PQ, NK, KT, TC, they form'd the Right-angled Triangles AEP, AFC, PDQ, CGM, by drawing EF, GC, PD, at Right Angles to the Parallels passing thro' Q and M, and parallel to the Meridian Line QM; and the same they did on the other Side the Figure, as is there represented.

19. Having thus measured the several Lines, they were found as follows.

		On the other Side.
PD = 9350,45		Nd = 13297,88
AE = 14213,24		KL = 24995,83
AF = 8566,08		Kg = 16651,05
CG = 22810,62		
Total, 54940,39	—— ——	54944,76
		54940,39

Therefore at a Mean the Meridian Line is QM = 54942,57

20. By very accurate Methods they deduced the Length of the Line qm = 55020,09 Toises, and the still more correct Distance qu = 55023,47. But this Distance or Arch qu, by the nicest Astronomical Observations and Corrections, was found to be equal to 57′ 28″,67 of a Degree. Therefore, As 57′ 28″,67 is to 55023,47 Toises, so is 60′, or 1 Degree, to 57437,9 Toises in one Degree at the *Arctic* Circle.

Maps

Maps of the World: And the several Circles, and Parts of the Surface of one Hemisphere, are so delineated on the said Plane, as they would ap-

21. If therefore from the Length of a Degree here, *viz.* at the *Arctic* Circle, —— —— —— 57437,9
you subduct the Length of a Degree at *Paris*, by *Picard*, —— —— 56925,7

the Difference will be —— —— 512,2 Toises, or 3282,878 Feet of *English* Measure.

22. Hence, having the Length of a Degree, the Radius of Curvature is found for any Part of the Elliptic Meridian. For let R denote that Radius, then it is $3,1416 : 1 :: 360 : 2R :: 180 : R$; therefore $R = \frac{180}{3,1416}$, or $R = \frac{180 \times 57437,9}{3,1416}$ Toises, for the Curvature of the Earth's Surface in the Latitude 66° 20′ at *Lapland*; and in *France* the Radius of the Earth's Curvature is $R = \frac{180 \times 56925,7}{3,1416}$ Toises; or in *English* Miles those Radii are $R = 3994$, and $R = 3958,4$.

23. We are now prepared to assign the Proportion of the Axis of the Earth to the Diameter of the Equator from this actual Mensuration; in order to which we must first of all premise some *Theorems*, which result from the Properties of the Ellipse. Therefore let EP*e* denote the Elliptic Surface of the Earth, E*e* the Diameter of the Equator, and CP the Semi-axis of the Earth. Let AI be the Radius of Curvature to any Point I of the Ellipsis, IF a Tangent; and draw HI and ID perpendicular to CP and CE. Take the Arch $Ii = 1$ Degree, and draw Ai and the Perpendicular id; there A is the Center of a Circle touching the Ellipsis in the Points I, *i*: The Angle $IBE = DIF$ is the Latitude of the Place I. Now put $z = CE$, $u = CP$, $x = CD = HI$, $y = DI$; and let 1, *t*, *s*, denote the Radius, Tangent, and Secant of the Angle IBD; and lastly, let $AI = r$.

Plate LXVIII. Fig. 6.

24. Therefore	1	$tt + 1 = s^2$, and so $t^2 - s^2 = 1$. *Theor.* I.
Let — —	2	$z^2 : u^2 :: 1 : a$, or $\frac{u^2}{z^2} = a$. *Theor.* II.
Hence —	3	$\frac{u^2}{z} = az = p$, the Parameter. *Theor.* III.
Again ——	4	$1 : s :: y : IF = sy$. *Theor.* IV.

pear

pear thereon to an Eye placed in the Pole, or middle Point, of the other Hemisphere. Hence it will come to pass, that the Stars and Constel-

Also ——	5	$1 : t :: y : DF = ty$. *Theor.* V.
And ——	6	$t : 1 :: IF = sy : IB = \frac{sy}{t}$. *Theor.* VI.
And ——	7	$t : 1 :: y : DB = \frac{y}{t}$. *Theor.* VII.
Then *per Conics*	8	$ED \times De : DI^2 :: CE^2 : CP^2$.
That is,	9	$zz - xx : yy :: zz : uu :: 1 : a$. *Theor.* VIII.
Also because	10	$CD : CE :: CE : CF$, *per Conics*,
That is,	11	$x : z :: z : x + ty$,
We have	12	$zz = x^2 + tyx$, or $z^2 - x^2 = tyx$.
Therefore (9)	13	$1 : a :: tyx : y^2 :: tx : y$.
Hence —	14	$y = atx$, or $x = \frac{y}{at}$. *Theor.* IX.
And also	15	$ax = \frac{y}{t} = DB$. *Theor.* X.
Since (14)	16	$y = atx$, $\therefore z^2 = x^2 + xty = x^2 + at^2x^2$.
Wherefore	17	$z^2 = x^2 \times \overline{1 + att}$, $\therefore z = x\sqrt{1 + att}$. *Th.* XI.
Whence also	18	$x^2 = \frac{z^2}{1 + att}$, $\therefore x = \frac{z}{\sqrt{1 + att}}$. *Theor.* XII.
Therefore also	19	$y = \frac{atz}{\sqrt{1 + att}}$. *Theor.* XIII.
Because	20	$DC : DB :: x : ax :: 1 : a :: z^2 : u^2$.
Theref. Conv.	21	$CD : CB :: 1 : 1 - a :: Cd : Cb$.
Whence	22	$Dd (= gI) : Bb :: 1 : 1 - a$.
Also we have	23	$GI : gI :: BF : DF$, because $IGi : FBI$.
Wh. conjointly	24	$GI : Bb :: BF : DF - aDF$.
Again —	25	$GI : Bb :: AI : AB :: BF : DF - aDF$.
Conv. and Inv.	26	$IB : AI :: BD + aDF : BF = BD + DF$.
In Species	27	$\frac{sy}{t} : r :: \frac{y}{t} + aty : \frac{y}{t} + ty$.
That is,	28	$\frac{sy}{t} : r :: 1 + att : 1 + tt = ss$.
Whence (15, 18)	29	$r = \frac{s^3y}{t + at^3} = \frac{s^3ax}{1 + att} = \frac{s^3az}{\overline{1 + att}^{\frac{3}{2}}}$. *Th.* XIV.
Whence also	30	$z = \frac{r \times \overline{1 + att}^{\frac{3}{2}}}{s^3a}$, and $zz = \frac{r^2 \times \overline{1 + att}^{3}}{s^6a^2}$. *T.* XV.

lations

lations of the Hemispheres, and the Parts of Land and Water in the Maps, are not represented in their natural and just Distances, and in their

Therefore	31	$z^{\frac{2}{3}} = \frac{r^{\frac{2}{3}} + r^{\frac{2}{3}} att}{s^2 a^{\frac{2}{3}}} = \frac{\mathrm{r}^{\frac{2}{3}} + \mathrm{r}^{\frac{2}{3}} a\mathrm{tt}}{a^{\frac{2}{3}} + a^{\frac{2}{3}} \mathrm{tt}}$. *Theor.* XVI.
Whence also	32	$tt = \frac{r^{\frac{2}{3}} - a^{\frac{2}{3}} z^{\frac{2}{3}}}{a^{\frac{2}{3}} z^{\frac{2}{3}} - a r^{\frac{2}{3}}}$. *Theor.* XVII.
For any other Lat. we have	33	$zz = \frac{r^2 \times \overline{1 + att}^3}{s^6 a^2} = \frac{\mathrm{r}^2 \times \overline{1 + a\mathrm{tt}}^3}{\mathrm{s}^6 a^2}$.
Whence it is	34	$r^{\frac{2}{3}} \mathrm{ss} + r^{\frac{2}{3}} att\mathrm{ss} = \mathrm{r}^{\frac{2}{3}} ss + \mathrm{r}^{\frac{2}{3}} a\mathrm{tt} ss$.
Therefore lastly	35	$a = \frac{\mathrm{r}^{\frac{2}{3}} ss - r^{\frac{2}{3}} \mathrm{ss}}{r^{\frac{2}{3}} tt\mathrm{ss} - \mathrm{r}^{\frac{2}{3}} \mathrm{tt} ss}$. *Theor.* XVIII.

25. From these Theorems we can calculate whatever relates to the Figure and Magnitude of the Earth; and first to determine the Value of a, or the Ratio of z^2 to u^2, that is, of CE to CP. In order to this, we have r, s, t, for the Latitude 66° 20′ at *Lapland*; and r, s, t, for the Latitude of 49° 22′, being the Middle of the Degree measured in *France*. (See *Art.* 22.) For having $r = 3994$, and $\mathrm{r} = 3958{,}4$; whence by Logarithms we have $r^{\frac{2}{3}}\mathrm{ss} = 593{,}6$, and $\mathrm{r}^{\frac{2}{3}} ss = 1552{,}9$; also $r^{\frac{2}{3}} tt\mathrm{ss} = 3090{,}1$, and $\mathrm{r}^{\frac{2}{3}}\mathrm{tt} ss = 2109$. Therefore $a = \frac{959{,}3}{981{,}1} = \frac{u^2}{z^2}$. Whence we get $z : u ::$ $313{,}22 : 309{,}72 ::$ CE : CP. Therefore by Mensuration it appears, that CE exceeds CP in a greater Proportion than that of 230 to 229, as was observed in the *Scholium* of *Annot.* XXXIV.

26. Hence we have $z = \mathrm{CE} = \frac{r \times \overline{1 + att}^{\frac{3}{2}}}{s^3 a}$, (by Logarithms) equal to 3971,1 Miles; and so the Diameter of the Equator is equal to 7942,2 Miles. Whence, because $\frac{u^2}{z^2} = a$, $u = z\sqrt{a} = \mathrm{CP} = 3926{,}2$ Miles; and so the Axis of the Earth is equal to 7852,4 Miles; so that the Equatoral Diameter exceeds the Axis by 89,8 or 90 Miles,

due

due Magnitudes and Forms, as on the Globes themselves: Yet most of the Problems of either

which is near three times as much as the Theory gave it. See *Annot.* XXXIV. 36.

27. In any given Latitude the Radius of Curvature is found by *Theorem* XIV, *viz.* $r = \frac{as^3 z}{\overline{1+att}^{\frac{3}{2}}}$; and, because under the Pole P the Angle IBE is a Right one, s and t will in that Case become infinite and equal; and therefore $r = \frac{z}{\sqrt{a}} = 4016{,}6$ Miles, which is the greatest of all: And under the Equator that Angle vanishes, and there $s = 1$, and $t = 0$; and so $r = az = 3881{,}8$ Miles, the least of all.

28. The Radius of Convexity being known, we find the Length of a Degree in any Latitude by this Analogy; As $180 : 3{,}1416 :: r : \frac{3{,}1416 r}{180} =$ the Length of the Degree required. Thus under the Equator we have $\frac{3{,}1416}{180} \times 3881{,}8 = 67\frac{3}{4}$ Miles, for the least Degree; and under the Pole we have $\frac{3{,}1416}{180} \times 4016{,}6 = 70\frac{1}{10}$ Miles, for the greatest Degree of Latitude: A *mean Degree* therefore is 68,92 Miles. Thus also a Degree in the Latitude 49° 22′ is $\frac{3{,}1416}{180} \times 3858{,}4 = 69{,}087$ Miles; and in the Latitude 66° 20′ it is $\frac{3{,}1416}{180} \times 3994{,}1 = 69{,}709$ Miles.

29. If the Length of a Degree be known, the Radius of Convexity may be determined, and thence the Latitude of the Place by *Theor.* XVII. $tt = \frac{r^{\frac{2}{3}} - a^{\frac{2}{3}} z^{\frac{2}{3}}}{a^{\frac{2}{3}} z^{\frac{2}{3}} - a r^{\frac{2}{3}}}$; for if the Tangent of an Angle be known, the Angle itself, that is, the Latitude, is known also.

30. Hence also the Radius of any Parallel of Latitude may be discover'd; for, by *Theorem* XII, $HI = CD = x = \frac{z}{\sqrt{1+att}}$; and $180 : 3{,}1416 :: x :$ a Degree of Longitude in the given Parallel. In the Equator $x = z$; hence $\frac{3{,}1416}{180}$

Globe

Globe are performable on these artificial Pro-

$\times$ 3971,1 = 69,309 Miles, the Length of a Degree in the Equator.

31. Hence the Circumference of the Earth under the Equator is 360 $\times$ 69,309 = 24951 Miles. I might now proceed to calculate the Surface and Solidity of the Earth as a Spheroid; but the Process would be tedious, and answer no great Purpose, enough having been said for any Person to form a proper Idea of the Magnitude and Figure of the Earth. I conclude with observing, that there is about $2\frac{1}{4}$ Miles between the greatest and least Degree of Latitude within the Compass of our common Charts: *Quære then, If our Theory of Navigation, founded upon an Hypothesis of their being all equal, be not very erroneous; and if it be not necessary to have one corrected according to the foregoing Measures?*

SCHOLIUM.

32. Since writing the above, I have met with a Treatise on this Subject by the Reverend Mr. MURDOCH, who has determined the Terrestrial Spheroid nearly the same as above; the Difference between the Square of the Semidiameter of the Equator and Semiaxis being by his Calculation 22, and by mine 21,8 (*Art.* 25.) And as he has given us a Table of the Degrees in the Quadrantal Arch of the Meridian both in the *Sphere* and *Spheroid*, with their Differences, I have here inserted it for the Reader's Satisfaction and Curiosity.

33. *A* TABLE *of Arcs of the Meridian to the Spheroid, in Minutes of the Equator.*

D.	*Spheroid.*	*Sphere.*	*Diff.*	D.	*Spheroid.*	*Sphere.*	*Diff.*
1	58.7	60.0	1.3	12	704.5	720.0	15.5
2	117.3	120.0	2.7	13	763.3	780.0	16.7
3	176.0	180.0	4.0	14	822.1	840.0	17.9
4	234.7	240.0	5.3	15	880.9	900.0	19.1
5	293.4	300.0	6.6	16	939.7	960.0	20.3
6	352.1	360.0	7.9	17	998.5	1020.0	21.5
7	410.8	420.0	9.2	18	1057.4	1080.0	22.6
8	469.6	480.0	10.4	19	1116.3	1140.0	23.7
9	528.3	540.0	11.7	20	1175.2	1200.0	24.8
10	587.0	600.0	13.0	21	1234.1	1260.0	25.9
11	645.8	660.0	14.2	22	1293.0	1320.0	27.0

jections,

jections, by those who understand their Nature

D.	*Spheroid.*	*Sphere.*	*Diff.*	D.	*Spheroid.*	*Sphere.*	*Diff.*
23	1352.0	1380.0	28.0	57	3370.0	3420.0	45.0
24	1411.0	1440.0	29.0	58	3435.1	3480.0	44.9
25	1470.0	1500.0	30.0	59	3495.2	3540.0	44.8
26	1529.0	1560.0	31.0	60	3555.3	3600.0	44.7
27	1588.1	1620.0	31.9	61	3615.5	3660.0	44.5
28	1647.2	1680.0	32.8	62	3675.7	3720.0	44.3
29	1706.3	1740.0	33.7	63	3736.0	3780.0	44.0
30	1765.5	1800.0	34.5	64	3796.2	3840.0	43.8
31	1824.7	1860.0	35.3	65	3856.5	3900.0	43.5
32	1883.9	1920.0	36.1	66	3916.8	3960.0	43.2
33	1943.1	1980.0	36.9	67	3977.2	4020.0	42.8
34	2002.4	2040.0	37.6	68	4037.5	4080.0	42.5
35	2061.7	2100.0	38.3	69	4097.9	4140.0	42.1
36	2121.0	2160.0	39.0	70	4158.4	4200.0	41.6
37	2180.4	2220.0	39.6	71	4218.8	4260.0	41.2
38	2239.8	2280.0	40.2	72	4279.3	4320.0	40.7
39	2299.2	2340.0	40.8	73	4339.8	4380.0	40.2
40	2358.7	2400.0	41.3	74	4400.3	4440.0	39.7
41	2418.2	2460.0	41.8	75	4460.8	4500.0	39.2
42	2477.7	2520.0	42.3	76	4521.3	4560.0	38.7
43	2537.3	2580.0	42.7	77	4581.9	4620.0	38.1
44	2596.8	2640.0	43.2	78	4642.5	4680.0	37.5
45	2656.6	2700.0	43.4	79	4703.1	4740.0	36.9
46	2716.4	2760.0	43.6	80	4763.7	4800.0	36.3
47	2776.2	2820.0	43.8	81	4824.3	4860.0	35.7
48	2835.9	2880.0	44.1	82	4884.9	4920.0	35.1
49	2895.5	2940.0	44.5	83	4945.5	4980.0	34.5
50	2955.3	3000.0	44.7	84	5006.2	5040.0	33.8
51	3015.2	3060.0	44.8	85	5066.8	5100.0	33.2
52	3075.0	3120.0	44.0	86	5127.5	5160.0	32.5
53	3135.0	3180.0	45.0	87	5188.2	5220.0	31.8
54	3194.9	3240.0	45.1	88	5248.8	5280.0	31.2
55	3254.9	3300.0	45.1	89	5309.5	5340.0	30.5
56	3314.9	3360.0	45.1	90	5370.2	5400.0	29.8

and

and Use. But these Things will be best understood from a View of those Prints, and a Specimen of the Praxis of their Use (CXLIX).

(CXLIX) 1. The Solution of most of these GEOGRAPHICAL PROBLEMS may be perform'd by a *Trigonometrical Calculation*, as is evident from the original Diagram we before made use of for the Solution of *Astronomical Problems*. Thus if A and Z be any two Places on the Surface of the Globe, then in the Triangle AZN we have

Plate LXVIII. Fig. 1.

ZN the *Co-Latitude* of the Place Z.
AN the *Co-Latitude* of the Place A.
ZA the *Distance* of the Places A and Z from one another.
ZNA the *Difference of Longitude*.
AZN the Angle of *Position*, or *Bearing* of A from Z.
ZAN the Angle of *Position*, or *Bearing* of Z from A.

2. After the same manner may Problems of NAVIGATION be solved; and indeed the only true and natural Way of SAILING is upon the *Arch* of a *Great Circle*, which gives the nearest Distance between any two Places on the Surface of the Globe; and therefore the nearer a Ship keeps to the Arch of a Great Circle, the shorter will her Way or Passage be from one Place to another. Thus in the same Triangle ZNA, if it be proposed to sail from Z to A, the Ship ought to be directed upon the Arch ZA. But in order to be acquainted with this Method of Sailing, the Doctrine of the Sphere must be well understood; therefore I shall refer the Reader who desires it, to Vol. II. of my *Young Trigonometer's Guide*.

3. However, I shall here subjoin the *Philosophical Principles* of all Kinds of Geographical and Nautical MAPS and CHARTS: And first I shall shew the Nature of what is call'd the ORTHOGRAPHIC PROJECTION of the SPHERE. Let ABDE be the Primitive Circle, or Plane of the Projection, which we may suppose to be a Meridian; and let AED be a Great Circle elevated above the Plane in any Angle BAE. Suppose this Circle to be projected on the Plane into the Curve AFD, by Perpendiculars passing through every Point thereof; it is required to find the Nature of the projected Curve AFD.

Fig. 7.

4. In order to this, let EF and IG be two Perpendiculars; draw GI parallel to CE, and HI parallel to CF, and Gg to CI; and from g let fall the Perpendicular gh; then is the Right-angled Triangle GHI equal and similar to ghC,

and

and gbC is similar to EFC. Wherefore putting AC = EC = a, CI = x, GI = y, CF = b, and HI = y; then by the Property of the Circle we have AI × ID = GI², that is, $yy = aa - xx$, and $y = \sqrt{a^2 - x^2} = $ GI; but GI : HI :: (gC : bC ::) EC : FC, that is, $y : \mathrm{y} :: a : b$; therefore $\mathrm{y} = \frac{by}{a} = \frac{b}{a}\sqrt{a^2 - x^2}$, which shews the Curve AFD to be an *Ellipse*, whose Semi-axes are AC and CF.

5. Hence the Circles of a Sphere view'd at an infinite Distance are projected into *Ellipses*. Thus the *Circle of Illumination* on the Disk of the Moon is an *Ellipsis*, as observed *Annot*. CXXXV. 23. Thus also a Sphere set in the Sun-Beams will have its Circles all projected into elliptic Shadows. And hence it is we construct the ORTHOGRAPHIC PROJECTION, call'd the ANALEMMA; which see in my fore-cited Book.

6. Now because CE : CF :: Radius : Co-sine of ECF, it appears that the Semidiameter CE of every Circle is projected into the Co-sine FC of its Elevation above the Plane of Projection. Hence also it appears, that in this Projection the same Number of Degrees in a Right Circle, as BCE, will be projected into very different Portions of the Diameter of the Plane BE. Thus 10 Degrees from the Pole of the Primitive will be projected into the Arch CK, but 10 Degrees from the Periphery will be projected into EM. But CK is to EM as the *Right Sine* of 10 Degrees to the *Versed Sine* of the same; that is, as 1736 to 152, or nearly as 12 to 1. Hence the Reason why the Spots in the Sun appear to move so much faster over the middle Parts of the Disk than on the Outside, and why their Motion is always unequal; with other Phænomena of the like Nature.

7. The STEREOGRAPHIC PROJECTION of the SPHERE is that on which our Maps are commonly made, and depends on this Principle, That if the Plane of any Meridian be supposed the Plane of the Projection, then an Eye placed in one Pole of that Meridian will project all the Circles in the opposite Hemisphere into circular Arches on the said Plane. Thus let AGDE be any Meridian; then the Diameter AD, dividing it into the upper and nether Hemisphere, is call'd the *Line of Measures*; and an Eye placed at the Pole E will project every Point B, F, G, in the opposite Semicircle into the Points H, I, C, into the Line of Measures AD, by the Visual Rays EB, EF, EG. Pl. LXX. Fig. 1.

8. Hence if the Arch AB = FG = 10 Degrees, then will their Representatives in the Line of Measures be AH and

IC; and the Points H and I are those through which the Circles of 10 Degrees and of 80 Degrees do pass in Projection, *viz.* the Circles GHE and GIE, as is evident from considering the Figure. Hence the Reason why the Meridians do all lie nearer to each other in the middle Parts of the Map than on the Outsides; and consequently, why the several Parts of the Earth cannot be duly represented on such Maps, either in respect of Magnitude or Position.

9. On E as a Center describe the Arch CM, and draw the Line EK; the Arch GK will be projected into the Line CL, which is the Tangent of the Angle CEL. But the Angle CEL is equal to Half the Angle GCK, or Arch GK; therefore any Arch GK is projected into a Line CL equal to the Tangent of Half that Arch. Hence the Line CD is call'd the *Line of Half-Tangents* in respect of the Quadrant GKD.

10. On this Projection are usually made all the Maps of the World in two Hemispheres; there is also another call'd the GLOBULAR PROJECTION, wherein all the Meridians are equally distant, as they are on the Globe itself. They are circular Arches here, as in the last Projection, and are drawn after the same manner, but are not projected by the Eye on the Surface as they are. By this Sort of Maps the several Parts of the Earth have their proper Proportion of Magnitude, Distance, and Situation assign'd nearly as on the Globe itself. As this Sort of Map is for that Reason very useful, and not common, I have given one here for the Reader's Use, corrected from the latest Observations. Plate LXXIV.

11. Besides the foregoing, there is another very useful Projection, generally made use of for Charts, and sometimes for Maps; it goes by the Name of MERCATOR'S PROJECTION, but was first invented by Mr. *Wright* long before. In this the Meridians and Parallels are strait Lines, and the former equidistant from each other. Hence in this Way the Degrees of Longitude in every Parallel are the same, and equal to those in the Equator; also the Degrees of Latitude are all unequal; both which are contrary to what they are on the Globe. Therefore Maps of this Sort do not exhibit the true Dimensions or Proportions of the several Parts of the Earth; however, they are very useful on divers Accounts; and that which I have given from Dr. *Halley* to illustrate the Account of the Winds is of this Kind.

12. But the greatest Use of this Projection is in SAILING; I shall therefore shew how it is constructed in the following Manner. Let AB be an Arch of the Equator contain'd between any two Meridians AP, BP, meeting in the Pole P of the

the Sphere, whose Center is C. Upon the Points A and B let there be erected the Perpendiculars AH and BI, and let DE represent an Arch of any Parallel between the same Meridians; draw CA and CB, KD and KE perpendicular to PC; through D and E draw CF, CG, and join FG; lastly, let fall the Perpendicular DL.

13. Now the Arch AB in the Equator is to the similar Arch of the Parallel DE as AC to DK, or as Radius to the Co-sine of the Latitude AD. Suppose now the Meridians AP, BP, to be in part projected into the Perpendiculars AH and BI; then will the Arch DE be projected into FG = AB; but in this Case DE, *the natural Length of the Arch, is to* FG *its protracted Length, as the Radius* CD *to the Secant* CF *of the Latitude,* or *as the Co-sine* LC *to the Radius* CD; for CF : (CD =) AC :: DC : LC.

14. But in whatever Proportion the Degrees of any Parallel are increased or diminished by a Projection *in Plano*, in the same Ratio ought the Degrees of Latitude also to be increased or diminished; otherwise the *true Bearing and Distances* of Places would be lost, as in the Case of the *Plain Chart*, where the Degrees of Latitude are all equal. *The Degrees*, therefore, *of Latitude in* Mercator's *Chart increase in Proportion of the Secant of the Latitude to the Radius.*

15. But that the Reader may see how such a Meridian is projected, let RCH be a Quadrant of the Primitive Circle, and RQ a Diameter; draw QS; then will the Arch SH be projected into HI, and RS into AI; but AI is the Tangent of $\frac{1}{2}$ RS (by *Art.* 9.) Let ST and CI be perpendicular to AH, and draw the Tangents SV, CK, to the Points S and C, meeting AH produced in V and K. And let $HI = x$, $HS = z$, and $AH = 1$. Pl. LXX. Fig. 3.

16. Then because AT : AH :: AH : AV, it is $AT \times AV = AH^2$; for the same Reason it is $AI \times AK = AH^2 = AT \times AV$. Wherefore AV : AK :: AI : AT (= SB) :: QI : QS. Let Q*s* be drawn infinitely near to QS, then $Ss = \dot{z}$, and $Ii = \dot{x}$; and because the Angle AIQ = TIS = ISV = Q*s*S, therefore the Triangles QI*i* and QS*s* are (in their nascent State) similar, and therefore QI : QS :: I*i* : S*s* :: $\dot{x} : \dot{z}$ :: AV : AK; consequently, it is $AV \times \dot{z} = AK \times \dot{x}$.

17. But $AK \times \dot{x}$ is the Fluxionary Rectangle of what is call'd a *Figure of Secants*, which may be thus explain'd. Let RCH be a Quadrant as before, HC an Arch, of which let the Secant be equal to IN, rightly applied as an Ordinate to the Absciss $HI = x$; and if this be conceived to be done for every Point in the Quadrant, we shall have a Curve BNP Fig. 4.

described by the Point N, which appears to be a *rectangular Hyperbola* by compleating the Square AB. Now drawing in infinitely near IN, we shall have $IN \times in = IN \times \dot{x}$ ($= AK \times \dot{x}$) $=$ Fluxion of the Area IHBN, which is composed of all the Secants belonging to the Arch HC, and is therefore call'd a *Figure of Secants*.

18. Now the Fluxion of the Area IHBN is to the Fluxion of the Rectangle IHBD as $IN \times \dot{x}$ to $ID \times \dot{x}$, that is, as IN to $ID = AR$, *viz.* as the Secant to the Radius. Therefore the Areas themselves are in the same Ratio; that is, the Area $IHB : R \times x :: S : R :: Z : z$, supposing Z represents the Arch z protracted. In the same manner it is shewn, that the Fluent or Area belonging to the Fluxion $AV \times \dot{z}$ is to $R \times z$ as $Z : z$; but this latter Fluent of $AV \times \dot{z}$ is equal to the Area IHBN, because their Fluxions are equal (by *Art.* 16.) Therefore $IHBN : R \times z :: Z : z$; consequently, $IHBN \times z = R \times z \times Z$; whence $IHBN = Z$ when $R = 1$.

19. But the hyperbolical Area INBH is the Logarithm or Measure of the Ratio of AH to AI, that is, of $\frac{1}{1-x} = \frac{1}{t}$, supposing $t =$ Tangent of $\frac{1}{2}$ the Complement of z. But any Hyperbolical Logarithm is to the Tabular Logarithm of the same Ratio, as 2,302585, &c. to 1; therefore the Tabular Logarithm of $\frac{1}{t} \times 2,302585 = INBH = Z$ gives the Length of the protracted Meridional Arch, answering to the Natural Arch z or HS.

20. Therefore, if A and a denote a Greater and a Lesser Arch, beginning from the Equator; then the Length of their Difference $A - a$ will be $\frac{2,302585}{T} - \frac{2,302585}{t}$, or $2,302585 \times \overline{\frac{1}{T} - \frac{1}{t}}$, or $2,302585 \times \overline{t - T}$. That is, *From the Tabular Logarithm of* $\frac{1}{2}$ *the Complement of the Lesser Arch a, subduct that of the Greater Arch* A; *the Difference multiplied by* 2,302585 *will give the Meridional Parts of the Arch* $A - a$.

21. As I am upon a Subject of this Nature, it will be proper to observe, that since the Ship's Course is or ought to be upon a *Rhumb-Line*, which makes equal Angles with every Meridian, therefore the Differences of Longitude will be the Logarithms of the Tangents of the Half-Complements of the Latitudes,

Latitudes, as may be thus shewn. Let ÆQ be a Quadrant of the Equator, P the Pole of the World; PÆ, PA, PB, &c. the several Meridians projected *in Plano*, and Æ*abc*, &c. a Rhumb-Line making equal Angles Æ*a*A, Æ*b*B, &c. with every Meridian. Pl. LXX. Fig. 5.

22. Then if we make ÆA = AB = BC, &c. and very small, then may the Triangles ÆP*a*, ÆP*b*, ÆP*c*, &c. be esteem'd rectilineal, and will be similar; and therefore ÆP : P*a* :: P*a* : P*b* :: P*b* : P*c*, and so on. Now if ÆA expound the Ratio of *a*P to ÆP, then because the Ratio of *b*P to PÆ is double the Ratio of *a*P to PÆ, and ÆB = 2ÆA, therefore ÆB will expound the Ratio of *b*P to ÆP. Again, because $cP : ÆP = 3 \times \overline{aP : ÆP}$, and ÆC = 3ÆA, therefore ÆC expounds the Ratio of *c*P to ÆP; and so of the rest.

23. Therefore the Arches ÆA, ÆB, ÆC, &c. are the Logarithms of *a*P, *b*P, *c*P, &c. in respect of PÆ. But ÆA, ÆB, &c. are the Differences of Longitude made in sailing from Æ to *a*, or *b*, &c.; and *a*P, *b*P, &c. are Tangents of half the Complements of the Latitudes A*a*, B*b*, &c. (See *Art*. 9.) *Therefore the Differences of Longitude in sailing on any Rhumb are the Logarithms of the Tangents of the Half-Co-Latitudes.*

24. Hence the Rhumb-Line has acquir'd the Name of the *Logarithmic Spiral*. Hence also it follows, that *any Table of Logarithmic Tangents is a Scale of the Differences of Longitude on some Rhumb or other*. Thus the Tabular Logarithms of Tangents in present Use are Differences of Longitude on that Rhumb which makes an Angle of 51° 38′ 9″; and the Rhumb which makes an Angle of 71° 1′ 42″, is the same for *Neper*'s Logarithmic Tangents. They who would see the Demonstration of this, as also how a Table of Meridional Parts is from hence constructed, and likewise how all the Problems of Navigation may be solved by the common Table of Logarithmic Tangents only, may consult my LOGARITHMOLOGIA. See also *Philosophical Transactions*, N° 219, where the Theory is given at large by its Inventor Dr. *Halley*.

SCHOLIUM.

25. I have here added a Table of *Meridional Parts*, calculated for the Oblate Spheroid by the Rev. Mr. *Murdoch*, in his new and learned Treatise of *Mercator's Sailing applied to the true Figure of the Earth*. By this the Reader will be enabled to project a true CHART for any Part of the Earth's Surface, and to solve thereby the several Problems of Sailing;

to delineate Maps of Countries, and to apply them for various other Purposes of *Navigation*, *Geography*, and *Astronomy*. Nor are the Errors of the common Spherical Projections so very small in many Cases, as to be inconsiderable and not dangerous. For Instance, if a Ship sails from South Latitude 25° to North Latitude 30°, and the Angle of the Course be 43°; then the Difference of Longitude by the common Table would be 3206′, exceeding the true Difference 3141 by 65′ or Miles. Also the Distance sail'd would be 4512, exceeding the true Distance 4423, by 89′ or Miles: Which Differences are too great to be neglected. For other Instances of such a Correction of the Charts, I refer to the Author's admirable Book above-mention'd. (See SCHOL. in *Annot*. CXLVIII.)

27. *A* TABLE *of Meridional Parts to the Spheroid and Sphere, with their Differences.*

D.	*Spheroid.*	*Sphere.*	*Diff.*	*D.*	*Spheroid.*	*Sphere.*	*Diff.*
1	58.7	60.0	1.3	22	1325.3	1353.7	28.4
2	117.3	120 0	2.7	23	1389.0	1418.6	29.6
3	176.1	180.1	4.0	24	1453.3	1484.1	30.8
4	234.9	240.2	5.3	25	1518.0	1550.0	32.0
5	293.8	300.4	6.6	26	1583.3	1616.5	33.2
6	352.7	360.6	7.9	27	1649.1	1683.5	34.4
7	411.8	421.0	9.2	28	1715.6	1751.2	35.6
8	471.0	481.5	10.5	29	1782.7	1819.5	36.8
9	530.4	542.2	11.8	30	1850.5	1888.4	37.9
10	589.9	603.0	13.1	31	1919.0	1958.0	39.0
11	649.7	664.1	14.4	32	1988.2	2028.3	40.1
12	709.6	725.3	15.7	33	2058.3	2099.5	41.2
13	769.8	786.8	17.0	34	2129.0	2171.4	42.3
14	830.2	848.5	18.3	35	2200.8	2244.2	43.4
15	890.9	910.5	19.6	36	2273.4	2317.9	44.5
16	951.8	972.7	20.9	37	2347.0	2392.6	45.6
17	1013.1	1035.3	22.2	38	2421.6	2468.3	46.7
18	1074.8	1098.3	23.5	39	2497.2	2544.9	47.7
19	1136.8	1161.6	24.8	40	2573.9	2622.6	48.7
20	1199.2	1225.2	26.0	41	2651.8	2701.5	49.7
21	1262.0	1289.2	27.2	42	2730.9	2781.6	50.7

D.	*Spheroid.*	*Sphere.*	*Diff.*	D.	*Spheroid.*	*Sphere.*	*Diff.*
43	2811.3	2863.0	51.7	67	5403.9	5474.0	70.1
44	2893.1	2945.8	52.7	68	5560.2	5630.8	70.6
45	2976.2	3029.9	53.7	69	5723.5	5794.6	71.1
46	3060.9	3115.5	54.6	70	5894.4	5965.9	71.5
47	3147.2	3202.7	55.5	71	6073.7	6145.6	71.9
48	3235.1	3291.5	56.4	72	6262.4	6334.7	72.3
49	3324.8	3382.1	57.3	73	6461.6	6534.3	72.7
50	3416.3	3474.5	58.2	74	6672.6	6745.7	73.1
51	3509.7	3568.8	59.1	75	6896.8	6970.3	73.5
52	3605.3	3665.2	59.9	76	7136.2	7210.0	73.8
53	3703.1	3763.8	60.7	77	7393.0	7467.1	74.1
54	3803.1	3864.6	61.5	78	7670.1	7744.5	74.4
55	3905.7	3968.0	62.3	79	7970.9	8045.6	74.7
59	4010.9	4073.9	63.0	80	8300.2	8375.2	75.0
57	4118.9	4182.6	63.7	81	8663.8	8739.0	75.2
58	4229.8	4294.2	64.4	82	9070.0	9145.4	75.4
59	4344.0	4409.1	65.1	83	9530.2	9605.8	75.6
60	4461.5	4527.3	65.8	84	10061.1	10136.9	75.8
61	4582.7	4649.2	66.5	85	10688.7	10764.6	75.9
62	4707.8	4775.0	67.2	86	11456.5	11532.5	76.0
63	4837.1	4904.9	67.8	87	12446.0	12522.1	76.1
64	4971.0	5039.4	68.4	88	13840.4	13916.4	76.0
65	5109.8	5178.8	69.0	89	16223.8	16299.5	75.7
66	5254.0	5323.6	69.6	90			37.75

APPENDIX:

CONTAINING A

Physico-Mathematical Theory

OF

LUNAR Motions and Irregularities,

OF THE

MOTION of the EARTH'S AXIS,

AND

PRECESSION of the EQUINOXES;

AND THE

Computation of the QUANTITY of Matter, DENSITY, WEIGHT of BODIES, &c.

On the SURFACE of the

SUN, EARTH, JUPITER, and *SATURN.*

AS the Subjects treated of in the ensuing Appendix *could not well be brought into the Body of the Book among the* Annotations, *and are the most important Part of the* Newtonian Philosophy, *they could not on any Account be omitted, and therefore I have here annexed them, to compleat a System of that Science. And I have taken such a Method as I hope will be found not only more natural and concise, but much more adapted to render those difficult and intricate Ideas easy to be apprehended by the intelligent Reader.*

APPENDIX.

1. THE Motion of the Moon about the Earth is ſimilar to that of the Waters of the Ocean revolving about the Earth's Center. To ſhew this, ſome Things muſt be premiſed; as, firſt, *That the Attraction of the Earth upon any Particle of Water is the ſame as it would be, were the whole Quantity of Matter contracted into a Point in its Center.* For let ABGO be the Earth, C its Center, P a Particle at any Diſtance PA from its Surface; let PG be drawn through P and C, and BO the Diameter of any Circle BDQE, or Section of the Sphere perpendicular to the Axis PG. Plate LXX. Fig. 6.

2. Now put $PN = a$, $BP = x$; then $PB^2 - PN^2 = x^2 - a^2 = BN^2$, which is as the Area of the Circle BDQE; the attracting Force whereof is $2x\dot{x}$, and is proportional to the Quantity of Matter or Number of Particles which act on the Corpuſcle P in the Periphery of the Area, and in Directions ſimilar to PB. And ſince the Force of Attraction is as the Number of Particles $(2x\dot{x})$ multiplied by the Force of each Particle, which is as ſome Power (n) of the Diſtance (x), therefore $2x\dot{x} \times x^n = 2\dot{x}x^{n+1}$ will be as the whole

or

or absolute Force of those Particles, that is, in the Directions PB.

3. BUT the Force represented by PB is resolvable into two Forces PN and BN, of which the former only causes the Corpuscle at P to approach the Sphere. Therefore as PB : PN :: $x : a :: 2\dot{x}x^{n+1} : 2a\dot{x}x^n$, the Force with which the Particle at P is attracted in the Direction PN; the Fluent of which $\frac{2ax^{n+1}}{n+1}$ (when corrected) is the whole Force of all the Particles in the Area of the Section BDOE, to attract the Corpuscle P in the Direction PC.

4. I say, the Fluent $\frac{2ax^{n+1}}{n+1}$ must be corrected, for it is at present too great; because when the Area of the Section becomes a Point, or $x = a$, then this Fluent has the Value $\frac{2a^{n+2}}{n+1}$, which therefore must be deducted from the general Fluent $\frac{2ax^{n+1}}{n+1}$, and their Difference $\frac{2ax^{n+1} - 2a^{n+2}}{n+1}$, or $\frac{ax^{n+1} - a^{n+2}}{n+1} = \frac{PN \times PB^{n+1} - PN^{n+2}}{n+1}$, will be as the Forces of any circular Areas BDOE attracting the Corpuscle P in the Direction of its Axis PC.

5. Now since in Natural Bodies this Power in any single Particle is *inversely as the Squares of the Distance*, therefore $n = -2$, and so the above Expression

Expression of the Force will become $1-\frac{PN}{PB}$.

6. Now if we put $AC=r$, $PA=c$, $PC=c+r=\frac{b}{2}$, $PB=c+x$, and $PN=y$; then $AG\times AN=2r\times\overline{y-c}=AO^2$, and $PA^2+AO^2+2PA\times AN=(cc+2ry-2rc+2cy-2cc=)\ PO^2=c^2+2cx+x^2$. Hence $2ry+2cy=2c^2+2rc+2cx+x^2$, and $y=c+\frac{2cx+x^2}{2c+2r}=\frac{cb+2cx+x^2}{b}$ (because $b=2r+2c$). And because the Force of Attraction in the circular Plane whose Diameter is BO is $1-\frac{PN}{PB}=1-\frac{cb+2cx+x^2}{cb+bx}=\frac{2rx+x^2}{b\times\overline{c+x}}$, if we multiply this by the Fluxion of the Distance, *viz.* $\dot{y}=\frac{2c\dot{x}+2x\dot{x}}{b}$, we shall have $\frac{4rx\dot{x}+2xx\dot{x}}{bb}$, whose Fluent $\frac{2rx^2+\frac{2}{3}x^3}{bb}$ is proportional to the Attraction of any Segment BAO of the Globe upon the Particle P.

7. HENCE, when $x=2r$, the Expression will become $\frac{5r^3}{3bb}$, or simply $\frac{r^3}{bb}$, for the Attraction of the whole Globe. Whence it appears, that *the attractive Forces of spherical Bodies are to one another in a Ratio compounded of their Quantities of Matter directly, and as the Squares of the Distances from their Centers inversely.* And therefore since the

the Number of Particles only, and their Distance from the Center, enter the Expression of the Force, it is plain the Effect will be the same upon a Corpuscle P placed any where without the Surface of the Globe, as if the whole Mass of Matter were contracted into a Point at its Center. *Q. E. D.*

8. To apply this: If all the Matter of the Earth were contracted into the Center, and the Waters of the Ocean were to continue their diurnal Rotation the same as they now do, they would then be affected in the same manner by the Earth and Moon as they now are, and have all the same Phænomena. And therefore if a Body, instead of revolving at the Distance of the Earth's Surface about its Center, were to revolve at the Distance of the Moon, every thing would happen in a similar Manner, and the Effects of the Earth and Sun in disturbing the Motion of the Satellite would be like those which are produced in the Motion of the Water by the Earth and Moon, but only in a less Degree.

9. ANOTHER Thing to be premised is, that the Moon revolves not about the Center of the Earth as the Center of its Motion; and therefore in order to consider its Motion in the best Manner, we must determine the Distance to which the Moon must be removed from the Center of the Earth at Rest, (and consider'd as the Center of its Motion) that it may revolve about it in the same periodical Time that it takes up now, together

together with the Earth, in revolving about the common *Center of Gravity*. See *Annot.* XXXVI.

10. In order to this, let D be the Distance of the Moon from the common Center of Gravity, and d that of the Earth from it; then will $D+d$ be the Distance of the Moon from the Earth; which, at a Mean, is $60\frac{1}{2}$ Semidiameters. Now let x = Distance required; then because the attracting Forces (F and f) in any two different Distances are as the Squares of those Distances inversely, we have $F : f :: x^2 : \overline{D+d}^2$. Again, because the periodical Time is given, or the same in both Cases, we have the Forces proportional to the Distances from the Centers of Motion; (See *Annot.* XXXIV.) therefore $F : f :: D : x$. Consequently $D : x :: x^2 : \overline{D+d}^2$, therefore $x^3 = \overline{D+d}^2 \times D$; and multiplying by $D+d$, we have $x^3 \times \overline{D+d} = \overline{D+d}^3 \times D$; whence $D+d : D :: \overline{D+d}^3 : x^3$; therefore $\sqrt[3]{D+d} : \sqrt[3]{D} :: D+d : x$. But $D+d : D ::$ the Quantity of Matter in the Earth and Moon together : the Quantity of Matter in the Earth alone; that is, as 40,31 to 39,31. Whence $\sqrt[3]{40,31} : \sqrt[3]{39,31} :: 60,5 : 60 = x$, *the Distance at which the Moon would revolve about the Earth at Rest in the same Time it now does.*

11. THESE Things premised, let S be the Sun, T the Earth, and P a Satellite revolving about it, and let SK be the Mean Distance of the Satellite or Moon from the Sun; and expound the accelerative Force, by which it is attracted to- Pl. LXX. Fig. 7.

wards

wards the Sun S. And take $SL : SK :: SK^2 : SP^2$, and $Sl : Sk :: S^2k : Sp^2$; then shall SL, or Sl, expound the accelerative Attraction in any Distance of the Satellite SP or Sp. That is, the Force at P is to the Force at p as SL is to Sl; for $SK = Sk$, and $SK^3 = SL \times SP^2 = Sl \times Sp^2$; therefore $SL : Sl :: Sp^2 : SP^2$.

Pl. LXX. Fig. 8. 12. JOIN PT and pT and draw parallel thereto the Lines LM and lm, meeting ST in M and m. And the Attraction SL, Sl, is resolvable into two others SM and LM, and Sm and lm. Hence the Body P is urged with a *Threefold* Force; *viz.* (1.) That by which it is attracted or tends towards T, arising from the mutual Attraction of the Bodies T and P. (2.) The Force LM, or lm, by which it is likewise urged towards T. (3.) The Force SM, Sm, by which it is urged towards S, or attracted in Directions always parallel to ST.

13. BY the first of these Forces the Satellite ought to describe an Ellipsis about T in one of its *Foci*, and therefore Areas proportional to the Time, as is evident from what was demonstrated in *Annotat.* CXL. This is upon Supposition the Body T was fix'd; but the Case is the same, supposing it moveable with the Body P about a common Center (which is really the Case of the *Earth* and *Moon*) as Sir *Isaac Newton* has shewn in *Theor.* XX. and XXI. *Lib.* 1. of the *Principia*.

14. THE second Force LM, as it conspires to impel the Body in the Direction PT, is to be added to the former, and causes that the Body shall

ſhall ſtill deſcribe *Areas proportional to the Time.* But becauſe this Force is not in the inverſe Ratio of the Square of the Diſtance, it will, compounded with the former, cauſe the Curve which the Satellite deſcribes to deviate from an *Elliptic Form*; and the more ſo, *cæteris paribus*, the greater the Proportion is which this Force bears to the former. Theſe Forces LM, *lm*, have been ſhewn (*Annot.* LXXXIV. 9.) to be as $\frac{PT}{SP^3}$ and $\frac{pT}{Sp^3}$; and therefore increaſe and decreaſe with the Diſtance PT or pT.

15. LASTLY, the third Force SM, impelling the Body P in Directions parallel to TS, will compound a Force with the former two, that is not directed from P to T, and ſo will cauſe that the Body P ſhall no longer *deſcribe Areas proportional to the Times* (as we have ſhewn.) It will alſo augment the Aberration of the Orbit from an Elliptic Form, on a double Account, *viz.* both becauſe it is not directed from P to T, and alſo becauſe it is not inverſely as the Square of the Diſtance PT. For the Forces $SM : Sm :: \frac{1}{SP^3} : \frac{1}{Sp^3} :: Sp^3 : SP^3$. Theſe Errors therefore are leaſt of all when the ſecond and third Forces (eſpecially the third) are ſo, the firſt Force remaining the ſame.

16. LET SN expound the Force by which the Body T is accelerated towards S; then if the

Forces SM and SN are equal, they, by attracting the Bodies T and P equally, and in parallel Lines, will cause no Alteration in their Site or Positions in respect of each other. But when the Force SM is greater, or S*m* lesser than the Force SN, the Difference NM, or N*m*, will be that alone by which the Proportionality of the Times and Areas, and also the Elliptic Form of the Orbit, will be disturbed. Hence when NM or N*m* is nothing, or least of all, the aforesaid Perturbations will vanish, that is, when the Body P is nearly in the Points C and D of its Orbit.

17. WE have hitherto supposed the Body P revolving about T in the same Plane with S; let us now suppose it to revolve in a different Plane, and let the Semi-Orbit CAD be above, and CDB below the Plane, in which are the Bodies S and T. In this case the Force LM will have the same Effect as before, *viz.* will only tend or impel the Body P from P to T. But the other Force NM, by acting in a Direction parallel to ST, and therefore (when the Body P is not in the Nodes C, D,) inclined to the Plane of the Orbit PAB, will, besides the above-mention'd Error in Longitude, induce an Error in Latitude, or disturb the Inclination of the Orbit.

Pl. LXX. Fig. 9.

18. FOR let P*q* be drawn parallel to NM, and let P*p* be the Space through which the Satellite P would move in its Orbit in a small Particle of Time, exclusive of the Force NM; and by the Force NM alone suppose it in the same Time moved through the Space P*r*; then com-

compleating the Parallelogram P*rsp*, and drawing the Diagonal P*s*, that will represent the real Motion, and *s* the true Place of the Satellite at the End of the said Time: But 'tis evident the *Lineola* P*s* is not in the Plane of the Orbit CAD.

19. HENCE it follows, that by the Force NM the Body P will be accelerated in its Motion from C to A, and from D to B; and retarded as it passes from A to D, and from B to C: For let P*q* be drawn parallel to NM, and expound that Force, then continuing TP to *r*, and drawing *qr* perpendicular thereto, the Force P*q* becomes resolved into the two Forces *r*P and r*q*; of which the former, acting in the Direction PT, does not disturb the Planet's Motion in Longitude, nor the equable Description of Areas: But the other Part *rq*, acting in the Direction *rq*, conspires with the Motion of the Satellite P in its Orbit, (as being parallel to the Tangent *ab*) and therefore accelerates its Motion in Longitude. By the same way of reasoning, by making the like Construction between A and D, it will appear that the Motion of the Satellite will be there retarded, the Force *rq* being on that Side in a contrary Direction. Pl.LXXI. Fig. 1.

20. AGAIN, as the Planet passes from D to B, it will be again accelerated; for let *ps* here express the Force N*m*, which is now negative, or acts in a contrary Direction to the former NM, that is, from *p* to *s*, supposing *ps* parallel to N*m*; for then *ts* conspires with the Motion of the Planet in the Direction of the Tangent *cd*. In

the ſame manner it is ſhewn the Planet is retarded in going from B to C, by the contrary Direction of *t s*.

21. HENCE alſo it appears, that ſince the Body P is conſtantly accelerated from C to A, and from D to B, the Velocity of the Satellite will be greater in the Points A and B (*cæteris paribus*) than in the Points C and D.

22. THE Orbit alſo will (*cæteris paribus*) be more convex in C and D than in the Points A and B; for the ſwifter Bodies move, the leſs they deflect from a Right-Line Courſe in a given Time. Moreover, in the Points A and B, the Force LM and NM are directly contrary to each other, and their Difference NM—LM=KL, will be as the Force which draws the Body from T towards S; and ſince this Force KL is greater when the Body is at A, than when at C or D, the Body will there be leſs urged towards T, and ſo will leſs deflect from a Right Line. The ſame may be ſhewn when the Body is in the Point B by the Force *kl*. See *Annot*. LXXXIV. 20.

23. WHENCE the Body P will (*cæteris paribus*) recede farther from T in the Points C and D than at A and B; as is eaſy to obſerve from the Figure of the Orbit, which is leſs curved, and therefore nearer to T at A and B, than at C and D. What is here ſaid is upon Suppoſition that the Orbit (excluſive of the perturbating Forces) is a *Circle*, and not an *Ellipſis*, which Caſe will be conſider'd by and by.

24. BE-

24. BECAUSE the centripetal Force of the central Body T, by which the Body P is retain'd in its Orbit, is augmented in the Points C and D by the addititious Force LM, and diminish'd in the Points A and B by the Force KL; and because KL is always greater than LM from C to A (and double thereto at A, See *Annot.* LXXXIV. 20, 21, 22,) and from A to D, (where it becomes equal to it,) therefore the centripetal Force (F) is upon the whole diminish'd by the Action of the Body S. And because $F : \frac{a}{P^2}$, by *Annot.* XXXIV. 6.) therefore the Radius TP (a) remaining the same, the Periodical Time (P) of the Planet will be augmented by the Action of the Power KL; and because in that Case $P : \frac{1}{\sqrt{F}}$, it appears the Periodical Time will be increased in the *Subduplicate Ratio* by which the Force F is decreased.

25. AGAIN, supposing the centripetal Force F to remain the same, (as we may when KL is very small with respect to it, *Annot.* LXXXIV. 22.) then however the Distance PT (a) may vary, we have $r P^2 : a^3$, or $P : a\sqrt{a}$. Therefore when neither the Distance (a) nor the centripetal Force F are constant, we have $P : \frac{a\sqrt{a}}{\sqrt{F}} : \sqrt{\frac{a^3}{F}}$; that is, the Periodical Time (P) will be in a Ratio compounded of the Sesquiplicate Ratio of the Di-

ſtance $\sqrt{a^3}$, and the Ratio $\frac{1}{\sqrt{F}}$, which is ſubduplicate of that by which the central Force F is increaſed or diminiſhed by the Decreaſe or Increaſe of the Action of the diſtant Body S.

26. FROM what has been ſaid, it follows alſo, that the Axis of the Ellipſis deſcribed by the Body P, or Line of the *Apſides*, has an angular Motion backwards and forwards by Turns, but its *Progreſs* exceeds the *Regreſs*; and by that Exceſs it is upon the whole carried forwards, or *in Conſequentia*. For the Force by which the Body P is urged towards T in the Points C and D, where the Force MN vaniſhes, is compounded of the Force LM, and the centripetal Force or Attraction of the Body T. The former Force LM, if the Diſtance PT be increaſed, increaſes nearly in the ſame Ratio; and the latter Force (F) is inverſely as the Square of that Diſtance, *viz.* as $\frac{1}{PT^2}$; wherefore the whole Force is as $PT + \frac{1}{PT^2}$.

27. Now $F : TP + \frac{1}{PT^2}$ is a leſs Ratio than $F : \frac{1}{PT^2}$. For Example: Let the Ellipſis (when the Satellite is in the Quadratures) be APB, and the Axis be AB; the Diſtances from the central Body T let be $PT : CT :: 6 : 5$; then the centripetal Force at P will be to that at C as 25 to

36; but the additional Force (LM) at P is as $PT=6$, and at C as $TC=5$; therefore the compound Forces at P and C are as $25+6:36+5$; or as 31 to 41, which is a less Ratio than $25:36$. For as $25:36::31:\frac{36\times 31}{25}=44,64$. But the Ratio $31:41$ is less than the Ratio $31:44,64$.

28. HENCE, since when the Force at P is as $\frac{1}{PT^2}$, the Planet describes an *Ellipsis* PAB; and when the said Force is as $\frac{1}{PT^3}$, the Curve is the *Equiangular Spiral* P *r s*, (by *Annot.* CXL,) 'tis evident the Satellite will with a Force as $PT+\frac{1}{PT^2}$ describe an Oval P *a q* still more curved than the Ellipsis, and therefore will lie within it. Now were the Planet P to set out from any Point P (in which the Radius T P cuts all the three Curves in one common oblique Angle) and to proceed first in the Spiral Path from P towards *s*, the Radius T P would constantly intersect the said Curve in the same Angle as at P. But secondly, if it proceeded in the Elliptic Arch from P towards A, the Angle TPA would continually be altering and approaching nearer to a Right Angle, which it would make when it arrived in the Point A. Lastly, if it set out in the Oval P *aq*, the said Angle TP*q* would alter much faster and approach more quickly to a Right Angle, Pl. LXXI. Fig. 2. Fig. 3.

which happens in the Point a, becauſe of its greater Curvity, or Deviation from the Spiral Ps.

29. THEREFORE by this compound Force the higheſt Apſis A will be removed backwards to a, or the Axis of the Ellipſis AB will recede into the Poſition ab; and this will be the Caſe every Time the Line of the Apſides comes into Square with the Sun.

30. ON the other hand, when the Satellite is in the Syzygical Line CP, it is urged with a Force in the lower Apſis C, which is equal to the Difference between the centripetal Force and that expreſſed by KL; and in the upper Apſis it is equal to the Difference between the central Force and kl; which kl is as PT or AT, as being double thereof; therefore the compound Force about the upper Apſis is as $\frac{1}{PT^2} - PT$, which is a greater Ratio than that of F : $\frac{1}{PT^2}$; or, in Numbers, $25 - 6 : 36 - 5 :: 19 : 31$. But 19 : 31 is a greater Ratio than 25 to 36; whence the Path of the Satellite Paq is not ſo much curved as the Ellipſis PAB, and therefore lies between it and the Spiral Prs; and therefore as the Radius moves from P towards A, it ſooner makes a Right Angle with the Ellipſe at A, than with the Oval Paq, which happens at a. The Line of the Apſides AB therefore goes forwards in this Caſe, and becomes ab.

31. AND becauſe the ablatitious Part kl is twice as great as the addititious Part lm for the upper

upper Apsis, and $KL = 2LM$ for the lower; therefore the Ratio of the compound Forces, which is greater than the Ratio of the Squares of the Distances inversely, will upon the whole prevail, and cause a progressive angular Motion of the Line of the Apsides.

32. HENCE 'tis evident there is a certain Point between the Quadratures and Syzygies, where the Apsides are quiescent; to find which, let P be the Place of the Satellite in the Apsis required; through P draw Pq equal and parallel to NM or TM, and produce it to K, then is $Pq = 3PK$. (*Annot.* LXXXIV. 21.) From q let fall the Perpendicular qr upon TP produced, and the Force Pq is resolved into two others Pr and qr; of which qr, by acting perpendicularly to the Radius, does neither accelerate nor retard the Motion of P towards T; but the other Part Pr, acting directly contrary thereto from P towards r, diminishes the central Force of P towards T. But the Force LM or PT augments it; the Point therefore where $Pr = PT$ is that required. Now because of similar Triangles TPK and qPr, we have $PT : PK :: Pq\ (= 3PK) : Pr = PT$, in the Case proposed. Therefore $3PK^2 = PT^2$; whence $PT : PK :: \sqrt{3} : 1$.

33. HENCE we have this Analogy; As $\sqrt{3} : 1 ::$ Radius PT : Sine PK of the Arch $CP = 35^\circ\ 16'$. The Point, then, where the central Force is neither increased nor diminished by the Force of the Sun, and consequently where the

Apsides

Apſides are at Reſt, is at $35°\ 16'$ on each Side the Quadratures, or at $54°\ 44'$ from the Syzygies on each Side; ſo that the Apſides do in each Revolution of the Planet (*cæteris paribus*) go backwards through $141°\ 4'$, and forwards through $218°\ 56'$.

Pl. LXXI. Fig. 4. 34. SINCE the Progreſs or Regreſs of the Apſides depends on the Decrement of the central Force in a greater or leſſer Ratio than that which is duplicate of the Diſtance in going from the lower Apſis A to the upper one B, and alſo on a ſimilar Increment in returning from B to A, and is therefore greateſt when this Proportion of the Force in the upper Apſis to that in the lower Apſis does moſt of all recede from the inverſe duplicate Ratio of the Diſtances; it is evident that the Apſides in their Syzygies by the ablative Force KL will go forwards more ſwiftly, and more ſlowly in their Quadratures by the addititious Force LM.

35. FOR let the abſolute Force of Attraction in T be $= a$, then becauſe this is every where in the Ratio of $\frac{1}{PT^2}$ at the Body P, the Force by which the Body P is attracted towards T will be as $\frac{a}{PT^2}$. Again, if the Satellite P be within 54 Degrees of the Syzygies A or B, its Force is diſturbed by an extraneous Force (b), which is every where as KL or kl; therefore this perturbating Force is $b \times KL$, or $b \times kl$; ſo that the Force

Force upon P in the Points P and p (within that Limit) is every where in the Ratio of $\frac{a}{PT^2} - b \times KL$ to $\frac{a}{pT^2} - b \times kl$; which Ratio, when P is in the Points A and B, becomes $\frac{a}{AT^2} - b \times AT$ to $\frac{a}{BT^2} - b \times BT$, (because then $KL : kl :: AT : pT$, and $TP = AT$, and $pT = BT$, as has been shewn.) Now this reduced to a common Denomination is $\overline{TB^2 \times a} - \overline{b \times AT^3}$ to $\overline{AT^2 \times a} - \overline{b \times TB^3}$.

36. Now this Ratio recedes so much the more from the Ratio of TB^2 to AT^2, or $\frac{1}{AT^2}$ to $\frac{1}{TB^2}$, by how much $\overline{a - b \times AT^3}$ recedes from an Equality with $\overline{a - b \times TB^3}$, or by how much AT is less than TB; that is, when the Line of the Apsides is in the Syzygies, as in *Fig.* 5. In this Position therefore the Apsides will go forwards swifter than in any other. Pl. LXXI.

37. But when (in this Case) the Body P is in the Quadratures C and D, the additional Force LM becoming equal to $CT = TD$, and $CT + TD$ being here less than in any other Situation of the Apsides, (as *Fig.* 4.) from the Nature of an Ellipsis, therefore the Ratio or Quantity of the perturbating Force thence arising will be least of all; and consequently the Apsides will recede slower

ſlower in this than in any other Situation. Hence, upon the Whole, the Exceſs in the Progreſs of the Apſides will in this Situation be greater than in any other.

Pl. LXXI. Fig. 6. 38. If the Line of the Apſides be ſituated in the Quadratures, then for juſt contrary Cauſes the contrary Phænomena will happen; that is, they will recede moſt ſwiftly when the Satellite is in the Quadratures, and proceed moſt ſlowly when it is in the Syzygies. So that in this Caſe the Regreſs might exceed the Progreſs, and the Apſides upon the Whole be moved *in Antecedentia*, were it not that the Force KL, by which they go forwards at A, is near twice as great as LM, by which they go backwards when the Body is at C. See *Art.* 24.

39. The Exceſs of the progreſſive above the regreſſive Motion of the Apſides will be augmented, if the Bodies P and S move both towards the ſame Parts; for then the Apſides will continue a longer Time in and near the Syzygies, than if the Body S were fix'd: And on the contrary, as their Motion would be contrary to that of S when P is in the Quadratures, ſo the Time of the Regreſs will be ſhorter; therefore the Time by which they go forwards will, upon the Whole, be from hence very much increaſed.

40. From what we have demonſtrated (*Art.* 28, 29, 30.) it is evident, that if a Body in deſcending from the upper to the lower Apſis be urged by a centripetal Force, which increaſes more than in a duplicate Ratio of the diminiſh'd

Fig. 7.

Diſtance

Distance from the Center, it will describe a Curve Acb interior to the Ellipse ACB, and consequently more eccentric, inasmuch as the Ratio of TB to TA is increased by being changed to the Ratio of Tb to TA.

41. ON the contrary, if a Body sets out from the lower Apsis B towards the upper A, and is attracted every where with a Force that decreases more than in the duplicate Ratio of the increasing Distance; then, being less attracted than it would be in the Ellipse, it will describe an Orbit exterior to the Ellipse, as Bda; which also is more eccentric than the Ellipse, because Ta to TB is a greater Ratio than TA to TB.

42. BY the same Way of Reasoning we shew, that if in the Descent the Force be increased in a Ratio less than that of the Square of the diminish'd Distance, or in the Ascent it be diminish'd in a Ratio less than the Square of the increased Distance, the Orbit described will be less eccentric than the Ellipse.

43. THEREFORE when the Satellite P is in the Quadratures C and D, if the absolute central Force be to the absolute additional Force as a to n, we shall have the whole Forces at C and D, in the Ratio of $\frac{a}{CT^2} + n \times CT$ to $\frac{a}{TD^2} + n \times TD$; which is as $\overline{TD^2 \times a + n \times \overline{CT^3}}$ to $\overline{TC^2 \times a + n \times \overline{TD^3}}$. But this is a less Ratio than that of TD^2 to TC^2, because CT is greater than TD. Therefore in that Part of the Orbit where

the

the addititious Force L M takes place, the Eccentricity will be diminished, by *Art.* 42.

44. AGAIN; supposing the Satellite in the Syzygies P Q, then the Force in Q will be to that at P as $\overline{TP^2 \times a - b \times TQ^3}$ to $\overline{TQ^2 \times a - b \times TP^3}$, which Ratio is greater than that of TP^2 to TQ^2, because T Q is less than T P; wherefore in and near the Syzygies the *Eccentricity* of the Orbit will be increased, (by *Art.* 40, 41.) The Eccentricity therefore of the Orbit will be twice changed in every Revolution of the Satellite.

Pl.LXXI. Fig. 6. 45. IF the Apsides be situated in the Quadratures, then, because the Ratio of T D to T C is greatest of all, the Eccentricity of the Orbit will be least of all, (*Art.* 43.) Again; when the Apsides are in the Syzygies, the Eccentricity is the greatest of all for the same Reason, *viz.* the greatest Disparity of A T and T B. Hence the Eccentricity of the Orbit is continually increasing as the Apsides pass from the Quadratures to the Syzygies, and *vice versa.* Fig. 5.

46. IT has been already shewn, (*Art.* 17, 18.) that if the Plane of the Satellite's Orbit be inclined to the Plane in which are the Bodies S and T, the Motion of the Satellite in Latitude will in no wise be disturbed by the Part L M of the extraneous Force, but only by the other Part N M, and not by that neither when the Nodes are in the Syzygies; but when they are in the Quadratures this Perturbation is greatest of all.

47. FOR

47. For let P be the Satellite in its Orbit CAD, inclined to the immoveable Plane CFD in any Angle ADF; and let PS expound the Force of the Body S, attracting the Satellite in the Direction PS. From P let fall the Perpendicular Pq to the Plane CFD, and draw the Right Line TqS; then the Force PS is resolvable into the Forces Sq and Pq; of which the former, being in the said Plane CFD, does not disturb the Satellite's Motion in Latitude; but the other Force Pq, being perpendicular to the Plane CFD, is wholly spent in drawing the Satellite from its Orbit CAD towards it, and therefore is proportional to the Force by which the Motion in Latitude is disturbed. But the Force Pq is evidently greatest when STD is a Right Angle, and is nothing when that Angle vanishes. Pl.LXXI. Fig. 8.

48. When the Nodes are in the Quadratures C, D, as the Satellite P passes from the Quadratures to the Syzygies the Inclination of the Orbit is diminish'd, and it is increased in going from the Syzygies to the Quadratures. For let Pq (as before) represent the extraneous Force NM, and the Direction of its Action; we have shewn that the Body P will describe the *Lineola* Ps in a small Particle of Time by the compound Force. which *Lineola* Ps is not in the Plane of the Orbit CPD, but deflects from it towards Pq; so that the Satellite really moves in the Plane TPs, which produced will not meet the Plane ECF in C, but in another Point c, towards the Opposition B.

49. For

Plate LXXII. Fig. 1.

49. For with the Radius TP describe the Circle ECFD in the fix'd Plane passing through T and S, and in the Plane TP*s* the Arch of a Circle P*c* intersecting the other in *c*. Now because the Force NM is very small compared with the central Force, therefore the Angle CP*c*, the Inclination of the Planes CPT and *cs*T, is exceeding small, and the Arch C*c* an infinitesimal Quantity; therefore since PA is a finite Quantity, the Sum of the two Arches PC + P*c* is less than CA + AD, or a Semicircle; and hence in the spherical Triangle CP*c* the external Angle PCF is greater than the internal opposite Angle P*c*C. (See my *Young Trigonometer's Guide*, Vol. II.) That is, the Inclination of the Plane CAD to the Plane CFD is greater than the Inclination of the Plane *c*PT thereto; which was the first Thing to be shewn.

50. In like manner we prove, that as the Body P goes from the Conjunction A to the Quadrature D, the Inclination of the Orbit will be increased; for if, in this Case, through the Points P and *s* we describe an Arch of a Circle in the Plane TP*s*, the said Arch P*sd* will meet the Plane CFD in the Point *d* between F and D; and the exterior Angle P*d*F, the new Inclination of its Orbit, will be greater than the interior opposite Angle PDF, which was the Inclination when the Satellite was at A; which was the second Thing to be shewn.

51. Hence 'tis evident, that in this Situation of the Nodes the Inclination of the Orbit is least

of

of all when the Satellite is in the Syzygies at A, and that it returns to its former Magnitude at the next Node; for the same Things are in the same Manner shewn when the Satellite passes through the remoter Part of its Orbit DBC.

52. HENCE also the Nodes in this Situation have a *retrograde Motion*, or are carried backwards from the Site DC to *dc*, in half a Revolution of the Satellite; and they recede as much more during the other Half-Revolution.

53. IF the Nodes K, L, are in the Octants after the Quadratures C and D, then, (1.) The Inclination of the Plane will be constantly diminish'd in passing from the Node K to the 90th Degree at H or G. (2.) It will be increased during the Motion from that Point to the next Quadrature D or C. (3.) During both these Transits, or the Motion from K to D, or from L to C, the Nodes go backwards. In passing from the Quadratures to the next Node the Inclination of the Orbit is diminish'd, and the Nodes go forwards. The first, second, and third are shewn as before, (*Art.* 49—52.) and the fourth is thus demonstrated. Plate LXXII. Fig. 2.

54. WHEN the Satellite P has pass'd the Quadrature D, the Power NM becomes negative, or acts in a contrary Direction with respect to T, and hence the *Lineola* P*s* described by the compound Motion deflects from the Arch of the Orbit P*p* towards the Side BA; therefore 'tis plain, the Arch of a Circle P*sl*, described with the Radius TP in the Plane TP*s*, will meet the

Circle F L B in a Point *l* between L and B; then, as before, we shew the Angle P*l*F is less than the Angle PLF; and the Node L has, during the Motion through D*l*, gone forwards to *l*. The same Things happen in the Transit from C to K.

55. FROM what we have demonstrated it appears, that during the whole Transit from the Node K to the Node L, the Inclination of the Orbit is more diminish'd than increased, and the same Thing happens on the other Side in going from L to K; therefore the Inclination is always less in the subsequent than in the preceding Node. And this will be the Case, more or less, wherever the Node K is placed between R and S.

Plate LXXII. Fig. 3.

56. WHEN the Nodes are in the other Octants, *viz.* between S and V, and R and W; then, (1.) While the Body P is passing from the Node to the next Quadrature, the Inclination of the Orbit is increased, and the Nodes go forwards. (2.) In passing from the Quadrature to the 90th Degree from the Node H or G, the Inclination is diminish'd, and the Nodes go backwards. (3.) In passing from thence to the next Node, the Inclination is increased, and the Nodes still go backwards. The second and third are demonstrated altogether as before, (*Art.* 49.) and the first is thus shewn.

57. THE Satellite being at P, between K and C, the Direction of the Force NM is that of P*q*; whence the *Lineola* P*s*, described by the compound Force, will deflect from the Arch P*p* of the

the Orbit towards the Side V R; and consequently a circular Arch described on the Center T through the Points *s* and P, in the Plane T *s* P, will meet the primitive Circle V S R in a Point *k* between K and S. Therefore the Angle *s k* F is greater than the Angle P K F; and the Node K is carried *in Consequentia* from K to *k*. The same Thing is shewn for the other Part of the Orbit L G K.

58. HENCE it appears, that since the Nodes go forwards only while the Satellite is between the Node and the next Quadrature, and backwards while it passes from thence to the next Node, *the Nodes in each Revolution go backwards more than forwards*; and therefore, upon the Whole, *the Motion of the Nodes is absolutely backwards*, unless they happen to be in the Syzygies, where they are quiescent; because in that Case the Motion in Latitude is not at all disturb'd by the Force N M, and consequently where the Inclination of the Orbit is the greatest of all. (See *Art.* 46.)

59. ALL the Errors in the Satellite's Motion hitherto described are a little greater in the Conjunction of the Bodies P and S, than in their Opposition; because the generating Forces N M and L M in the former Case are greater than N *m* and *l m* in the latter; as we have shewn in *Annotation* LXXXIV. *Art.* 9, 10, 11, 12. Also it is there shewn, that each of the disturbing Forces N M and L M is inversely as the Cube of the Distance, and therefore become greater when the Distance

ST is leſs, viz. *in Perihelio*, and leſs as the Diſtance increaſes, viz. *in Aphelio*.

60. Of theſe diſturbing Forces, ſince NM is near twice as great as LM, therefore the Diminution of the central Force will exceed its Augmentation doubly; and ſo, upon the Whole, the Satellite P will be leſs attracted towards T by the joint Forces of S and T, than by the Body T alone; conſequently the Satellite deſcribes a larger Orbit, and it's Period of Revolution is greater.

61. In all that has been ſaid, if S be the Sun, T the Earth, and P the Moon, the Theory of the Lunar Motions and Irregularities is contain'd in the foregoing Articles. And as this Theory reſults from the Laws of Attraction, and was firſt excogitated by Sir *Iſaac Newton* by reaſoning *à Priori*; ſo it is found no leſs conſonant to the Experience and Obſervations of Aſtronomers: For from thence it appears, (1.) That the Moon deſcribes not a *Circle* but an *Ellipſe* about the Earth. (2.) That the Eccentricity of the Lunar Orbit is variable, being when leaſt but 43619; when mean, 55237; and when greateſt, 66854 of ſuch Parts as the Radius contains 1000000. (3.) That the Moon's Apogee goes forwards in the Syzygies, and backwards in the Quadratures; but upon the whole it goes forwards, ſo as to compleat a Revolution in about nine Years. (4.) That the Moon's Orbit is inclined to the Plane of the Ecliptic in a certain Angle. (5.) That this Inclination of the Lunar Orbit is variable,

variable, being when leaſt 5°, and when greateſt 5° 18′. (6.) That the Nodes of the Moon go ſometimes backwards, ſometimes forwards, and are in the Syzygies quieſcent. (7.) That the Motion of the Lunar Nodes is upon the whole *backwards*, at the Rate of 20° *per Annum*, and ſo as to compleat a Revolution in about 18 Years and a half. Such is the *ſurprizing Harmony of the* Newtonian *Theory with Aſtronomical Obſervation, even in this moſt difficult Part*, that *Halley* might well ſay,

Intima panduntur victi penetralia Cæli,
Nec latet extremos quæ vis circumrotat Orbes.

And,

Diſcimus hinc tandem quâ cauſâ argentea Phœbe
Paſſibus haud æquis graditur; cur ſubdita nulli
Hactenus Aſtronomo numerorum fræna recuſet;
Cur remeant Nodi, curque Anges progrediuntur.

62. THE ſame Method of Reaſoning, by which we have explain'd the *Tides*, and the *Lunar Theory*, does alſo furniſh us with a *Phyſical Explication* of the *Motion of the Earth's Axis*. For let us conceive numerous Bodies, ſuch as P, to revolve about the Earth T, at an equal Diſtance, in equal Times, and in a Plane inclined to the Plane of the Ecliptic, 'tis evident each one will be affected with the ſame Motions as the Body P. Again, Let us ſuppoſe their Number ſo increaſed as that they become contiguous to each other, and there- Plate LXXIII. Fig. 1.

by form a fluid *Annulus* or Ring of cohering Bodies.

63. THEN ſince each Part of the Ring obſerves the ſame Laws of Motion with P, and becauſe while one Part is ſo attracted as to augment the Inclination of the Plane, the contrary Part is affected by a contrary Force to diminiſh it, therefore the Inclination of the Plane will always be variable, and govern'd by the Difference of the Forces which act upon it in contrary Parts.

64. THEREFORE ſince the greater Force always prevails, the Parts of the Ring which are in the Conjunction and Oppoſition will move more ſwiftly, and accede nearer to the Body T than thoſe in the Quadratures (by *Article* 21, 22.) And the Nodes of this Ring will be quieſcent in the Syzygies, but in any other Situation will go backwards, and ſwifteſt of all in the Quadratures (by *Article* 47—58.) Laſtly, the Inclination of the Ring will be every where analogous to that of the Lunar Orbit; and conſequently its Axis will in each Revolution oſcillate to and from the Axis of the Ecliptic, and be carried backward by the Retroceſſion of the Line of Nodes.

65. IF the Quantity of Matter in the Ring were to be diminiſh'd in any Ratio, the Motions would all remain the ſame, as depending on the attractive Force of the central Body T, which is ſtill the ſame. If the Diameter of the Ring be diminiſh'd, the Motions will be in the ſame Ratio diminiſh'd alſo; for Effects will be as their Cauſes.

Causes. But LM : $\frac{PT}{TS^3}$; and, because TS is constant, LM is as PT. Also MS $= \frac{ST \times LM}{PT}$ $= ST$; therefore MS is as ST, a given Quantity. (See *Annot.* LXXXIV. 9, 11.) Consequently the Motions of the Ring will be every where as the diminish'd Distance P T.

66. SUPPOSE therefore the Diameter of the Ring to be diminish'd so far as to be equal only to the Diameter of the Earth, and the Body T to be spherical, and every way enlarged till it equall'd the Bulk of the Earth; then would the Ring of Bodies coincide with and be contiguous to the Surface of the Earth, and would also cohere to it. And suppose the Plane of the Ring made an Angle with the Plane of the Ecliptic of 23 Degrees and a half, then would all the Motions of the Ring continue, only in a lesser Degree; and would be communicated to the Earth; because it adheres firmly thereto; for the Earth equilibrated in Æther will yield to any Motion impress'd upon it from without. But the Motions of the Ring being now communicated to the Body of the Earth, will be farther diminish'd in Proportion as the Mass of Matter to be moved is augmented. Plate LXXIII. Fig. 2.

67. Now this Circle or Ring of Bodies encompassing the Earth by Supposition is actually the true State of the Earth; for we have shewn its Diameter through the Equator ÆQ exceeds

the Length of the Axis ND, (*Annot.* CXLVIII.) and therefore it is surrounded by a Zone of Matter upon the Equator analogous to this feign'd Ring of Bodies, and which must of course produce the same Effects.

68. HENCE in the Equinoxes, that is, when the Earth's Nodes are in the Syzygies, or when the Line of the Nodes (*viz.* the Equinoxes) pass through the Earth and Sun, the Inclination of the Equator and Ecliptic, that is, the Angle ÆTE or FTH, is greatest of all; and from this Time it grows less till the Sun arrives at the 90th Degree, (or *Solstice*) when the Line of Nodes are in the Quadratures, and then it is least of all.

69. THEREFORE twice in a Year the Inclination of the Ecliptic and Equator is diminish'd, and twice again restored; and the Nodes (or Equinoxes) constantly go backwards, and carry the Axis of the Earth TH with a retrograde Motion about the Axis of the Ecliptic TF, tracing out the Circle, or rather vermicular Curve HIGR in the Heavens among the Fix'd Stars.

70. AGAIN; the Plane of the Equator is inclined to the Plane of the Moon's Orbit, for the latter makes an Angle of but about 5 Degrees with the Plane of the Ecliptic; and therefore the Moon (though a less Body than the Sun, yet being nearer) produces a greater Effect than the Sun on the Equatorial Ring or Zone of Matter, and so augments all the aforesaid Motions of the Earth's

Earth's Plane and Axis. Sir *Isaac Newton* has shewn (*Prop.* XXXIX. *Lib.* III.) that the Part of the annual *Recession of the Equinoxes*, which is owing to the Sun, is $9'' \; 7''' \; 20''''$, and that which is owing to the Moon is $40'' \; 52''' \; 52''''$; therefore by the joint Influence of the Sun and Moon the Equinoxes recede yearly about $50'' \; 00''' \; 12''''$; which is likewise verified by the Observations of Astronomers for 2000 Years past. See *Annotation* CXLI.

71. I SHALL now explain the Method used by Philosophers for computing the Quantities of Matter, Densities, Weight of Bodies, &c. in the *Sun*, the *Earth*, *Jupiter*, and *Saturn*, by means of Satellites revolving about them. In order to this let Q, q, express the Quantities of Matter in the two Bodies A, B; also let G, g, be the respective Forces of Gravity at the equal Distances AC and BD. Let T, $\mathcal{T}$, be the Periodical Times of Bodies revolving about A and B at those equal Distances; and let T, t, be the Periodical Times of Bodies revolving at the unequal Distances AC and BE, which call D and d. Plate LXXIII. Fig. 3.

72. THEN in the given Distances AC, BD, we have $Q : q :: G : g$ (*Art.* 7.) But $G : g :: \frac{1}{T^2} : \frac{1}{\mathcal{T}^2}$ (by *Annot.* XXXIV. 6.) Whence $Q : q :: \frac{1}{T^2} : \frac{1}{\mathcal{T}^2}$; and multiplying the latter Ratio by D^3, we have $Q : q :: \frac{D^3}{T^2} : \frac{D^3}{\mathcal{T}^2}$. But because $\mathcal{T}^2$

: t^2

$: t^2 :: D^3 : d^3$, (ibid. *Art.* 11.) therefore $\frac{D^3}{T^2} = \frac{d^3}{t^2}$; consequently, $Q : q :: \frac{D^3}{T^2} : \frac{d^3}{t^2}$. That is, *The Quantities of Matter in any two Bodies are in the compound Ratio of the Cubes of the Distances directly, and Squares of the Periodical Times inversely, of Bodies revolving about them.*

73. In this Calculation the Bodies A and B are supposed at Rest. We consider the Sun at Rest with respect to *Venus*, and *Jupiter* and *Saturn* in respect of their Secondaries; and we have reduced the Distance of the Moon to 60 Semidiameters, at which she would revolve about the Earth at Rest. Now let the Distance of the Earth from the Sun be put — — 1000
then *Venus* revolves about the Sun at Distance 723
the 4th Satellite of *Jupiter* at the Distance 12,4775
the 4th Satellite of *Saturn* at the Distance 8,5107
the Moon at the Distance 3,054

The Periodical Time	of *Venus* is	19414160″
	of the *Jovian* Sat.	1441929″
	of the *Saturnian* Sat.	1377674″
	of the Moon,	2360580″

74. Now suppose the Quantity of Matter in the Sun be 10000, then for that in *Jupiter* say,

$$\text{As } \frac{\overline{723}^3}{\overline{19414160''}^2} : \frac{\overline{12,4775}^3}{\overline{1441929''}^2} :: 10000 : 9,305,$$

(by *Art.* 71.) the Density of *Jupiter* compared with that of the Sun. By the same Analogy the rest

rest are found, and in each they are as follow.

In the *Sun*,	*Jupiter*,	*Saturn*,	*Earth*,	*Moon*.
10000.	9,305.	3,250.	0,0512.	0,0013.

75. Now if these Quantities of Matter are divided by the Squares of the Diameters of these Bodies, the Quotients will be as the *Weight of Bodies on their Superficies*, (by *Annot*. XIX. 3.) The Diameters of the Sun and Planets see in *Annot*. CXXXV. Then these Gravities will be as follow.

In the *Sun*,	*Jupiter*,	*Saturn*,	*Earth*,	*Moon*.
10000.	936.	519.	431.	146.

76. IN homogeneous, unequal, spherical Bodies, the *Gravities on their Surfaces are as the Diameters*, if the Densities are equal (*Annotation* XIX. 3.) But if the Bodies be equal, *the Gravities will be as the Densities*, because they will be as the Quantities of Matter, which in this Case are as the Densities (*Annot*. XVII.) Therefore in Bodies of unequal Bulks and Densities, *the Gravities will be in a compound Ratio of the Diameters and Densities*. Consequently, *the Densities will be as the Gravities divided by the Diameters*; and therefore in the several Bodies as follows.

In the *Sun*,	*Jupiter*,	*Saturn*,	*Earth*,	*Moon*.
10000.	9385.	6567.	39539.	48911.

77. As it is not likely that these Bodies are homogeneal, the Densities here determin'd are not to be supposed the *true*, but rather *mean Densities*, or such as the Bodies would have if they were

were homogeneal, and of the same Mass of Matter and Magnitude.

78. LET F, f, be the Forces of the Sun and Moon to move the Sea; D, d, their Distances from the Earth; then $F : f :: \frac{Q}{D^3} : \frac{q}{d^3}$. (See *p.* 46. Vol. I. and *Annot.* LXXXIV. 9, 11.) Let B, b, be the Bulks; R, r, the Diameters; and N, n, the Densities of the Sun and Moon; then will $Q : q :: BN : bn :: R^3N : r^3n$; (*Annot.* XVII. and XIX.) wherefore $F : f :: \frac{R^3N}{D^3} : \frac{r^3n}{d^3}$. Lastly, let A, a, be the apparent Diameters of the Sun and Moon; then will $A : a :: \frac{R}{D} : \frac{r}{d}$; because any Body appears larger the bigger it is, and less in proportion to the increasing Distance; therefore $A^3 : a^3 :: \frac{R^3}{D^3} : \frac{r^3}{d^3}$. Hence $F : f :: A^3N : a^3n$. Consequently, $N : n :: Fa^3 : fA^3 :: \frac{F}{A^3} : \frac{f}{a^3}$.

79. BUT (according to Sir *Isaac Newton*) $F : f :: 1 : 4{,}4815$. (See *Annotat.* LXXXIV. 28.) And $A : a :: 32' \, 12'' : 31' \, 16\frac{1}{2}''$ (at a Mean, by Observation). That is, $A : a :: 3864 : 3753$. Therefore $N : n :: \frac{1}{3864^3} : \frac{4{,}4815}{3753^3} :: 10000 : 48911$, the Ratio of the Density of the Sun and Moon, as above shewn, *Art.* 76.

80. THE

80. THE Quantities of Matter being $Q : q :: R^3 N . r^3 n$, (*Art.* 78.) and with respect to the Earth and Moon, $N : n :: 39539 : 48911$; and $R : r :: 109 : 30$, (*Annot.* CXXXVI. 4.) therefore $Q : q :: 109^3 \times 39539 : 30^3 \times 48911 :: 39,31 : 1 :: 0,0512 : 0,0013$, as determined in *Art.* 74.

81. THE Weight of Bodies on the Surface of the Earth and Moon are *in the compound Ratio of the Diameters and Densities*, (*Art.* 76.) that is, in the Ratio of 109×39539 to 30×48911, or as 431 to 146, (as *per Art.* 75.) or as 3 to 1 nearly.

82. HAVING the Quantities of Matter in the Earth and Moon, the Distance of the common Center of Gravity is determined: For the Distance of the Moon from the Earth's Center is to this Distance as 40,31 to 1; which Ratio is more accurate than that of 41 to 1, made use of in *Annot.* XXXVI. *Art.* 2.

83. THE Theory we have here been explaining is applicable to any System of three or more Bodies, as well as to the *Sun*, the *Earth*, and *Moon*. Thus the perturbating Forces and Irregularities of Motion in the System of the *Sun*, *Jupiter*, and any of his Moons, may be estimated in nearly the same Manner, (*mutatis mutandis*) as also those of the *Sun*, *Saturn*, and his *Satellites*; and lastly, between the *Sun* and *primary Planets*, by putting the Case more generally, (as Sir *Isaac* does) in supposing both S and P to revolve about the fix'd central

tral Body T, which we may suppose to be the Sun, and S and P any two of the Planets at pleasure. Therefore, to use the Author's own Words (in another Case) for a Conclusion: *Usus igitur hujus Theoriæ latissimè patet, & latè patendo Veritatem ejus evincit.*

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